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Exact Reliability for Consecutive k-out-of-r-from-n: F System with Equal and Unequal Components Probabilities

[1*]Yaser Amirian, [2]Ahmad Khodadadi and [3]Omid Chatrabgoun

[1,2] Department of Statistics
Faculty of Mathematical Sciences and Statistics
Shahid Beheshti University
Tehran, Iran
[1]y_amirian@sbu.ac.ir; [2]a_khodadadi@sbu.ac.ir;

[3]Department of Statistics
Faculty of Mathematical Sciences and Statistics
Malayer University
Malayer, Iran
omid.6423@yahoo.com;

*Corresponding Author

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Abstract

A consecutive k-out-of-r-from-n: F system consists of n linear ordered components such that the system fails if and only if there exist a set of r consecutive linear component that contains at least k failed components. Consecutive k-out-of-r-from-n: F system has been considered in many fields such as reliability analysis. All recent efforts in this area have been focused on acquiring band or approximation for their reliability such that less attention has been paid to their closed form and exact reliability in the literature. In the present paper, with designing an innovative algorithm the exact reliability for extensive class of consecutive k-out-of-r-from-n: F system is obtained. Specially this task for equal and unequal components probabilities is done. Finally, the numerical results for calculating the exact reliability in extensive class of this strategic systems were applied.

Keywords: Exact reliability; Consecutive k-out-of-r-from-n: F system; Equal components probabilities; Unequal components probabilities; Innovative algorithm

MSC 2010 No.: 90B25, 60K10, 62N05

1. Introduction

Table 1. Notations

k	Minimum number of components in r consecutive that failure will lead to failure of the system $k \leq r$
r	A window of r consecutive out of n components $r \leq n$
n	Number of components in the system
p	Equal probability components
p_i	Probability of component i, $i = 1, 2, ..., n$
X_i	Situation of component i
$\mathbf{X}$	Vector of component states X_i, $i = 1, 2, ..., n$
Y_i	The sum of r component $X_i + X_{i+1} + ... + X_{i+r-1}$
$\mathbf{Y}$	The vector of Y_i, $i = 1, 2, ..., n - r + 1$
t_i	Sample values of component i
$\mathbf{t}$	Sample value of vector t_i, $i = 1, 2, ..., n$
C	Producer algorithms of vectors $\mathbf{t}$
$R_{n,r,k,p}$	The reliability of the system for n, r, k, p

Industry development requires a need to design update systems and their optimal management, particularly the systems that are frequently used at the macro level and strategic conditions. One of the known systems in this area is consecutive k-out-of-r-from-n: F system that was presented by Tong (1985). A consecutive k-out-of-r-from-n: F system consists of n linear ordered components such that the system fails if and only if there exist a set of r consecutive linear component that contains at least k failed components.

Such a system can be used in many applied sciences. For example their applications are in the radar detection, in quality control and in inspection procedures. The two systems consecutive-k-out-of-r-from-n: F and consecutive $(r-k+1)$-out-of-r-from-n: G have the same performance (Kuo et al., 1990). The consecutive k-out-of-r-from-n: F system involves a consecutive k-out-of-n: F system (when $k = r$) and k-out-of-n: F system (when $r = n$) as special cases. This system consists of many subsets which have many applications (see e.g. Chiang and Niu (1981), Kao (1982), Chao and Lin (1984), Chiang and Chiang (1986), Hwang (1989), Shen and Zuo (1994), Kuo et al. (1990), Kuo and Zuo (2003)).

Obtaining the reliability has been one of the most important goal of this system in the recent years (See e.g. Radwan et al. (2011), Habib et al. (2007), Eryilmaz (2009), Habib and Szantai (2000), Daus and Beiu (2015), Gokdere et al. (2016), Kounias and Sfakianakis (1991), Koutras and Papastavridis (1993), Malinowski and Preuss (1995, 1995), Papastavridis (1988), Papastavridis and Koutras (1993), Papastavridis and Sfakianakis (1991), Sfakianakis (1993), Sfakianakis et al. (1992), Szantai and Aziz(1997)). Because of disability in obtaining exact reliability, all efforts have been attracted to the approximation of reliability and getting the band. On the other hand, most existing researches also considered systems with equal probabilities component while this could be expanded to unequal probabilities. Indeed, the problem of obtaining exact reliability under more general conditions has been less considered.

