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On the Lucas Difference Sequence Spaces Defined by Modulus Function

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Abstract

In this paper, firstly, we define the Lucas difference sequence spaces by the help of Lucas sequence and a sequence of modulus function. Besides, we give some inclusion relations and examine geometrical properties such as Banach-Saks type p, weak fixed point property.

Keywords: Difference sequence space; Lucas numbers; Modulus function; Sequence space; Banach-Saks property; Weak fixed point property; Infinite matrix

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1. Introduction

Let w be the space of all real and complex valued sequences. Each linear subspace of w is called sequence space. Throughout the paper $\ell_\infty, \ell_p (1 \leq p < \infty), c$ and c_0 denote the spaces of all bounded, p-absolutely summable, convergent and null sequences, respectively. If X is a complete linear metric space, then a K-space X is called an FK-space. An FK-space whose topology is normable is called BK-space.

An infinite matrix is a double sequence $A = (a_{nk})$ of real or complex numbers defined by a function A from the set $\mathbb{N} \times \mathbb{N}$ into the complex field $\mathbb{C}$ $(or\ \mathbb{R})$ where $\mathbb{N} = \{0,1,2,\dots\}$. The treatment of infinite matrices is absolutely different from finite matrices. There are various reasons for this. In some instances, the most general linear operator among two sequence spaces is presented by an infinite matrix. Let X and Y be any two sequence spaces. A defines a matrix mapping from X into Y, if $Ax = \{(Ax)_n\} \in Y$ for every $x = (x_k) \in X$ where

$$(Ax)_n = \sum_k a_{nk} x_k. \tag{1}$$

The class of all matrices A such that $A: X \to Y$ is symbolized by $(X: Y)$. In this way, $A \in (X: Y)$ iff the series on the right hand side of (1) converges for each $n \in \mathbb{N}$ and every $x \in X$, and we get $Ax = \{(Ax)_n\} \in Y$ for all $x \in X$.

The concept of matrix domain is important for our study. For an infinite matrix A, the matrix domain ϕ_A in a sequence space ϕ is defined by

$$\phi_A = \{x \in \omega: Ax \in \phi\}, \tag{2}$$

which is a sequence space [Başar (2011)].

Recently, so many authors have made use of the approach of constructing a new sequence space using matrix domain for a triangle infinite matrix, e.g., Başar and Altay (2003), Altay and Basar (2005), Kirisci and Basar (2010), Mursaleen and Noman (2010), Mursaleen and Noman (2011), Kara and Basarir (2012), Kara (2013), (Debnath and Saha 2014), (Debnath et al. 2015), Karakas (2015).

In the literature, the matrix domain λ_Δ is called the difference sequence space if λ is a normed or paranormed sequence space where Δ symbolizes the following backward difference matrix $\Delta = (\Delta_{nk})$ and $\Delta' = (\Delta'_{nk})$ symbolizes the following transpoze of the matrix Δ, the forward difference matrix. For $\lambda = \ell_{\mathrm{p}}$, this space is called as the space of sequences of $p-$bounded variation, that is, bv_p. Also, it is clear that $bv_p = \left(\ell_{\mathrm{p}}\right)_\Delta$.

$$\Delta_{nk} = \begin{cases} (-1)^{n-k}, & (n-1 \le k \le n), \\ 0, & (0 \le k < n-1 \text{ or } k > n), \end{cases} \quad \text{and} \quad \Delta'_{nk} = \begin{cases} (-1)^{n-k}, & (n \le k \le n+1), \\ 0, & (0 \le k < n \text{ or } k > n+1). \end{cases}$$

The notion of difference sequence spaces was first defined by Kizmaz (1981) in the form of $X(\Delta) = \{x \in \omega: x_k - x_{k+1} \in X\}$ for $X = \ell_\infty, c, c_0$. These spaces were generalized by Et and Çolak (1995) as $X(\Delta^r) = \{x \in \omega: \Delta^r x \in X\}, X = \ell_\infty, c, c_0$.

