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Numerical Solution of 3rd order ODE Using FDM: On a Moving Surface in MHD Flow of Sisko Fluid

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Abstract

A Similarity group theoretical technique is used to transform the governing nonlinear partial differential equations of two dimensional MHD boundary layer flow of Sisko fluid into nonlinear ordinary differential equations. Then the resulting third order nonlinear ordinary differential equation with corresponding boundary conditions is linearised by Quasi linearization method. Numerical solution of the linearised third order ODE is obtained using Finite Difference method (FDM). Graphical presentation of the solution is given.

Keywords: Finite difference method; Group theory; MATLAB; MHD flow; Moving plate; Quasi linearization; Sisko fluid model

MSC 2010 No: 65L12, 76A05, 76D05, 76D10, 76W05

Nomenclature:

u , v - Velocity components in X, Y directions respectively
U - Main stream velocities in X direction
$\bar{\tau}$ - Stress component
$\bar{\Delta}$ - Strain rate component
a, b ,n- Flow behaviour indices
G -Group notation
η - Similarity variable
f , g -Similarity functions
$A, \alpha_1, ..., \alpha_5$ - Real constants/parameters
σ - Electrical conductivity,
B_0 -Imposed magnetic induction.
N-Number of subintervals
h- Width of intervals

1. Introduction

For the past years, the problem of the classical boundary layer over a surface has been studied in two different types. First is a boundary layer flow past a stationary surface, while the other type is the problem of a boundary layer flow over a solid surface continuously moving in a fluid at rest such as hot rolling, metal forming and continuous casting as discussed by Altan et al. (1979), Fisher et al. (1976) and Tadmor et al. (1970). Number of engineering processes contains boundary layer conduct on a moving surface is an important type of flow that occurs. Boundary layer flows of a viscous incompressible fluid past a surface moving with a constant velocity in a Newtonian fluid is analytically studied by Sakiadis (1961) and experimentally applied by Tsou et al. (1967). Takhar et al. (1987) have obtained MHD asymmetric flow over a semi-infinite moving surface and numerical solution. Erickson et al. (1965) studied the cooling of a moving continuous flat sheet. Vajravelu et al. (1997) presented the analysis of heat and mass transfer characteristics in an electrically conducting fluid over a linearly stretching sheet with variable wall temperature. Acrivos et al. (1960) and Pakdemirli (1994) derived the boundary layer equations of power-fluids. Chiam (1993) derived MHD boundary layer flow over continuously flat plate. Kumari et al. (2001) presented the problem of MHD boundary layer flow of a non-Newtonian fluid over a continuously moving surface with a parallel free stream while the non-similar solution is obtained by Jeng et al. (1986).

Jayshri et al. (2016) have presented the similarity solution of Magneto hydro dynamic flow of Sisko fluid in semi-infinite flat plate. Akber (2014) elaborated the peristaltic Sisko fluid with nano particles over asymmetric channel. She recommended that material parameter inclines pressure in peristaltic pumping regions; on the other hand it decays pressure in augmented pumping region. Moallemi et al. (2011) explored the physical properties of Sisko fluid through pipe and calculated the solution with He's homotopy perturbation method. Mailk et al. (2015) and Munir et al. (2015) discussed the convective heat transfer of Sisko fluid. The influence of applied magnetic field on Sisko fluid over stretching cylinder was discussed by Mailk et al. (2016). They have examined the flow characteristics of MHD Sisko fluid over stretching cylinder under the impact of viscous dissipation. Recently, Manjunatha et al. (2015), Si et al. (2016) and Malik et al. (2016) investigated the fluid flows assuming varying thermal conductivity.

Recently, the numerical analysis of the time dependent free convective flow of Sisko fluid on flat plate moving through a binary mixture has been obtained by Olanrewaju et al. (2013). Also, Siddiqui et al. (2013) have examined the drainage of Sisko fluid film down a vertical belt. Asghar et al. (2014) have presented the equations for the peristaltic flow of MHD Sisko fluid in a channel.

Mathematically, Sisko Model can be written as (Jayshri et al. (2016)),

$$\bar{\tau} = -\left\{ a + b\left| \sqrt{\frac{1}{2}}\left(\bar{\Delta} : \bar{\Delta}\right) \right|^{(n-1)} \right\} \bar{\Delta} \ ,$$

where, $\bar{\tau}$ and $\bar{\Delta}$ are the stress tensor and the rate of deformation tensor respectively. *a*, *b* and *n* are constants defined differently for different fluids.