The main contributions of the present paper are as follows. At first, for the consecutive k-out- of-r-from-n: F system an exact distribution for the r dependent components is calculated, then with the use of it, exact reliability for this type of system that consists of many sub-systems is calculated. In fact, this work is done under realistic conditions which, to our knowledge allows, this issue has not been addressed previously. Also, in this paper a general innovative algorithm is designed that provides appropriate response for different situations of the consecutive k-out-of-r-from-n: F system.

For convenience in the contents of this article, signs and notations that are frequently used are presented in Table 1.

Then, the paper is organized as follows: In the second section, we deal with the exact reliability for general consecutive k-out-of-r-from-n: F system with equal probability component. In Section 3, similar to Section 2, we present the exact reliability for this system with unequal probability component. Section 4 is devoted to the numerical results and compare with existing results that explain performance of the obtained results in Section 2 and 3. Section 5 is dedicated to the application of our results in two real-world examples. Finally, in the last section some conclusions are stated.

2. Exact Reliability with Equal Components Probabilities

In this section, consecutive k-out-of-r-from-n: F systems with equal probability component have been considered. A consecutive k-out-of-r-from-n: F system consists of n linear ordered components such that the system fails if and only if there exist a set of r consecutive linear component that contains at least k failed components.

Let X_i, $i = 1, ..., n$, is system component, independent identically Bernoulli distribution with success probability p, $B(p)$, with equal probability features. In the considered system, active or inactive of the entire system depends on performance of r consecutive components and not to the each of the individual component. So, we decompose the system to possible r consecutive components as

$$Y_i = X_i + X_{i+1} + \cdots + X_{i+r-1}, \quad i = 1, 2, ..., n-r+1,$$

which Y_i is binomial r and p distribution, $Y_i \sim b(r, p)$. And Y_i for $i = 1, 2, ..., n-r+1$, in Y vector, $(Y_1, Y_2, ..., Y_{n-r+1})$, are dependent with binomial r and p distribution. The system works properly if each Y_i has at least $r-k+1$ active components. In order to find the exact reliability of the system, the joint distribution of $(Y_1, Y_2, ..., Y_{n-r+1})$ is calculated in the following theorem.

Theorem 2.1.

Let $X_i \sim B(p)$; for all $i = 1, 2, ...n$, is the system component, and accordingly Y_i for $i = 1, 2, ..., n-r+1$, is defined as follows:

$$Y_i = X_i + X_{i+1} + \cdots + X_{i+r-1}, \quad i = 1, 2, ..., n-r+1, \tag{1}$$

which Y_i are dependent and identically distributed with $b(r, p)$. The joint distribution of

$(Y_1, Y_2, ..., Y_{n-r+1})$ is

$$P_{Y_1,Y_2,...,Y_{n-r+1}}(y_1, y_2, ..., y_{n-r+1}) = \sum_C p^{\sum_{i=1}^{n} t_i}(1-p)^{n-\sum_{i=1}^{n} t_i},$$

which operator C determines considered $\mathbf{t} = (t_1, t_2, ..., t_n)$ using an innovative algorithm. Of course, for the sake of textual unity, we postpone exact definition C to the final step of proof.

Proof:

To calculate joint distribution $(Y_1, Y_2, ..., Y_{n-r+1})$, we can write

$$P_1 = P(Y_1 = y_1, Y_2 = y_2, ..., Y_{n-r+1} = y_{n-r+1}) = P_{Y_1,Y_2,...,Y_{n-r+1}}(y_1, y_2, ..., y_{n-r+1}).$$

We replace $X_i, \ \ i = 1, 2, ..., n,$

$$P_1 = P(X_1 + X_2 + \cdots + X_r = y_1, ..., X_{n-r+1} + X_{n-r+2} + \cdots + X_n = y_{n-r+1}).$$

