Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 14, Issue 1 (June 2019), pp. 359 – 369

Associated Matrix Polynomials with the Second Kind Chebyshev Matrix Polynomials

[1]M.S. Metwally, [2]M.T. Mohamed and [3,4]Ayman Shehata

[1]Department of Mathematics
Faculty of Science (Suez)
Suez Canal University, Egypt
met641958@yahoo.com

[2]Department of Mathematics
Faculty of Science (New Valley)
Assiut University
New Valley, EL-Kharga 72111, Egypt
tawfeek200944@yahoo.com

[3]Department of Mathematics
Faculty of Science
Assiut University, Assiut 71516, Egypt

[4]Department of Mathematics
College of Science and Arts
Unaizah Qassim University, Qassim, Saudi Arabia
drshehata2006@yahoo.com

Received: January 15, 2018; Accepted: September 7, 2018

Abstract

This paper deals with the study of the associated Chebyshev matrix polynomials. Associated matrix polynomials with the Chebyshev matrix polynomials are defined here. Some properties of the associated Chebyshev matrix polynomials are obtained here. Further, we prove that the associated Chebyshev matrix polynomials satisfy a matrix differential equation of the second order.

Keywords: Matrix functional calculus; Chebyshev matrix polynomials; Matrix recurrence; Relation; Chebyshev matrix differential equation

MSC 2010 No.: 33C01, 33C02, 33C45, 33E20

1. Introduction

Orthogonal matrix polynomials is an emergent field whose development is reaching an important result form both the theoretical and practical points of view. Some recent result is this field which can be found in Dattoli (2004). The important connection between orthogonal matrix polynomials and matrix differential equations appears in Defez and Jódar (1998), Defez and Jódar (2002), Jódar and Company (1996), Jódar et al. (1994), Jódar et al. (1996), and Metwally et al. (2015). Chebyshev matrix polynomials is closely related to Hermite, Humbert and Laguerre matrix polynomials, see Defez and Jódar (1998), Metwally et al. (2008), Kargin and Kurt (2013), Kargin and Kurt (2014a), Kargin and Kurt (2014b), Kargin and Kurt (2015), Kargin and Kurt (2017), Shehata (2016), Shehata (2017a), Shehata (2017b), Shehata (2017c), and Shehata and Çekim (2016). Recently, Chebyshev matrix polynomials have been introduced and studied in Defez and Jódar (2002), Metwally et al. (2015), Altin and Çekim (2012), Batahan (2006), Kargin and Kurt (2015), and Shehata (2018) for matrices in $\mathbb{C}^{N\times N}$ whose eigenvalues are all situated in the open right half-plane. The purpose of this paper is to present, investigate and define a matrix polynomials with the associated Chebyshev matrix polynomials and derive the associated Chebyshev matrix differential equation of the second order. Some properties of the associated Chebyshev matrix polynomials are obtained here.

If D_0 is the complex plane cut along the negative real axis and $\log(z)$ denotes the principal logarithm of z, then $z^{\frac{1}{2}}$ represents $\exp(\frac{1}{2}\log(z))$. If A is a matrix of $\mathbb{C}^{N\times N}$, it's two-norm denoted $||A||_2$ is defined by

$$||A||_2 = \frac{||Ax||_2}{||x||_2},$$

where for a vector y in $\mathbb{C}^N$, $||y||_2$ denotes the usual Euclidean norm of y, $||y||_2 = (y^T y)^{\frac{1}{2}}$. The set of all the eigenvalues of A is denoted by $\sigma(A)$. If $f(z)$ and $g(z)$ are holomorphic functions of the complex variable z, which are defined in an open set Ω of the complex plane, that is if A a matrix in $\mathbb{C}^{N\times N}$ such that $\sigma(A)\subset\Omega$, then from the properties of the matrix functional calculus, it follows that

$$f(A)g(A) = g(A)f(A). \tag{1}$$

If A is a matrix with $\sigma(A)\subset D_0$, then $A^{\frac{1}{2}} = \sqrt{A} = \exp(\frac{1}{2}\log(A))$ denotes the image by $z^{\frac{1}{2}} = \sqrt{z} = \exp(\frac{1}{2}\log(z))$ of the matrix functional calculus acting on the matrix A. Let A be a positive stable matrix in $\mathbb{C}^{N\times N}$ satisfying the condition (see Jódar et al. (1996))

$$Re(z) > 0, \quad \text{for all} \quad z\in\sigma(A). \tag{2}$$

Also, we recall that if $A(k,n)$ and $B(k,n)$ are matrices on $\mathbb{C}^{N\times N}$ for $n\geq 0$, $k\geq 0$, the relations are satisfied (see Defez and Jódar (1998))