The difference sequence space bv_p which is examined by Altay and Başar (2007) contains sequences (x_k) such that $(x_k - x_{k-1})$ for $0 < p < 1$. In the case $1 \le p \le \infty$, this space was studied by Çolak et al. (2004). Also, many authors analyzed the certain difference sequence spaces, see Ahmad (1987), Malkowsky (1989), Et (1993), Mursaleen (1996), Et and Basarir (1997), Tripathy (2003), Colak and Et (2005), Başar et al. (2008).

The main aim of this note is to introduce new sequence spaces $\ell\left(\hat{E}(r,s), \mathfrak{F}, p, u\right)$ and $\ell_\infty\left(\hat{E}(r,s), \mathfrak{F}, p, u\right)$ with the help of sequence of modulus functions, non-zero real numbers r and s, Lucas difference matrix and its matrix domain. In addition, we investigate some topological properties and also find out some inclusion relations concerning with these spaces. Finally, we work through geometrical properties of the space $\ell\left(\hat{E}(r,s), \mathfrak{F}, p, u\right)$.

2. Material and methods

In this part of our study, we inform about familiar concepts which are necessary for us later on the paper.

Definition 2.1.

A modulus f is a function from $[0,\infty)$ to $[0,\infty)$ such that $f(x)=0$ if and only if $x=0$; $f(x+y)\le f(x)+f(y)$ for $x,y\ge 0$; f is increasing; f is continuous from the right at 0. [Nakano (1953)].

It follows that f must be continuous everywhere on $[0,\infty)$. To construct some sequence spaces, many authors used a modulus function, see Ruckle (1973), Maddox (2008), Pehlivan and Fisher (1994), Altin (2009), Raj et al. (2015).

Definition 2.2.

A Banach space X possess Banach-Saks property if any bounded sequence in X approves a subsequence whose arithmetic mean converges in norm. In a similar vein, a Banach space X has weak Banach-Saks property if any weakly null sequence in X admits a subsequence whose arithmetic mean strongly converges in norm.

Definition 2.3.

Let Y be a Banach space. The coefficient $R(Y)$ was defined by

$$R(Y)=sup\left(\lim_{n\to\infty} inf\|y_n+y\|\right)$$

and also a Banach space Y with $R(Y)<2$ holds weak fixed point property. [Garcia-Falset (1994)].

Definition 2.4.

Let L_n be the nth term of a sequence such that $L_0=2, L_1=1$ and $L_n=L_{n-1}+L_{n-2}, n\ge 2$. The resulting sequence 1,3,4,7,11,18,… is called Lucas sequence.

We can find so many fundamental properties about Lucas sequences in Koshy (2017) and Vajda (1989). Some of them are as follows:

$$\sum_{k=1}^{n} L_k = L_{n+2}-3;\ \sum_{k=1}^{n} L_{2k-1} = L_{2n-2};\ n\ge 1,$$

$$\sum_{k=1}^{n} L_k^2 = L_nL_{n+1}-2;\ \sum_{k=1}^{2n} L_kL_{k-1} = L_{2n}^2-4;$$

$$L_{n+1}L_{n-1}-L_n^2=(-5)(-1)^n.$$

It can be easily derived by placing L_{n+1} in the last equality that

$$L_{n-1}^2+L_nL_{n-1}-L_n^2=-5(-1)^n.$$

By using above information, we define the following double band matrix

$$\hat{E}=(\hat{L}_{nk})=\begin{cases}-\frac{L_n}{L_{n-1}}, & (k=n-1),\\ \frac{L_{n-1}}{L_n}, & (k=n),\\ 0, & other,\end{cases}\quad ;\ n,k\in\mathbb{N}-\{0\}. \tag{3}$$