We use conditional probability

$$P_1 = \sum_{t_1=0}^{1} ... \sum_{t_n=0}^{1} P(X_1 + \cdots + X_r = y_1, ..., X_{n-r+1} + \cdots + X_n = y_{n-r+1} \,| X_1 = t_1, ..., X_n = t_n) \\ \times P(X_1 = t_1, ..., X_n = t_n),$$

which sum on t_i, $i = 1, 2, ..., n$ should follows conditions below

$$t_i + t_{i+1} + \cdots + t_{i+r-1} = y_i, \quad \forall i = 1, 2, ..., n-r+1, \tag{2}$$

so despite these conditions,

$$P_{Y_1,Y_2,...,Y_{n-r+1}}(y_1, y_2, ..., y_{n-r+1}) = \sum_{t_1=0}^{1} \sum_{t_2=0}^{1} ... \sum_{t_n=0}^{1} P(X_1 = t_1, X_2 = t_2, ..., X_n = t_n),$$

it has two states that can be considered as follows.

State 1: We start condition (2) such that $X_1, X_2, ..., X_{r-1}$, is equaled to their sample values. Then X_r is replaced with a function of these sample values, and then $X_{r+1}, X_{r+2}, ..., X_n$ is obtained as a function of the previous sample values.

$$P_1 = \sum_{t_1=0}^{1} ... \sum_{t_n=0}^{1} P(X_1 = t_1, ..., X_{r-1} = t_{r-1}, X_r = y_1 - (t_1 + t_2 + \cdots + t_{r-1}), X_{r+1} = y_2 - y_1 \\ + t_1, ..., X_n = y_{n-r+1} - y_{n-r} + t_{n-r}).$$

We use the independence of $X_i, \ \ i = 1, 2, ..., n$,

$$
\begin{aligned}
P_1 = & \sum_{t_1=0}^{1} ... \sum_{t_{r-1}=0}^{1} \ \sum_{y_1-(t_1+t_2+\cdots+t_{r-1})=0}^{1} \ \sum_{y_2-y_1+t_1=0}^{1} ... \sum_{y_{n-r+1}-y_{n-r}+t_{n-r}=0}^{1} P(X_1 = t_1)... \\
& \times P(X_{r-1} = t_{r-1}) P(X_r = y_1 - (t_1 + t_2 + \cdots + t_{r-1})) P(X_{r+1} = y_2 - y_1 + t_1)... \\
& \times P(X_n = y_{n-r+1} - y_{n-r} + t_{n-r}),
\end{aligned}
$$

and according to t_i,

$$
\begin{aligned}
P_1 = & \sum_{t_1=0}^{1} ... \sum_{t_{r-1}=0}^{1} \ \sum_{(t_1+t_2+\cdots+t_{r-1})=y_1-1}^{y_1} \ \sum_{t_1=y_1-y_2}^{y_1-y_2+1} ... \sum_{t_{n-r}=y_{n-r}-y_{n-r+1}}^{y_{n-r}-y_{n-r+1}+1} P(X_1 = t_1)... \\
& \times P(X_{r-1} = t_{r-1}) P(X_r = y_1 - (t_1 + \cdots + t_{r-1})) P(X_{r+1} = y_2 - y_1 + t_1)... \\
& \times P(X_n = y_{n-r+1} - y_{n-r} + t_{n-r}),
\end{aligned}
$$

we put the X_i distribution value, for $i = 1, 2, ..., n$,

$$
\begin{aligned}
P_1 = & \sum_{t_1=0}^{1} ... \sum_{t_{r-1}=0}^{1} \ \sum_{(t_1+\cdots+t_{r-1})=y_1-1}^{y_1} \ \sum_{t_1=y_1-y_2}^{y_1-y_2+1} .. \sum_{t_{n-r}=y_{n-r}-y_{n-r+1}}^{y_{n-r}-y_{n-r+1}+1} \left[p^{t_1} (1-p)^{1-t_1} \right] ... \\
& \times \left[p^{t_{r-1}} (1-p)^{1-t_{r-1}} \right] \left[p^{y_1-(t_1+t_2+\cdots+t_{r-1})} (1-p)^{1-y_1+(t_1+t_2+\cdots+t_{r-1})} \right] ... \\
& \times \left[p^{y_{n-r+1}-y_{n-r}+t_{n-r}} (1-p)^{1-y_{n-r+1}+y_{n-r}-t_{n-r}} \right].
\end{aligned}
$$