Ordinary linear differential equations are simple to solve comparing to nonlinear equations. The quasi linearization method (QLM) is the best tool to convert nonlinear equations to linear. The quasi linearization method (QLM) was first introduced by Bellman et al. (1959, 1965) as a generalization of the Newton-Raphson method to solve individual or systems of nonlinear ODE and PDE.

One of the approximations of a Taylor series expansion is finite difference method (FDM). A finite difference method is applied on ODE and PDE both. In which each derivative is replaced with an approximate difference formula. The computational domain is usually divided into small sub cells and the solution will be obtained at each nodal point. FDM is easy to understand when the physical grid is given in the Cartesian form. Application of FDM on higher order ODE is very rare in the literature. Recently, Carlos et al. (2011) has presented a numerical solution of the Falkner-Skan equation using high-order and high-order-compact FDM. After that an interesting work has been carried out by Noor et al. (2012). They introduced FDM in two steps for finding approximate solutions of system of 3rd order boundary value problems associated with odd-order obstacle problems.

In the present paper, an effective group theoretical technique is applied to transform the given nonlinear partial differential equations of steady two dimensional MHD boundary layer flow of non-Newtonian fluid. Sisko fluid model is considered for the stress-strain relationship. The obtained third order nonlinear ordinary differential equation with suitable boundary conditions is linearised by Quasi linearization method. Finite Difference method with the MATLAB coding is then applied for the numerical and graphical presentation of the solution.

2. **Fundamental of Finite Difference Method (FDM)**

The finite difference method for the solution of a two –point boundary value problem consists in replacing the derivatives occurring in the differential equation (and in the boundary condition as well) by means of their finite difference approximations and then solving the resulting system of equations by standard procedure.

A general form of the third order boundary value problems (BVPs) on the interval I=[x_0, x_n] as follows:

$$y'''(x) = F[x, y(x), y'(x), y''(x)], \qquad x \in \mathrm{I}. \tag{2.1}$$

Subject to the boundary conditions:

$$y(x_0) = \alpha\ , \quad y'(x_0) = \beta\ , \quad y'(x_n) = \gamma\ , \qquad x \in \mathrm{I}. \tag{2.2}$$

The prime denotes the differentiation with respect to ***x; α, β*** and ***γ*** are given constants.

To solve boundary value problem, we divide the range [$\boldsymbol{x_0}$, $\boldsymbol{x_n}$] into *N* equal subintervals of width ***h***, so that

$$x_i = x_0 + ih, \qquad i = 1, 2, 3, \cdots, N.$$

The corresponding values of ***y*** at each point are obtained by,

$$y(x_i) = y_i = y(x_0 + ih), \qquad i = 1, 2, 3, \cdots, N.$$

Using Taylor series expansion second order central difference formula

$$y_i'(x) = \frac{y_{i+1} - y_{i-1}}{2h} + O(h). \tag{2.3}$$

$$y_i''(x) = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} + O(h^2). \tag{2.4}$$

$$y_i'''(x) = \frac{y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2}}{2h^3} + O(h^3), \qquad i = 1, 2, 3, \cdots, \mathrm{N}. \tag{2.5}$$

Putting the above central difference formulas (equations (2.3) to (2.5)) in equation (2.1), the equation (2.1) reduces into the system of linear equations. This is further solved by the method of Finite difference along with boundary conditions given by equation (2.2). The increase in the number of sub-interval will give the solution of desired accuracy.