Karakaş and Karakaş (2017). The inverse of this matrix is

$$\hat{E}^{-1} = \begin{cases} \dfrac{L_n^2}{L_{k-1}L_k}, & 0 < k \leq n, \\ 0, & other, \end{cases}$$

and the $\hat{E}-$ transform of a sequence $x = (x_n)$ is defined by

$$y_n = \hat{E}_n(x) = \tfrac{L_{n-1}}{L_n} x_n - \tfrac{L_n}{L_{n-1}} x_{n-1};\ n \geq 1. \quad (4)$$

Later, the above Lucas matrix was generalized and constructed the matrix $E(r,s) = \big(L_{nk}(r,s)\big)$ for non-zero real numbers r and s as follows:

$$\hat{E}(r,s) = \begin{cases} s\dfrac{L_n}{L_{n-1}}, & (k = n-1), \\ r\dfrac{L_{n-1}}{L_n}, & (k = n), \\ 0, & (0 \leq k < n-1 \ or\ k > n), \end{cases};\quad n, k \in \mathbb{N} - \{0\}.$$

Now, by using the matrix $\hat{E}(r,s)$, we introduce the following Lucas difference sequence spaces:

$$\ell\big(\hat{E}(r,s), \mathfrak{F}, p, u\big) = \left\{x \in w: \sum_n \left[u_n F_n\left(\left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|\right)\right]^{p_n} < \infty\right\}$$

and

$$\ell_\infty\big(\hat{E}(r,s), \mathfrak{F}, p, u\big) = \left\{x \in w: sup_n \left[u_n F_n\left(\left|r\tfrac{L_{n-1}}{L_n}x_n + s\tfrac{L_n}{L_{n-1}}x_{n-1}\right|\right)\right]^{p_n} < \infty\right\},$$

where $\mathfrak{F} = (F_n)$ is a sequence of modulus functions, $p = (p_n)$ is any bounded sequence of positive real numbers and $u = (u_n)$ is a sequence of strictly positive real numbers. Also, for the inverse of the matrix $\hat{E}(r,s)$ and the $\hat{E}(r,s)$-transform of the sequence $x = (x_n)$, we'll use the following equalities:

$$\hat{E}^{-1}(r,s) = \begin{cases} \frac{1}{r}\left(-\frac{s}{r}\right)^{n-k} \frac{L_n^2}{L_{k-1}L_k}, & 0 < k \leq n, \\ 0, & k > n, \end{cases}$$

and

$$y_n = \hat{E}_n(r,s)(x) = r\tfrac{L_{n-1}}{L_n}x_n + s\tfrac{L_n}{L_{n-1}}x_{n-1};\ n \geq 1. \quad (5)$$

In addition, we will require the inequality

$$|a_k + b_k|^{p_k} \leq C(|a_k|^{p_k} + |b_k|^{p_k}), \quad (6)$$

where $C = max\{1, 2^{H-1}\}, H = supp_k$ and $p = (p_k)$ is a sequence of positive real numbers.

3. Results and discussion

The proofs of the following two theorems are easy, so we give them without proof.

Theorem 3.1.

$\ell\big(\hat{E}(r,s),\mathfrak{F},p,u\big)$ and $\ell_\infty\big(\hat{E}(r,s),\mathfrak{F},p,u\big)$ are linear spaces over $\mathbb{C}$.

Theorem 3.2.

$\ell\big(\hat{E}(r,s),\mathfrak{F},p,u\big)$ is a paranormed space for $M = \max(1,H)\,, H = sup p_n$ with paranorm

$$g(x) = sup_n\left(\sum_n\left[u_n F_n\left(\left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|\right)\right]^{p_n}\right)^{1/M}.$$

Theorem 3.3.

Let $\mathfrak{F} = (F_n)$ be a sequence of modulus functions, $p = (p_n)$ and $q = (q_n)$ be bounded sequences of positive real numbers. If $0 \le p_n \le q_n < \infty$ for each n, then $\ell\big(\hat{E}(r,s),\mathfrak{F},p,u\big) \subseteq \ell\big(\hat{E}(r,s),\mathfrak{F},q,u\big)$.