So,

$$
P_1 = \sum_{t_1=0}^{1} ... \sum_{t_{r-1}=0}^{1} \ \sum_{(t_1+t_2+\cdots+t_{r-1})=y_1-1}^{y_1} \ \sum_{t_1=y_1-y_2}^{y_1-y_2+1} ... \sum_{t_{n-r}=y_{n-r}-y_{n-r+1}}^{y_{n-r}-y_{n-r+1}+1} \left[p^{\sum_{i=1}^{n} t_i} (1-p)^{n-\sum_{i=1}^{n} t_i} \right].
$$

State 2: We start condition (2) such that $X_n, X_{n-1}, ..., X_{n-r+2}$, is equaled to their sample values, on the contrary trend State 1. Then X_{n-r+1} is replaced with a function of these sample values, and then $X_{n-r}, X_{n-r-1}, ..., X_1$ is obtained as a function of the previous sample values.

$$
\begin{aligned}
P_1 = & \sum_{t_1=0}^{1} ... \sum_{t_n=0}^{1} P(X_n = t_n, ..., X_{n-r+2} = t_{n-r+2}, X_{n-r+1} = y_{n-r+1} - (t_n + t_{n-1} + \cdots \\
& + t_{n-r+2}), X_{n-r} = y_{n-r} - y_{n-r+1} + t_n, ..., X_1 = y_1 - y_2 + t_{r+1}).
\end{aligned}
$$

We use the independence of $X_i, \ \ i = 1, 2, ..., n$,

$$P_1=\sum_{t_n=0}^{1}\cdots\sum_{t_{n-r+2}=0}^{1}\sum_{y_{n-r+1}-(t_{n-r+2}+t_{n-r+3}+\cdots+t_n)=0}^{1}\sum_{y_{n-r}-y_{n-r+1}+t_n=0}^{1}\cdots\sum_{y_1-y_2+t_{r+1}=0}^{1}P(X_n=t_n)...$$
$$\times P(X_{n-r+2}=t_{n-r+2})P(X_{n-r+1}=y_{n-r+1}-(t_{n-r+2}+t_{n-r+3}+\cdots+t_n))$$
$$\times P(X_{n-r}=y_{n-r}-y_{n-r+1}+t_n)...P(X_1=y_1-y_2+t_{r+1}),$$

and according to t_i,

$$P_1=\sum_{t_n=0}^{1}\cdots\sum_{t_{n-r+2}=0}^{1}\sum_{(t_{n-r+2}+t_{n-r+3}+\cdots+t_n)=y_{n-r+1}-1}^{y_{n-r+1}}\sum_{t_n=y_{n-r+1}-y_{n-r}}^{y_{n-r+1}-y_{n-r}+1}\cdots\sum_{t_{r+1}=y_2-y_1}^{y_2-y_1+1}P(X_n=t_n)...$$
$$\times P(X_{n-r+2}=t_{n-r+2})P(X_{n-r+1}=y_{n-r+1}-(t_{n-r+2}+t_{n-r+3}+\cdots+t_n))$$
$$\times P(X_{n-r}=y_{n-r}-y_{n-r+1}+t_n)...P(X_1=y_1-y_2+t_{r+1}),$$

we put the X_i distribution value, for $i=1,2,...,n$,

$$P_1=\sum_{t_n=0}^{1}\cdots\sum_{t_{n-r+2}=0}^{1}\sum_{(t_{n-r+2}+t_{n-r+3}+\cdots+t_n)=y_{n-r+1}-1}^{y_{n-r+1}}\sum_{t_n=y_{n-r+1}-y_{n-r}}^{y_{n-r+1}-y_{n-r}+1}\cdots\sum_{t_{r+1}=y_2-y_1}^{y_2-y_1+1}\left[p^{t_n}(1-p)^{1-t_n}\right]\cdots$$
$$\times\left[p^{t_{n-r+2}}(1-p)^{1-t_{n-r+2}}\right]\left[p^{y_{n-r+1}-(t_{n-r+2}+t_{n-r+3}+\cdots+t_n)}(1-p)^{1-y_{n-r+1}+(t_{n-r+2}+t_{n-r+3}+\cdots+t_n)}\right]$$
$$\times\left[p^{y_{n-r}-y_{n-r+1}+t_n}(1-p)^{1-y_{n-r}+y_{n-r+1}-t_n}\right]\cdots\left[p^{y_1-y_2+t_{r+1}}(1-p)^{1-y_1+y_2-t_{r+1}}\right].$$