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A(k,n) = \sum_{n=0}^{\infty}\sum_{k=0}^{[\frac{1}{2}n]} A(k,n-2k), \tag{3}$$

and

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} B(k,n) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} B(k,n-k). \tag{4}$$

Similarly, we can write

$$\sum_{n=0}^{\infty}\sum_{k=0}^{[\frac{1}{2}n]} A(k,n) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A(k,n+2k), \tag{5}$$

and

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n} B(k,n) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} B(k,n+k), \tag{6}$$

where $[x]$ denotes the greatest integer in x.

We recall the following specialized version of the definitions. Then, the Chebyshev matrix polynomials $U_n(x,A)$ is defined by (see Metwally et al. (2015))

$$U_n(x,A) = \sum_{k=0}^{[\frac{1}{2}n]} \frac{(-1)^k (n-k)!(x\sqrt{2A})^{n-2k}}{k!(n-2k)!}; \quad n \geq 0, \tag{7}$$

and satisfies the three terms recurrence matrix relation in the form

$$U_n(x,A) = x\sqrt{2A}U_{n-1}(x,A) - U_{n-2}(x,A), \tag{8}$$

and satisfies the differential recurrence matrix relation

$$(n+1)\sqrt{2A}U_n(x,A) = DU_{n+1}(x,A) - DU_{n-1}(x,A). \tag{9}$$

Also from Metwally et al. (2015), we have

$$(I - xt\sqrt{2A} + t^2 I)^{-1} = \sum_{n=0}^{\infty} U_n(x,A)t^n, \quad |t| < 1, \quad |x| \leq 1, \tag{10}$$

where I is the unite matrix in $\mathbb{C}^{N\times N}$, $I - xt\sqrt{2A} + t^2 I$ is an invertible matrix and $xt\sqrt{2A} - t^2 I$ is an invertible matrix.

The Chebyshev matrix polynomials $U_n(x,A)$ is a solution of the second order matrix differential equation in the form (see Metwally et al. (2015))

$$\begin{aligned}&(4I - (x\sqrt{2A})^2)D^2U_n(x,A) - 3x(\sqrt{2A})^2 DU_n(x,A)\\ &+ n(n+2)(\sqrt{2A})^2 U_n(x,A) = \mathbf{0},\end{aligned} \tag{11}$$

where $\mathbf{0}$ is the null matrix in $\mathbb{C}^{N\times N}$.

In the next section, we prove that from these matrix polynomials satisfy a second order matrix differential equation for the associated Chebyshev matrix polynomials and definition.

2. Definition of associated Chebyshev matrix polynomials $U_{n,m}(x,A)$ and their properties

Suppose that Z is a solution of Chebyshev matrix polynomials differential equation, i.e.

$$(4I - (x\sqrt{2A})^2)\frac{d^2Z}{dx^2} - 3x(\sqrt{2A})^2\frac{dZ}{dx} + n(n+2)(\sqrt{2A})^2 Z = \mathbf{0}. \tag{12}$$

By differentiating (12) m times with respect to x , we get

$$\tfrac{d^m}{dx^m}\left[(4I-(x\sqrt{2A})^2)\tfrac{d^2Z}{dx^2}\right]-3(\sqrt{2A})^2\tfrac{d^m}{dx^m}\left[x\tfrac{dZ}{dx}\right]+n(n+2)(\sqrt{2A})^2\frac{d^mZ}{dx^m}=\mathbf{0},$$

where m is an non-negative integer.