***Proof*:**

Let us take $x \in \ell\big(\hat{E}(r,s),\mathfrak{F},p,u\big)$. So, $\sum_n\left[u_n F_n\left(\left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|\right)\right]^{p_n} < \infty$. It means that $\left[u_n F_n\left(\left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|\right)\right]^{p_n} \le 1$ for $n \ge n_0$, that is, sufficiently large values of n. Hence, we obtain

$$\sum_{n\ge n_0}\left[u_n F_n\left(\left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|\right)\right]^{q_n}$$
$$\le \sum_{n\ge n_0}\left[u_n F_n\left(\left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|\right)\right]^{p_n} < \infty,$$

which implies that $x \in \ell\big(\hat{E}(r,s),\mathfrak{F},q,u\big)$ because of $p_n \le q_n$ and F_n is increasing.

Theorem 3.4.

If $\mathfrak{F} = (F_n)$ be a sequence of modulus functions and $\alpha = \lim_{q\to\infty}\frac{F_n(q)}{q} > 0$, then $\ell\big(\hat{E}(r,s),\mathfrak{F},p,u\big) \subseteq \ell\big(\hat{E}(r,s),p,u\big)$.

***Proof*:**

The definition of α gives us $F_n(q) \ge \alpha(q)$, for all $q > 0$. Herefrom, we see that $\frac{1}{\alpha}F_n(q) \ge q$, for all $q > 0$. Now, for $x \in \ell\big(\hat{E}(r,s),\mathfrak{F},p,u\big)$ we have

$$\sum_n\left[u_n\left(\left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|\right)\right]^{p_n} \le \frac{1}{\alpha}\sum_n\left[u_n F_n\left(\left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|\right)\right]^{p_n},$$

which shows that $x \in \ell(\hat{E}(r,s),p,u)$.

Theorem 3.5.

The inclusion $\ell(\hat{E}(r,s),\mathfrak{F}',p,u) \cap \ell(\hat{E}(r,s),\mathfrak{F}'',p,u) \subseteq \ell(\hat{E}(r,s),\mathfrak{F}'+\mathfrak{F}'',p,u)$ holds for sequences of modulus functions $\mathfrak{F}' = (F_n')$ and $\mathfrak{F}'' = (F_n'')$.

Proof:

Let $x \in \ell(\hat{E}(r,s),\mathfrak{F}',p,u) \cap \ell(\hat{E}(r,s),\mathfrak{F}'',p,u)$. Then, it can be easily seen that

$$\sum_n \left[u_n F_n'\left(\left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|\right)\right]^{p_n} < \infty;$$

and

$$\sum_n \left[u_n F_n''\left(\left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|\right)\right]^{p_n} < \infty.$$

From here, we have

$$\begin{aligned}\sum_n &\left[u_n (F_n' + F_n'')\left(\left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|\right)\right]^{p_n} \\ &\leq M\sum_n \left[u_n F_n'\left(\left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|\right)\right]^{p_n} \\ &+M\sum_n \left[u_n F_n''\left(\left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|\right)\right]^{p_n},\end{aligned}$$

which means that $x \in \ell(\hat{E}(r,s),\mathfrak{F}'+\mathfrak{F}'',p,u)$.