So,

$$P_1=\sum_{t_n=0}^{1}\cdots\sum_{t_{n-r+2}=0}^{1}\sum_{(t_{n-r+2}+t_{n-r+3}+\cdots+t_n)=y_{n-r+1}-1}^{y_{n-r+1}}\sum_{t_n=y_{n-r+1}-y_{n-r}}^{y_{n-r+1}-y_{n-r}+1}\cdots\sum_{t_{r+1}=y_2-y_1}^{y_2-y_1+1}\left[p^{\sum_{i=1}^{n}t_i}(1-p)^{n-\sum_{i=1}^{n}t_i}\right].$$

Other states are similar to these two states and do not create a new structure, and our response for joint distribution $(Y_1,Y_2,...,Y_{n-r+1})$ is subscription state 1 and state 2 such that leads to the following

$$P_{Y_1,Y_2,...,Y_{n-r+1}}(y_1,y_2,...,y_{n-r+1})=\sum_{C}p^{\sum_{i=1}^{n}t_i}(1-p)^{n-\sum_{i=1}^{n}t_i}, \tag{3}$$

which

$$C=\{(t_1,t_2,...,t_n)|\text{Algorithm } 1\},$$

Algorithm 1.

1) $\forall i, \quad t_i \in \{0,1\}$,

2) $t_1 \in \{y_1 - y_2, y_1 - y_2 + 1\}$,
$t_2 \in \{y_2 - y_3, y_2 - y_3 + 1\}, ...,$
$t_r \in \{y_r - y_{r+1}, y_r - y_{r+1} + 1\}$,

3) $t_{n-r+1} \in \{y_{n-2r+2} - y_{n-2r+1}, y_{n-2r+2} - y_{n-2r+1} + 1\}$,
$t_{n-r+2} \in \{y_{n-2r+3} - y_{n-2r+2}, y_{n-2r+3} - y_{n-2r+2} + 1\}, ...,$
$t_n \in \{y_{n-r+1} - y_{n-r}, y_{n-r+1} - y_{n-r} + 1\}$,

4) $t_{r+1} \in \{\{y_{r+1} - y_{r+2}, y_{r+1} - y_{r+2} + 1\} \cap \{y_2 - y_1, y_2 - y_1 + 1\}\}$,
$t_{r+2} \in \{\{y_{r+2} - y_{r+3}, y_{r+2} - y_{r+3} + 1\} \cap \{y_3 - y_2, y_3 - y_2 + 1\}\}, ...,$
$t_{n-r} \in \{\{y_{n-r} - y_{n-r+1}, y_{n-r} - y_{n-r+1} + 1\} \cap \{y_{n-2r+1} - y_{n-2r}, y_{n-2r+1} - y_{n-2r} + 1\}\}$,

5) $t_i + t_{i+1} + \cdots + t_{i+r-1} = y_i; \quad \forall i = 1, 2, ..., n - r + 1.$

Algorithm 1 provides different vectors $\mathbf{t} = (t_1, t_2, ..., t_n)$ related to k and n, and resulted vectors $\mathbf{t}$ lie on the (3) and completes proof. ■

Now, the system will be active if and only if for each $Y_i,\ i = 1, 2, ..., n - r + 1$ at least there exist $r - k + 1$ active components. So, the reliability function equals to

$$R_{n,r,k,p} = P_{Y_1,Y_2,...,Y_{n-r+1}}(\forall i = 1, 2, ..., n-r+1; \quad y_i \geq r-k+1) = P_{Y_1,Y_2,...,Y_{n-r+1}}(\forall i; \quad y_i > r-k). \quad (4)$$

This reliability can be calculated exactly using Theorem (2.1).

Note that if $k = r$, the system will be changed to consecutive k-out-of-n: F system with same structure such that its reliability function as a special case (4) is as

$$R_{n,r,k,p} = P_{Y_1,Y_2,...,Y_{n-r+1}}(\forall i; \quad y_i > 0).$$

Also, if $r = n$, the system will be changed to k-out-of-n: F system which its reliability function again is as

$$R_{n,r,k,p} = P_{Y_1}(y_1 \geq n - k + 1) = P(\sum_{i=1}^{n} X_i \geq n - k + 1),$$

which it is usual and known reliability function in the literature.