3. Governing Equations

The flow considered here is parallel to X-direction and Y-axis is normal to it. The governing equations of two dimensional MHD boundary layer flow of Sisko fluid past a semi-infinite moving plate are (Jayshri et al. (2016)):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{3.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \frac{\partial}{\partial y}\left\{\left[a + b\left(\frac{\partial u}{\partial y}\right)^{n-1}\right]\frac{\partial u}{\partial y}\right\} + \sigma B_0^2(\mathrm{U} - \mathrm{u}). \tag{3.2}$$

Boundary conditions:

$$y = 0\ ;\ u = 1, v = 1. \tag{3.3}$$

$$y \to \infty\ ;\ u = U(x). \tag{3.4}$$

By taking one parameter scaling Group Transformation to transform the co-ordinates (x,y,u,v,U) into the co-ordinates ($\bar{x},\bar{y},\bar{u},\bar{v},\bar{U}$) as:

$$y = A^{\alpha_1}\bar{y},\ x = A^{\alpha_2}\bar{x},\ u = A^{\alpha_3}\bar{u},\ v = A^{\alpha_4}\bar{v},\ U = A^{\alpha_5}\bar{U}. \tag{3.5}$$

Introducing equation (3.5) in equations (3.1)-(3.2) and above set of equations remain invariant provided we get relation between α 's Jayshri et al. (2016).

$$\alpha_3 - \alpha_2 = \alpha_4 - \alpha_1.$$
$$2\alpha_3 - \alpha_2 = \alpha_4 + \alpha_3 - \alpha_1 = 2\alpha_5 - \alpha_2 = n\alpha_3 - (n+1)\alpha_1 = \alpha_5 = \alpha_3.$$

Solving above relations for α 's we get:

$$3\alpha_1 = \alpha_2 = 3\alpha_3 = -3\alpha_4 = 3\alpha_5.$$

Put $\alpha = \dfrac{\alpha_2}{\alpha_1} = \dfrac{1}{3}.$

$$\alpha = \alpha_3 = -\alpha_4 = \alpha_5.$$

Following Seshadri et al. (1985) one can derive the absolute invariants, so called similarity independent variable η and similarity dependent variables $f(\eta)$, $g(\eta)$ and $h(\eta)$ as follows:

$$\eta = \frac{y}{x^{\alpha}} = x^{-\frac{1}{3}}y. \tag{3.6}$$

$$f'(\eta) = \frac{u}{x^{\frac{1}{3}}}\ . \tag{3.7}$$

$$g(\eta) = \frac{v}{x^{-\frac{1}{3}}}\ . \tag{3.8}$$

$$h(\eta) = \frac{U}{x^{\frac{1}{3}}} = c_1\ . \tag{3.9}$$

Using equations (3.6)-(3.9), equation (3.1) and equation (3.2) transformed into the following:

$$f'(\eta) - \eta f''(\eta) + 3g'(\eta) = 0. \tag{3.10}$$

$$f'^2(\eta) - \eta f'(\eta) f''(\eta) + 3g(\eta) f''(\eta) = c_1^2 + 3af'''(\eta) + 3nb\left(f''(\eta)\right)^{n-1} f'''(\eta) + 3\sigma B_0^2 (c_1 - f'(\eta)). \tag{3.11}$$

Now, integrating 'g' from equation (3.10) to obtain ***g*** (i.e., $g = \frac{1}{3}\left[\eta f'(\eta) - 2f(\eta)\right]$) and substituting the value of g in equation (3.11) so that ordinary differential equation with two variables (f and g), will be reduced to single variable f ,

$$f'^2(\eta) - 2f(\eta)f''(\eta) - c_1^2 - 3af'''(\eta) - 3nb\left(f''(\eta)\right)^{n-1} f'''(\eta) - 3\sigma B_0^2(c_1 - f'(\eta)) = 0. \quad (3.12)$$

With the boundary conditions equation (3.3) and equation (3.4):

$$f(0) = 1,\ f'(0) = 1,\ f'(1) = 0. \quad (3.13)$$

Put $\sigma B_0^2 = M_0$ and $C_1 = 1$.

$$f'^2(\eta) - 2f(\eta)f''(\eta) - 3af'''(\eta) - 3nb\left(f''(\eta)\right)^{n-1} f'''(\eta) - 3M_0(1 - f'(\eta)) - 1 = 0. \quad (3.14)$$