Theorem 3.6. $\ell(\hat{E}(r,s),\mathfrak{F}',p,u) \subseteq \ell(\hat{E}(r,s),\mathfrak{F}\circ\mathfrak{F}',p,u)$

Proof:

Let us take $\varepsilon > 0, 0 < \delta < 1$ such that $F_n(q) < \varepsilon$, for $0 \leq q \leq \delta$. Choose $y_n = u_n F_n'\left(\left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|\right)$ and consider

$$\sum_n [F_n(y_n)]^{p_n} = \sum_{y_n \leq \delta}[F_n(y_n)]^{p_n} + \sum_{y_n > \delta}[F_n(y_n)]^{p_n}.$$

In view of the continuity of F_n, we have

$$\sum_{y_n \leq \delta}[F_n(y_n)]^{p_n} < \varepsilon^H , \tag{7}$$

and $y_n > \delta \Rightarrow y_n < \frac{y_n}{\delta} \leq 1 + \frac{y_n}{\delta}$. For $y_n > \delta$, we obtain by the definition that

$$F_n(y_n) < 2F_n(1)\frac{y_n}{\delta}$$

and so

$$\sum_{y_n>\delta}[F_n(y_n)]^{p_n} < max\{1,(2F_n(1)\delta^{-1})^H\}\sum_n[y_n]^{p_n}. \tag{8}$$

Equations (7) and (8) give the fact that $\ell\big(\hat{E}(r,s),\mathfrak{F}',p,u\big) \subseteq \ell\big(\hat{E}(r,s),\mathfrak{F}\circ\mathfrak{F}',p,u\big)$.

Theorem 3.7.

$\ell\big(\hat{E}(r,s),\mathfrak{F},p,u\big)$ and $\ell_\infty\big(\hat{E}(r,s),\mathfrak{F},p,u\big)$ are normed spaces with

$$\|x\|_{\ell(\hat{E}(r,s),\mathfrak{F},p,u)} = \left(\sum_n\left[u_kF_k\left(\left|\hat{E}_n(r,s)(x)\right|\right)\right]^{p_k}\right)^{1/M},$$

and

$$\|x\|_{\ell_\infty(\hat{E}(r,s),\mathfrak{F},p,u)} = sup_n\left[u_kF_k\left(\left|\hat{E}_n(r,s)(x)\right|\right)\right]^{p_k},$$

for $1 \le p_k \le H \le \infty$ for all $k \in \mathbb{N}$.

***Proof*:**

It can be proved by standard technic in Raj et al. (2015).

$\ell\big(\hat{E}(r,s),\mathfrak{F},p,u\big)$ and $\ell_\infty\big(\hat{E}(r,s),\mathfrak{F},p,u\big)$ are sequence spaces of non-absolute type. Indeed,

$$\|x\|_{\ell(\hat{E}(r,s),\mathfrak{F},p,u)} \neq \||x|\|_{\ell(\hat{E}(r,s),\mathfrak{F},p,u)} \text{ and } \|x\|_{\ell_\infty(\hat{E}(r,s),\mathfrak{F},p,u)} \neq \||x|\|_{\ell_\infty(\hat{E}(r,s),\mathfrak{F},p,u)};$$

which means that the absolute property is not valid on the spaces $\ell\big(\hat{E}(r,s),\mathfrak{F},p,u\big)$ and $\ell_\infty\big(\hat{E}(r,s),\mathfrak{F},p,u\big)$ for at least one sequence $x=(x_k)$.

Theorem 3.8.

The sequence space $\ell\big(\hat{E}(r,s),\mathfrak{F},p,u\big)$ of non-absolute type is linearly isomorphic to the space ℓ_p for $1 \le p_k \le H < \infty$ for all $k \in \mathbb{N}$, that is, $\ell\big(\hat{E}(r,s),\mathfrak{F},p,u\big) \cong \ell_p$.