By Leibniz's rule for the m^{th} derivative of a product and the property (1) of the matrix functional calculus, and taking into account that the higher derivatives of $(4I-(x\sqrt{2A})^2)$ and x vanish, one gets

$$\begin{aligned}&(4I-(x\sqrt{2A})^2)\frac{d^{m+2}Z}{dx^{m+2}}-2mx(\sqrt{2A})^2\frac{d^{m+1}Z}{dx^{m+1}}-m(m-1)(\sqrt{2A})^2\frac{d^mZ}{dx^m}\\&-3(\sqrt{2A})^2x\frac{d^{m+1}Z}{dx^{m+1}}-3m(\sqrt{2A})^2\frac{d^mZ}{dx^m}+n(n+2)(\sqrt{2A})^2\frac{d^mZ}{dx^m}=\mathbf{0}.\end{aligned}$$

Collecting terms in $\frac{d^{m+2}Z}{dx^{m+2}}$, $\frac{d^{m+1}Z}{dx^{m+1}}$ and $\frac{d^mZ}{dx^m}$, we get becomes

$$\begin{aligned}&(4I-(x\sqrt{2A})^2)\frac{d^{m+2}Z}{dx^{m+2}}-(2m+3)(\sqrt{2A})^2x\frac{d^{m+1}Z}{dx^{m+1}}\\&+(n^2-m^2+2(n-m))(\sqrt{2A})^2\frac{d^mZ}{dx^m}=\mathbf{0}.\end{aligned}\tag{13}$$

By taking $Z_1=\frac{d^mZ}{dx^m}$ in (13), we obtain

$$\begin{aligned}&(4I-(x\sqrt{2A})^2)\frac{d^2Z_1}{dx^2}-(2m+3)(\sqrt{2A})^2x\frac{dZ_1}{dx}\\&+(n^2-m^2+2(n-m))(\sqrt{2A})^2Z_1=\mathbf{0}.\end{aligned}\tag{14}$$

If we write

$$Z_2=\sqrt{(4I-(x\sqrt{2A})^2)^m}Z_1=\sqrt{(4I-(x\sqrt{2A})^2)^m}\frac{d^mZ}{dx^m},\ \left\|\frac{Ax^2}{2}\right\|<1,$$

where $4I-(x\sqrt{2A})^2$ is an invertible matrix, then (14) can be written in the form

$$\begin{aligned}&(4I-(x\sqrt{2A})^2)\frac{d^2}{dx^2}\left[(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}}Z_2\right]\\&-x(2m+3)(\sqrt{2A})^2\frac{d}{dx}\left[(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}}Z_2\right]\\&+(n^2-m^2+2(n-m))(\sqrt{2A})^2\left[(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}}Z_2\right]=\mathbf{0}.\end{aligned}\tag{15}$$

Since

$$\begin{aligned}\frac{d}{dx}\left[(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}}Z_2\right]&=(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}}\frac{dZ_2}{dx}\\&\quad+mx(\sqrt{2A})^2(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}-1}Z_2,\end{aligned}$$

and

$$\frac{d^2}{dx^2}\left[(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}}Z_2\right] = (4I-(x\sqrt{2A})^2)^{-\frac{m}{2}}\frac{d^2Z_2}{dx^2} + 2mx(\sqrt{2A})^2(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}-1} \frac{dZ_2}{dx} + m(\sqrt{2A})^2(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}-1}\frac{dZ_2}{dx} + m(m+2)x^2(\sqrt{2A})^4Z_2(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}-2},$$

then equation (15) can be written in the form

$$\begin{aligned}&(4I-(x\sqrt{2A})^2)\Bigg[(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}}\frac{d^2Z_2}{dx^2}\\&+2mx(\sqrt{2A})^2(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}-1}\frac{dZ_2}{dx}\\&+m(\sqrt{2A})^2(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}-1}Z_2\\&+m(m+2)x^2(\sqrt{2A})^4Z_2(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}-2}\Bigg]\\&-(2m+3)x(\sqrt{2A})^2\Bigg[(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}}\frac{dZ_2}{dx}\\&+mx(\sqrt{2A})^2(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}-1}Z_2\Bigg]\\&+(n^2-m^2+2(n-m))(\sqrt{2A})^2\left[(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}}Z_2\right]=\mathbf{0}.\end{aligned}$$