Equation (3.14) is a nonlinear ordinary differential equation. Applying quasi linearization method to convert equation (3.14) into linear equation:

$$\begin{aligned} &f_n'^2(\eta) - 2f_n(\eta)f_n''(\eta) - 3af_n'''(\eta) - 3nb\left(f_n''(\eta)\right)^{n-1} f_n'''(\eta) - 3M_0(1 - f_n'(\eta)) - 1 \\ &-2f_n''(\eta)(f_{n+1}(\eta) - f_n(\eta)) + 2f_n'(\eta)(f_{n+1}'(\eta) - f_n'(\eta)) + 3M_0(f_{n+1}'(\eta) - f_n'(\eta)) \\ &-2f_n(\eta)(f_{n+1}''(\eta) - f_n''(\eta)) - 3af_{n+1}'''(\eta) + 3af_n'''(\eta) - 3n(n-1)b\left(f_n''(\eta)\right)^{n-2} \\ &(f_{n+1}''(\eta) - f_{n+1}''(\eta))f_n'''(\eta) - 3nb\left(f_n''(\eta)\right)^{n-1}(f_{n+1}'''(\eta) - f_{n+1}'''(\eta)) = 0. \end{aligned}$$

Simplifying above equation we have,

$$\begin{aligned} &-f_n'^2 - 3M_0 - 1 - 2f_n''(\eta)f_{n+1}(\eta) + 2f_n'(\eta)f_{n+1}'(\eta) + 3M_0 f_{n+1}'(\eta) - 2f_n(\eta)f_{n+1}''(\eta) \\ &+2f_n(\eta)f_n''(\eta) - 3af_{n+1}'''(\eta) - 3n(n-1)b\left(f_n''(\eta)\right)^{n-2} f_{n+1}''(\eta)f_n'''(\eta) \\ &+3n(n-1)b\left(f_n''(\eta)\right)^{n-1} f_n'''(\eta) - 3nb\left(f_n''(\eta)\right)^{n-1} f_{n+1}'''(\eta) = 0. \end{aligned} \quad (3.15)$$

To fit the curve, consider the solution

$$f_n = Ax^2 + Bx + C, \quad (3.16)$$

with boundary conditions in equation (3.13). We have constants A = -0.4995, B = 1, C = 1. So, equation (3.16) can be written as,

$$f_n = -0.4995x^2 + x + 1. \quad (3.17a)$$

Hence,

$$f_n' = -0.999x + 1, \quad (3.17b)$$

$$f_n'' = -0.999, \quad (3.17c)$$

$$f_n''' = 0. \quad (3.17d)$$

Substitute the values of equation (3.17*a*)-(3.17*d*) in equation (3.15)

$$[-3\mathrm{a}-3nb(-0.999)^{n-1}]\,f_{n+1}^{'''}+[0.999x^2-2x-2]\,f_{n+1}^{''}+(2+3M_0-1.9998x)\,f_{n+1}^{'}$$
$$+1.998\,f_{n+1}=3.998+3M_0. \qquad (3.18)$$

Now, Equation (3.18) is linear ordinary differential Equation. To find the numerical solution of the linear ordinary differential Equation, we will apply the Finite difference method. Substituting Equation (2.1)-(2.3) in Equation (3.18) at the ith node we have,

$$[-3a-3nb(-0.99)^{n-1}]\,f_{i+2}+[6a+6nb(-0.99)^{n-1}+1.99hx^2-4hx-4h+3h^2M_0+2h^2]$$
$$-1.998h^2x]\,f_{i+1}+(-3.996hx^2+8hx+8h+3.996h^3)\,f_i+[-6a-6nb(-0.999)^{n-1}$$
$$+1.998hx^2-4hx-4h-3h^2M_0-2h^2+1.9h^2x]\,f_{i-1}[3a+3nb(-0.99)^{n-1}]\,f_{i-2}$$
$$=(3.998+3M_0)2h^3. \qquad (3.19)$$

This gives the system of linear Equations. To solve the system of Equations using MATLAB ODE solver, we have divided the interval [0, 1] into 1000 subinterval having length h = 0.001.

Case I. Non-MHD Sisko Fluid

If we take M_0=0 and a=b=0.5 in Equation (3.19), then it will reduced in the case of Non-MHD fluid. Then the system of linear Equations (3.19) will reduced in the following Equation (3.20) with the same boundary conditions given in Equation (3.13):

$$[-1.5-1.5n(-0.999)^{n-1}]\,f_{i+2}+[3+3n(-0.999)^{n-1}+1.998hx^2-4hx-4h+2h^2$$
$$-1.998h^2x]\,f_{i+1}+[-3.996hx^2+8hx+8h+3.996h^3]\,f_i+[-3-3n(-0.999)^{n-1}+1.998hx^2$$
$$-4hx-4h-2h^2+1.998h^2x]\,f_{i-1}+[1.5+1.5n(-0.999)^{n-1}]\,f_{i-2}=7.996h^3. \qquad (3.20)$$