***Proof*:**

Let us take into consideration the transformation $Z\colon \ell\big(\hat{E}(r,s),\mathfrak{F},p,u\big) \to \ell_p$ defined by $x \to y = Zx$, with equality (5). Then, for $x \in \ell\big(\hat{E}(r,s),\mathfrak{F},p,u\big)$, we have $Zx = y = \hat{E}(r,s)x \in \ell_p$. So, it is obvious that Z is linear. Also, it can be easily shown that $Zx = 0 \Rightarrow x = 0$ and thus Z is injective. Now, let us consider $y=(y_k) \in \ell_p$ and identify the sequence $x=(x_k)$ for $1 \le p_k \le H < \infty$ for all $k \in \mathbb{N}$ as follows:

$$x_k = \frac{1}{r}\sum_{j=1}^{k}\left(-\frac{s}{r}\right)^{k-j}\frac{L_k^2}{L_{j-1}L_j}y_j.$$

Hence, we obtain for $1 \le p_k \le H < \infty$ for all $k \in \mathbb{N}$ and $p=\infty$,

$$\|x\|_{\ell(\hat{E}(r,s),\mathfrak{F},p,u)} = \left(\sum_k \left[u_k F_k\left(\left|r\frac{L_{k-1}}{L_k}x_k + s\frac{L_k}{L_{k-1}}x_{k-1}\right|\right)\right]^{p_k}\right)^{1/M}$$
$$= \left(\sum_k \left[u_k F_k\left(\left|r\frac{L_{k-1}}{L_k}\frac{1}{r}\sum_{j=1}^{k}\left(-\frac{s}{r}\right)^{k-j}\frac{L_k^2}{L_{j-1}L_j}y_j \right.\right.\right.\right.$$
$$\left.\left.\left.\left.+ s\frac{L_k}{L_{k-1}}\frac{1}{r}\sum_{j=1}^{k-1}\left(-\frac{s}{r}\right)^{k-j}\frac{L_k^2}{L_{j-1}L_j}y_j\right|\right)\right]^{p_k}\right)^{1/M}$$
$$= \|y\|_{\ell_p} < \infty,$$

and

$$\|x\|_{\ell_\infty(\hat{E}(r,s),\mathfrak{F},p,u)} = \sup_k\left[u_k F_k\left(\left|\hat{E}_k(r,s)(x)\right|\right)\right]^{p_k} = \|y\|_{\ell_\infty} < \infty.$$

This gives the fact that Z is linear bijection and so the proof is completed. Now, we'll investigate the geometric structure of the space $\ell\left(\hat{E}(r,s),\mathfrak{F},p,u\right)$, that is, we'll examine whether the space $\ell\left(\hat{E}(r,s),\mathfrak{F},p,u\right)$ has Banach-Saks property of type p and the weak fixed point property or not.

Theorem 3.9.

Let $1 < p_k \leq H < \infty$ for all $k \in \mathbb{N}$. The space $\ell\left(\hat{E}(r,s),\mathfrak{F},p,u\right)$ has Banach-Saks property of type p.

***Proof*:**

It can be demonstrated by standard technic which can be seen in Et et al. (2014).
Since the space $\ell\left(\hat{E}(r,s),\mathfrak{F},p,u\right)$ is linearly isomorphic to space ℓ_p, we have $R\left(\ell\left(\hat{E}(r,s),\mathfrak{F},p,u\right)\right) = R\left(\ell_p\right) = 2^{1/p}$. In view of $R\left(\ell\left(\hat{E}(r,s),\mathfrak{F},p,u\right)\right) < 2$, we can give the following theorem with the help of definition 2.3.

Theorem 3.10.

The space $\ell\left(\hat{E}(r,s),\mathfrak{F},p,u\right)$ has weak fixed point property in the case $1 < p_k \leq H < \infty$ for all $k \in \mathbb{N}$.

4. Conclusion

Geometrical properties of Banach spaces have been studied by many authors. In recent years, one of the interesting topics is to examine topological and geometrical properties of difference sequence spaces defined by Fibonacci and Lucas numbers. In this paper, we apply the domain of infinite triangular matrix established by Lucas numbers and the modulus function to space ℓ_p. Then, we investigate topological and geometric structure of the obtained space. Since some of the geometric properties of Banach spaces play an important role in the fixed point theory, our results are interesting. However, the Lucas numbers and its properties can be considered in different fields of summability theory such as statistical convergence and its applications.

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