Canceling a common factor of $(4I-(x\sqrt{2A})^2)^{-\frac{m}{2}}$ and collecting terms, we find that

$$\begin{aligned}&(4I-(x\sqrt{2A})^2)\frac{d^2Z_2}{dx^2}+2mx(\sqrt{2A})^2\frac{dZ_2}{dx}\\&+m(\sqrt{2A})^2Z_2+m(m+2)x^2(\sqrt{2A})^4Z_2(4I-(x\sqrt{2A})^2)^{-1}\\&-(2m+3)x(\sqrt{2A})^2\left[\frac{dZ_2}{dx}+mx(\sqrt{2A})^2(4I-(x\sqrt{2A})^2)^{-1}Z_2\right]\\&+(n^2-m^2+2(n-m))(\sqrt{2A})^2Z_2=\mathbf{0},\end{aligned}$$

which can be written in the form

$$\begin{aligned}&(4I-(x\sqrt{2A})^2)\frac{d^2Z_2}{dx^2}-3x(\sqrt{2A})^2\frac{dZ_2}{dx}+\Bigg[(n^2-m^2\\&+2n-m)I-m(m+1)(x\sqrt{2A})^2\left(4I-(x\sqrt{2A})^2\right)^{-1}\Bigg](\sqrt{2A})^2Z_2=\mathbf{0}.\end{aligned}$$

Hence, we can state the following theorem.

Theorem 2.1.

If Z is a solution of associated Chebyshev matrix differential equation

$$(4I-(x\sqrt{2A})^2)\frac{dZ}{dx}-3x(\sqrt{2A})^2\frac{dZ}{dx}+n(n+2)(\sqrt{2A})^2Z=\mathbf{0},$$

then $Y = (4I - (x\sqrt{2A})^2)^{\frac{m}{2}} \frac{d^m Z}{dx^m}$ is a solution of the differential matrix equation

$$(4I - (x\sqrt{2A})^2)\frac{d^2Y}{dx^2} - 3x(\sqrt{2A})^2\frac{dY}{dx} + \Bigg[(n^2 - m^2 + 2n - m)I \\ - m(m+1)(x\sqrt{2A})^2\Big(4I - (x\sqrt{2A})^2\Big)^{-1}\Bigg](\sqrt{2A})^2 Y = \mathbf{0}, \tag{16}$$

where A is a positive stable matrix in $\mathbb{C}^{N\times N}$ satisfying the condition (2), and $4I - (x\sqrt{2A})^2$ is an invertible matrix for $\left\|\frac{Ax^2}{2}\right\| < 1$.

Equation (16) will be called associated Chebyshev matrix differential equation of the second order.

Since Chebyshev matrix polynomials $U_n(x, A)$ satisfy Chebyshev matrix differential equations, then we can derive the definition of the associated Chebyshev matrix polynomials $U_{n,m}(x, A)$ as follows

$$U_{n,m}(x, A) = \sqrt{\Big(4I - (x\sqrt{2A})^2\Big)^m} D^m U_n(x, A), \left\|\frac{Ax^2}{2}\right\| < 1, \tag{17}$$

where $4I - (x\sqrt{2A})^2$ is an invertible matrix.

Remark 2.1.

It is clear that $U_{n,0}(x, A) = U_n(x, A)$.

Remark 2.2.

It has seen that Chebyshev matrix polynomials $U_n(x, A)$ is a matrix polynomials of degree n. So that $U_{n,m}(x, A) = \mathbf{0}$, $m > n$.

3. Recurrence matrix relations

Here, some recurrence matrix relations are carried out on the associated Chebyshev matrix polynomials. Let us take $Z_1 = D^m U_n(x, A)$ in equation (14). Hence

$$(4I - (x\sqrt{2A})^2)\frac{d^2}{dx^2}D^m U_n(x, A) - (2m+3)(\sqrt{2A})^2 x\frac{d}{dx}D^m U_n(x, A) \\ + (n^2 - m^2 + 2(n-m))(\sqrt{2A})^2 D^m U_n(x, A) = \mathbf{0},$$

which becomes

$$(4I - (x\sqrt{2A})^2)D^{m+2}U_n(x, A) - (2m+3)(\sqrt{2A})^2 x D^{m+1}U_n(x, A) \\ + (n^2 - m^2 + 2(n-m))(\sqrt{2A})^2 D^m U_n(x, A) = \mathbf{0}. \tag{18}$$