Case II. MHD Power-Law Fluid

If we take $a=0$, b=1, M_0 non zero constant in Equation (3.19) , then it will reduced in the case of MHD Power-Law fluid. Then the system of linear Equations (3.19) will reduced in the following Equation (3.21) with the same boundary conditions given by Equation (3.13):

$$[-3n(-0.999)^{n-1}]\,f_{i+2}+[6n(-0.999)^{n-1}+1.998hx^2-4hx-4h+3h^2M_0+2h^2-1.998h^2x]\,f_{i+1}$$
$$+[-3.996hx^2+8hx+8h+3.996h^3]\,f_i+[-6n(-0.999)^{n-1}+1.998hx^2-4hx-4h$$
$$-3h^2M_0-2h^2+1.998h^2x]\,f_{i-1}+[3\mathrm{n}(-0.999)^{n-1}]\,f_{i-2}=[3.998+3M_0]2h^3. \qquad (3.21)$$

Case III. Non-MHD Power-Law Fluid

If we take $a=0$, $b=1$ and $Mo = 0$ in Equation (3.19), then it will reduced in the case of non-MHD Power-Law fluid. Then the system of linear Equations (3.19) will reduced in the following Equation (3.22) with the same boundary conditions given in Equation (3.13):

$$[-3n(-0.999)^{n-1}]\,f_{i+2}+[6\,\mathrm{n}(-0.999)^{n-1}+1.998hx^2-4hx-4h+2h^2-1.998h^2x]\,f_{i+1}$$
$$+[-3.996hx^2+8hx+8h+3.996h^3]\,f_i+[-6n(-0.999)^{n-1}+1.998hx^2-4hx$$

$$-4h-2h^2+1.998h^2x]f_{i-1}+[3\,\mathrm{n}(-0.999)^{n-1}]f_{i-2}=7.996h^3. \tag{3.22}$$

Case IV. MHD Newtonian Fluid

If we take n=1, a=1,b=0 and M_0 non zero constant in Equation (3.19) , then it will reduced in the case of MHD Newtonian fluid. Then the system of linear Equations (3.19) will reduced in the following Equation (3.23) with the same boundary conditions given in Equation (3.13):

$$\begin{aligned}-3f_{i+2}+[6+1.998hx^2-4hx-4h+3h^2M_0+2h^2-1.998h^2x]f_{i+1}+[-3.996hx^2\\ +8hx+8h+3.996h^3]f_i+[-6+1.998hx^2-4hx-4h-3h^2M_0-2h^2\\ +1.998h^2x]f_{i-1}+3f_{i-2}=[3.998+3M_0]2h^3.\end{aligned} \tag{3.23}$$

Case V. Non-MHD Newtonian Fluid.

If we take a=1, b=0, n=1and M_0=0 in Equation (3.19), then it will reduced in the case of non-MHD Newtonian fluid. Then the system of linear Equations (3.19) will reduced in the following Equation (3.24) with the same boundary conditions given in Equation (3.13):

$$\begin{aligned}-3f_{i+2}+[6+1.998hx^2-4hx-4h+2h^2-1.998h^2x]f_{i+1}+[-3.996hx^2+8hx+8h\\ +3.996h^3]f_i+[-6+1.998hx^2-4hx-4h-2h^2+1.998h^2x]f_{i-1}+3f_{i-2}=7.996h^3.\end{aligned} \tag{3.24}$$