3. Exact Reliability with Unequal Components Probabilities

In this section with the another point of view which may be realistic in applications, we consider the exact reliability of the system. Indeed we assume that components of the system have unequal probability to active. This condition is realistic than equal probability component since each of the components may be under certain conditions. For example consider waves transfer stations which are under different circumstances. Finally, we bring up following theorem similar to Theorem (2.1).

Theorem 3.1.

Let $X_i \sim B(p_i)$; for all $i = 1, 2, ...n$, is the system component with unequal probabilities, and Y_i for $i = 1, 2, ..., n-r+1$, is defined as (1) which Y_is are dependent, but not identically distributed. The joint distribution of $(Y_1, Y_2, ..., Y_{n-r+1})$ is

$$P_{Y_1,Y_2,...,Y_{n-r+1}}(y_1, y_2, ..., y_{n-r+1}) = \sum_C \left[\prod_{i=1}^{n} \left[p_i^{t_i}(1-p_i)^{1-t_i}\right]\right],$$

which operator C is same as previous in Theorem (2.1).

Proof:

Again we consider two states for proof.

State 1: The steps of this state is exactly similar to State 1 in Theorem (2.1) such that only unequal probabilities components replace, so

$$P_2 = P(Y_1 = y_1, Y_2 = y_2, ..., Y_{n-r+1} = y_{n-r+1}) = P_{Y_1,Y_2,...,Y_{n-r+1}}(y_1, y_2, ..., y_{n-r+1}).$$

We put the X_i distribution value, for $i = 1, 2, ..., n$,

$$P_2 = \sum_{t_1=0}^{1} \cdots \sum_{t_{r-1}=0}^{1} \sum_{(t_1+\cdots+t_{r-1})=y_1-1}^{y_1} \sum_{t_1=y_1-y_2}^{y_1-y_2+1} \cdots \sum_{t_{n-r}=y_{n-r}-y_{n-r+1}}^{y_{n-r}-y_{n-r+1}+1} \left[p_1^{t_1}(1-p_1)^{1-t_1}\right] \cdots \left[p_n^{t_n}(1-p_n)^{1-t_n}\right],$$

so,

$$P_2 = \sum_{t_1=0}^{1} \cdots \sum_{t_{r-1}=0}^{1} \sum_{(t_1+t_2+\cdots+t_{r-1})=y_1-1}^{y_1} \sum_{t_1=y_1-y_2}^{y_1-y_2+1} \cdots \sum_{t_{n-r}=y_{n-r}-y_{n-r+1}}^{y_{n-r}-y_{n-r+1}+1} \left[\prod_{i=1}^{n} \left[p_i^{t_i}(1-p_i)^{1-t_i}\right]\right].$$

State 2: Again this state unequal probabilities components replace, so

$$P_2 = P(Y_1 = y_1, Y_2 = y_2, ..., Y_{n-r+1} = y_{n-r+1}) = P_{Y_1,Y_2,...,Y_{n-r+1}}(y_1, y_2, ..., y_{n-r+1}).$$

We put the X_i distribution value, for $i = 1, 2, ..., n$,

$$P_2 = \sum_{t_n=0}^{1} \cdots \sum_{t_{n-r+2}=0}^{1} \sum_{(t_{n-r+2}+t_{n-r+3}+\cdots+t_n)=y_{n-r+1}-1}^{y_{n-r+1}} \sum_{t_n=y_{n-r+1}-y_{n-r}}^{y_{n-r+1}-y_{n-r}+1} \cdots \sum_{t_{r+1}=y_2-y_1}^{y_2-y_1+1} \left[p_1^{t_1}(1-p_1)^{1-t_1}\right] \cdots \times \left[p_n^{t_n}(1-p_n)^{1-t_n}\right],$$

so,

$$P_2 = \sum_{t_n=0}^{1} \cdots \sum_{t_{n-r+2}=0}^{1} \sum_{(t_{n-r+2}+\cdots+t_n)=y_{n-r+1}-1}^{y_{n-r+1}} \sum_{t_n=y_{n-r+1}-y_{n-r}}^{y_{n-r+1}-y_{n-r}+1} \cdots \sum_{t_{r+1}=y_2-y_1}^{y_2-y_1+1} \left[\prod_{i=1}^{n} \left[p_i^{t_i} (1-p_i)^{1-t_i} \right] \right].$$