Multiplying (18) by $\left(4I-(x\sqrt{2A})^2\right)^{\frac{m}{2}}$ gives

$$\begin{aligned}&\left(4I-(x\sqrt{2A})^2\right)^{\frac{m}{2}+1}D^{m+2}U_n(x,A)\\&-(2m+3)(\sqrt{2A})^2x\left(4-(x\sqrt{2A})^2\right)^{\frac{m}{2}}D^{m+1}U_n(x,A)\\&+(n^2-m^2+2(n-m))(\sqrt{2A})^2\left(4I-(x\sqrt{2A})^2\right)^{\frac{m}{2}}D^mU_n(x,A)=\mathbf{0}.\end{aligned}$$

By definition (17), we obtain

$$\begin{aligned}&U_{n,m+2}(x,A)-(2m+3)x(\sqrt{2A})^2\left(\sqrt{4I-(x\sqrt{2A})^2}\right)^{-1}U_{n,m+1}(x,A)\\&+(n^2-m^2+2(n-m))(\sqrt{2A})^2U_{n,m}(x,A)=\mathbf{0},\end{aligned}$$

which, by replacing m by $m-1$, yields

$$\begin{aligned}&U_{n,m+1}(x,A)-(2m+1)x(\sqrt{2A})^2\left(\sqrt{4I-(x\sqrt{2A})^2}\right)^{-1}U_{n,m}(x,A)\\&+(n^2-m^2+2n+1)(\sqrt{2A})^2U_{n,m-1}(x,A)=\mathbf{0}.\end{aligned}\tag{19}$$

The recurrence matrix relation (19) is the relationship between three associated Chebyshev matrix polynomials with equal n values and consecutive m values.

In the pure recurrence matrix relation of Chebyshev matrix polynomials (8), substituting $n+1$ for n gives

$$U_{n+1}(x,A)=x\sqrt{2A}U_n(x,A)-U_{n-1}(x,A).\tag{20}$$

By differentiating (20) m times with respect to x and using Leibniz's rule for the m^{th} derivative of the second term, we find

$$\begin{aligned}D^mU_{n+1}(x,A)=&x\sqrt{2A}D^mU_n(x,A)+m\sqrt{2A}D^{m-1}U_n(x,A)\\&-D^mU_{n-1}(x,A).\end{aligned}\tag{21}$$

Also, by differentiating the differential recurrence relation (9) $m-1$ times with respect to x, we see that

$$(n+1)\sqrt{2A}D^{m-1}U_n(x,A)=D^mU_{n+1}(x,A)-D^mU_{n-1}(x,A).\tag{22}$$

Substituting $D^{m-1}U_n(x,A)$ from (22) in (21), it follows that

$$\begin{aligned}(n+1)D^mU_{n+1}(x,A)=&(n+1)x\sqrt{2A}D^mU_n(x,A)+mD^mU_{n+1}(x,A)\\&-mD^mU_{n-1}(x,A)-(n+1)D^mU_{n-1}(x,A),\end{aligned}$$

i.e.,

$$\begin{aligned}(n-m+1)D^mU_{n+1}(x,A)=&(n+1)x\sqrt{2A}D^mU_n(x,A)\\&-(n+m+1)D^mU_{n-1}(x,A).\end{aligned}$$

Multiplying by $\left(4I-(x\sqrt{2A})^2\right)^{\frac{m}{2}}$ and using definition (17), we get

$$\begin{aligned}(n-m+1)U_{n+1,m}(x,A) =&(n+1)x\sqrt{2A}U_{n,m}(x,A)\\&-(n+m+1)U_{n-1,m}(x,A).\end{aligned}$$

Rearrangement of terms gives

$$\begin{aligned}(n-m+1)U_{n+1,m}(x,A)&-(n+1)x\sqrt{2A}U_{n,m}(x,A)\\&+(n+m+1)U_{n-1,m}(x,A)=\mathbf{0}.\end{aligned} \tag{23}$$

The recurrence matrix relation (23) is the relationship linking three associated Chebyshev matrix functions with the same m values and consecutive n values.