4. Results and discussions

Using QLM we obtained the third order linear ODE which is then solved by FDM. If we change the values of flow parameter like a, b, M_o, and the flow behaviour index n, the fluid we considered is change to Power law and Newtonian. Which is indicated as sub cases from Case I to Case V. We solve all sub cases and give graphical presentation of all. Our aim is to apply FDM on third-order ODE. Numerical values and graphs can now easily generated. In Figure 1, a=b=0.5 and the flow behaviour index n = 0.5 are constants and the magnetic no. Mo is varies, 0, 2, 5, 10. In Figure 2, a = b = 0.5 and the magnetic no. Mo=5 are constant and the flow behaviour index n is varies, 0.5, 1, 1.5, 2.5. In Figure 3, a=0.5, the magnetic number .Mo=10 and the flow behaviour index n=0.5 are constants and constant parameter b varies, 0.5, 1.5, 2.5, 5. In Figure 4, b = 0.5, the magnetic number. Mo =10 and the flow behaviour index n=0.5 are constants and constant parameter a varies, 0.5, 1.5, 2.5, 5. In Figure 5, a=b=0.5 and the magnetic no. Mo = 0 and the flow behaviour index n is varies, 0.1, 0.5, 1, 1.5. In Figure 6, a = 0,b = 1 and the magnetic no. Mo=5 is constant and the flow behaviour index n is varies n=0.5, 1, 1.5, 2.5. In Figure 7, a = 0, b = 1 and the magnetic no. Mo=0 is constant and the flow behaviour index n is varies n=0.5, 1, 1.5, 2.5. In Figure 8, a = 1, b = 0 and the flow behaviour index n is constant n = 1 and the magnetic no. Mo is varies Mo= 2, 5, 10, 15. In Figure 9, b=0, the flow behaviour index n is constant n=1 and the magnetic no. Mo =0 the constant a is varies a=0.5, 1, 1.5, 2.5.

5. Conclusion

In the present paper, the governing Equations of motion (partial differential Equations) of the laminar MHD boundary layer flow of non-Newtonian fluid are solved numerically using FDM. The velocity is decrease uniformly with the increase in η for variable fluid index, Magnetic

induction and fluid parameters. Increase in the magnetic number accelerate the fluid. Also for the case of Power-law fluid, shear thinning fluid velocity increase more rapidly than that of for shear thickening fluids.

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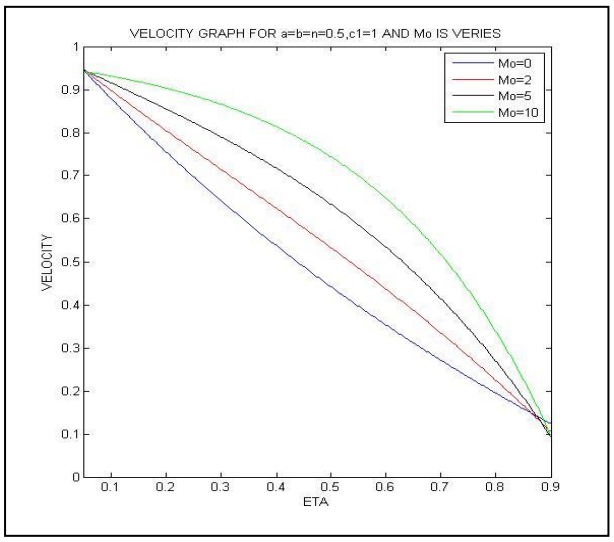