Now, multiplying equation (22) by $\left(4I-(x\sqrt{2A})^2\right)^{\frac{m}{2}}$ gives

$$\begin{aligned}&(n+1)\sqrt{2A}\left(4I-(x\sqrt{2A})^2\right)^{\frac{m}{2}}D^{m-1}U_n(x,A)\\&=\left(4I-(x\sqrt{2A})^2\right)^{\frac{m}{2}}D^{m}U_{n+1}(x,A)\\&-\left(4I-(x\sqrt{2A})^2\right)^{\frac{m}{2}}D^{m}U_{n-1}(x,A),\end{aligned}$$

which, by using definition (17), becomes

$$(n+1)\sqrt{2A}\sqrt{4I-(x\sqrt{2A})^2}U_{n,m-1}(x,A)=U_{n+1,m}(x,A)-U_{n-1,m}(x,A). \tag{24}$$

Substituting $(m+1)$ for m gives

$$(n+1)\sqrt{2A}\sqrt{4I-(x\sqrt{2A})^2}U_{n,m}(x,A)=U_{n+1,m+1}(x,A)-U_{n-1,m+1}(x,A), \tag{25}$$

which represents a recurrence matrix relation of associated Chebyshev matrix polynomials.

Finally, in (19), replacing $xU_{n,m}(x,A)$ from (23) by

$$\begin{aligned}xU_{n,m}(x,A)=&\frac{(\sqrt{2A})^{-1}}{n+1}\Big[(n-m+1)U_{n+1,m}(x,A)\\&+(n+m+1)U_{n-1,m}(x,A)\Big]=\frac{(\sqrt{2A})^{-2}\sqrt{4I-(x\sqrt{2A})^2}}{(2m+1)}\Big[U_{n,m+1}(x,A)\\&+(n^2-m^2+2n+1)(\sqrt{2A})^2U_{n,m-1}(x,A)\Big],\end{aligned}$$

and $U_{n,m-1}(x,A)$ from (24) by

$$U_{n,m-1}(x,A)=\frac{\left(\sqrt{2A}\sqrt{4I-(x\sqrt{2A})^2}\right)^{-1}}{n+1}\Big[U_{n+1,m}(x,A)-U_{n-1,m}(x,A)\Big],$$

which gives

$$U_{n,m+1}(x,A) = \frac{(2m+1)\sqrt{2A}\left(\sqrt{4I-(x\sqrt{2A})^2}\right)^{-1}}{n+1}$$
$$\times \Big[(n-m+1)U_{n+1,m}(x,A) + (n+m+1)U_{n-1,m}(x,A)\Big]$$
$$-\frac{\sqrt{2A}(n^2-m^2+2n+1)\left(\sqrt{4I-(x\sqrt{2A})^2}\right)^{-1}}{n+1}$$
$$\times \Big[U_{n+1,m}(x,A) - U_{n-1,m}(x,A)\Big].$$

Therefore, by rearrangement terms we obtain

$$\begin{aligned}&(n+1)\sqrt{4I-(x\sqrt{2A})^2}U_{n,m+1}(x,A)\\&= \Big[m-n+2mn-m^2-n^2\Big]\sqrt{2A}U_{n+1,m}(x,A)\\&-\Big[3m^2-n^2+3m-n+2mn\Big]\sqrt{2A}U_{n-1,m}(x,A).\end{aligned} \tag{26}$$

Summary of these results is given in the following theorem.

Theorem 3.1.

Let A be a positive stable matrix in $\mathbb{C}^{N\times N}$ satisfying the condition (2), and let $4I-(x\sqrt{2A})^2$ be an invertible matrix with $\left\|\frac{Ax^2}{2}\right\| < 1$, then the associated Chebyshev matrix polynomials satisfy the following matrix recurrence relations (19), (20), (25) and (26).

4. Conclusion

In this paper, several new associated matrix polynomials are introduced using Chebyshev matrix polynomials allowing the derivation of a wealth of relations involving these matrix polynomials. In a forthcoming investigation, we will extend this approach to derive achieved results involving other associated Chebyshev matrix polynomials and also to introduce new families of Chebyshev matrix polynomials which will be a problem for a further research.

Acknowledgement:

(a) The authors would like to thank the reviewers for their valuable comments and suggestions, which improve the readability of the paper.