Figure 1: **Velocity for different M_0**

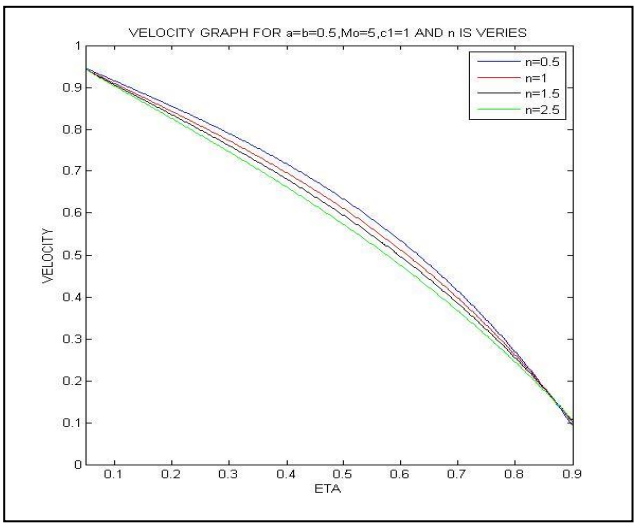

Figure 2: **Velocity for different *n***

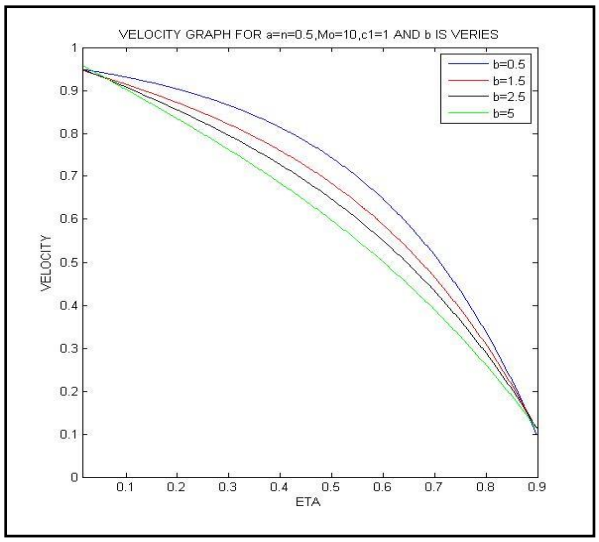

Figure 3: **Velocity for different *b***

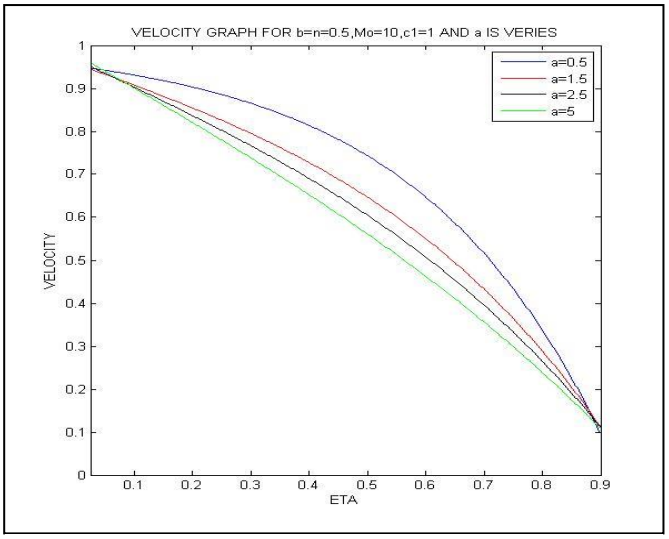

Figure 4: **Velocity for different a**

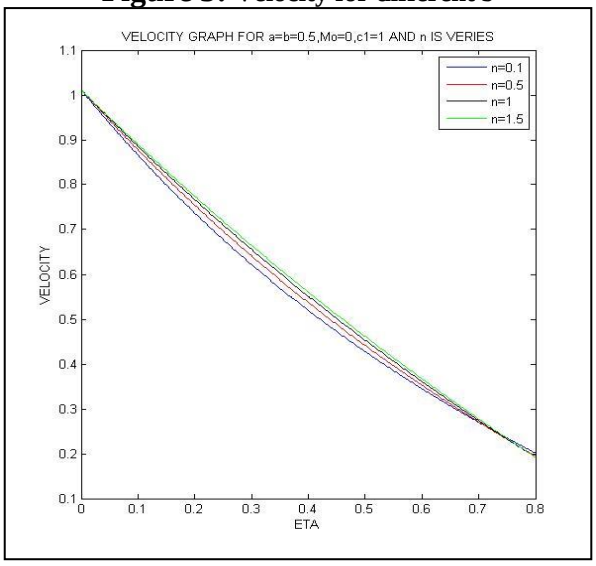

Figure 5: **Case-I Velocity for M_0=0**

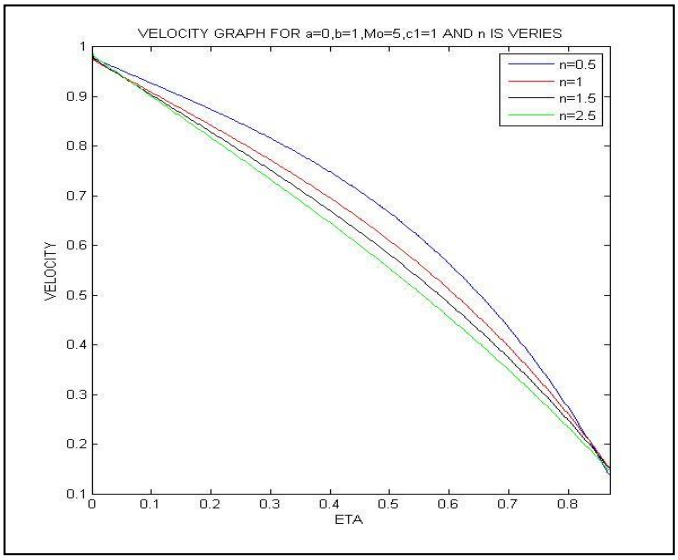

Figure 6: **Case-II Velocity for MHD Power Law**

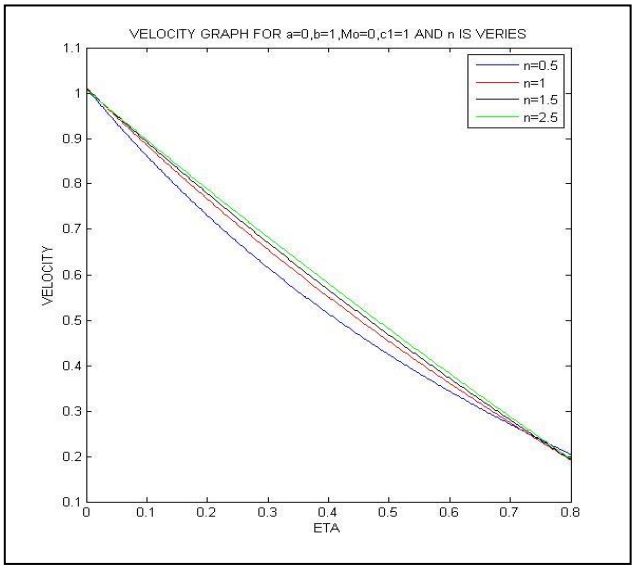

Figure 7: Case-III Velocity for Power Law (M_0=0)

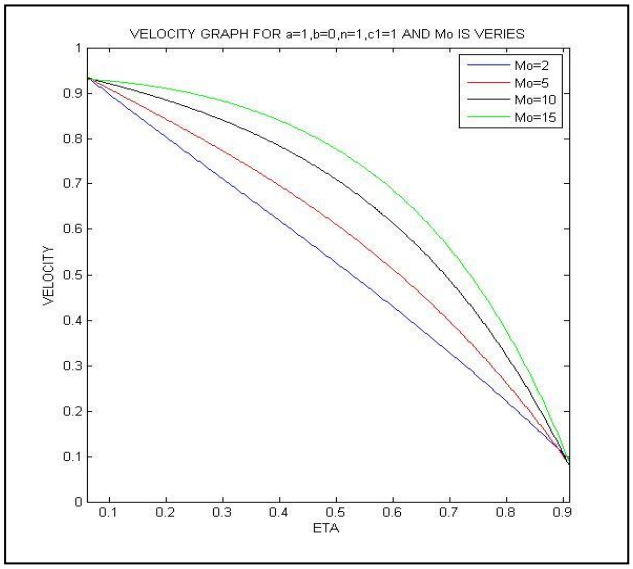

Figure 8: Case-IV Velocity for MHD Newtonian

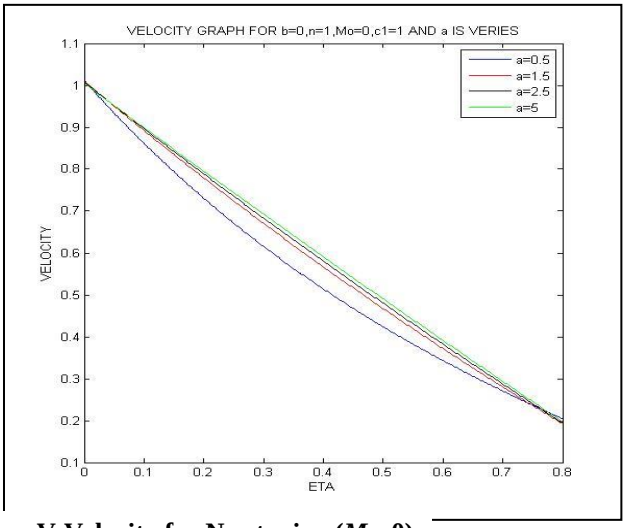

Figure 9: Case-V Velocity for Newtonian (M_0=0)