(b) The third Author (Ayman Shehata) expresses his sincere appreciation to Dr. Mohammed Eltayeb Elffaki Elasmaa (Assistant Prof. Department of English language, College of Science and Arts,

Unaizah, Qassim University, Qassim, Kingdom of Saudi Arabia) for his kinds interests, encouragements, help and correcting language errors for this paper.

REFERENCES

Altin, A. and Çekim, B. (2012). Generating matrix functions for Chebyshev matrix polynomials of the second kind, Hacettepe Journal of Mathematics and Statistics, Vol. 41, No. 1, pp. 25–32.

Batahan, R.S. (2006). A new extension of Hermite matrix polynomials and its applications, Linear Algebra and its Applications, Vol. 419, pp. 82–92.

Dattoli, G. (2004). Integral transforms and Chebyshev-like polynomials, Applied Mathematics and Computation, Vol. 148, pp. 225–234.

Defez, E. and Jódar, L. (1998). Some applications of the Hermite matrix polynomials series expansions, Journal of Computational and Applied Mathematics, Vol. 99, pp. 105–117.

Defez, E. and Jódar, L. (2002). Chebyshev matrix polynomials and second order matrix differential equations, Utilitas Mathematica, Vol. 61, pp. 107–123.

Jódar, L. and Company, R. (1996). Hermite matrix polynomials and second order matrix differential equations, Approximation Theory and its Applications, Vol. 12, pp. 20–30.

Jódar, L. Company, R. and Navarro, E. (1994). Laguerre matrix polynomials and systems of second order differential equations, Applied Numerical Mathematics, Vol. 15, pp. 53–63.

Jódar, L. Defez, E. and Ponsoda, E. (1996). Orthogonal matrix polynomials with respect to linear matrix moment functional: Theory and applications, Approximation Theory and its Applications, Vol. 12, No. 1, pp. 96–115.

Kargin, L. and Kurt, V. (2013). Some relations on Hermite matrix polynomials, Mathematical and Computational Applications, Vol. 18, No. 3, pp. 323–329.

Kargin, L. and Kurt, V. (2014a). On generalized Humbert matrix polynomials, Miskolc Mathematical Notes, Vol. 15, No.2, pp. 509–524.

Kargin, L. and Kurt, V. (2014b). Modified Laguerre matrix polynomials, Filomat, Vol. 28, No. 10, pp. 2069–2076.

Kargin, L. and Kurt, V. (2015). Chebyshev-type matrix polynomials and integral transforms, Hacettepe Journal of Mathematics and Statistics, Vol. 44, No. 2, pp. 341–350.

Kargin, L. and Kurt, V. (2017). On generalized two-index Hermite matrix polynomials, Miskolc Mathematical Notes, Vol. 18, No. 1, pp. 223–234.

Metwally, M.S. Mohamed, M.T. and Shehata, A. (2008). On Hermite-Hermite matrix polynomials, Mathematica Bohemica, Vol. 133, No. 4, pp. 421–434.

Metwally, M.S. Mohamed, M.T. and Shehata, A. (2015). On Chebyshev matrix polynomials, matrix differential equations and their properties, Afrika Matematika, Vol. 26, No. 5, pp. 1037–1047.

Shehata, A. (2016). Some relations on Humbert matrix polynomials, Mathematica Bohemica, Vol. 141, No. 4, pp. 407–429.

Shehata, A. (2017a). Some relations on generalized Rice's matrix polynomials, An International Journal Applications and Applied Mathematics, Vol. 12, No. 1, pp. 367–391.

Shehata, A. (2017b). Some relations on the generalized Bessel matrix polynomials, Asian Journal of Mathematics and Computer Research,Vol. 17, No. 1, pp. 1–25.

Shehata, A. (2017c). Extended biorthogonal matrix polynomials, Mathematica Moravica, Vol. 21, No. 2, pp. 27 – 50.

Shehata, A. (2018). Some new results for Chebshev matrix polynomials of first kind, International Bulletin of Mathematical Research (inpress).

Shehata, A. and Çekim, B. (2016). Some relations on Hermite-Hermite matrix polynomials, University Politechnica of Bucharest Scientific Bulletin, Series A: Applied Mathematics and Physics, Vol. 78. No. 1, pp. 181–194.