Other states are similar to these two states and do not create a new structure, and our response for joint distribution $(Y_1, Y_2, ..., Y_{n-r+1})$ is subscription state 1 and state 2 such that leads to the following

$$P_{Y_1,Y_2,...,Y_{n-r+1}}(y_1, y_2, ..., y_{n-r+1}) = \sum_{C} \left[\prod_{i=1}^{n} \left[p_i^{t_i} (1-p_i)^{1-t_i} \right] \right].$$

Note that with replacement of this joint probability distribution, we can obtain exact reliability formula similar to (4). ■

4. Numerical Results

This section set out to the numerical results to obtain the exact reliability in equal and unequal component probability conditions. Indeed, the numerical results have been applied to show performance and efficiency of the obtained results in the previous sections. In the equal component probability condition, the exact reliability is calculated for fixed n, r and p and different k in Tables 2 and 3. Also the exact reliability is calculated for fixed n, k and p and different r in Tables 4 and 5. Then the exact reliability is calculated for fixed n, r and k and different p in Tables 6, 7 and 8 . After that the exact reliability is calculated for fixed r, k and p and different n in Tables 9,10 and 11. Also the exact reliability is calculated for different n, r, k and p Table in 12. In the following ,the reliability graph against k is plotted to see k changes and is reported in Figure 1. The reliability graph against r is plotted to see r changes and is reported in Figure 2. Then the reliability graph against p is plotted to see p changes and is reported in Figure 3. Finally the reliability graph against n is plotted to see n changes and is reported in Figure 4. Also, the exact reliability with unequal component probability is calculated for different n, r, k and p vector, which is realistic than equal component probability. The resulted calculations is reported in Tables 13 and 14 such that components probability is chosen randomly. In the following, a brief comparison was made between the results and the results of the previous authors, and it is reported in Table 15. If interested readers wish to programming codes, the authors can provide them which written in MATLAB R2104a.

5. Application

In this section we apply obtained results in the present paper for two real-world examples.

Example 5.1.

This real-world application is based on the process of Safiran Chemical Paint Company plant in Kermanshah city, Islamic Republic of Iran. In the producing line, filling machine is configured such that when the box is placed under it, 1 Liter (L) paint is poured in the first box and then another boxes fill in the same way. Sometimes cutting tool of the machine causes problem and does not perform well ($1 - p = 0.05$), and 2 L paint is poured, but the capacity of each box is just 1.25 L. So, the remaining paint back to the tank. This situation occurs because 0.25 L extra capacity is provided for each box to mix with other materials in application. Then the boxes are closed by automatic machine, and for the final packaging stay on strap consecutively. The strap is designed such that can withstand up to 10 L, and at any moment 9 boxes is moving on it ($r = 9$). The strap stops when at least 5 boxes have over limit weight ($k = 5$). The company fills 100 boxes in each shift ($n = 100$). Based on the reliability function (4), $R_{100,9,5,0.95} = 0.998491$, and this is reliability function for consecutive 5-out-of-9-from-100: F system.

Example 5.2.

Suppose there are 12 electricity transfer stations between the two cities(A and B) which transfer electricity from A to B. These stations are in two different situations:

1)They are damaged which do not amplify the current, and only transmit part of it.

2)They are intact, which in addition to its current is completely strengthened.

To transmit current from A to B, at least two stations should be intact of each three alternative stations. The studies have shown that the probability that the station is damaged is 0.01; in fact, this equates to:

$$2 - out - of - 3 - from - 12 : \ F\, system,$$

which is at each station in two possible situations: damaged and intact, and component probability is 0.99.

The reliability of the system that transfer the current to B city is equal to

$$R_{12,3,2,0.99} = 0.9979465.$$

6. Conclusion

In the present article, we have tried to obtain the exact reliability, unlike existing research, for consecutive k-out-of-r-from n: F system in the equal and unequal components probability. In fact, with the help of presented pattern in this paper the exact and closed form reliability for such a system has been achieved that involves many sub-systems. Also, an algorithm to obtain possible values of dependent r consecutive is suggested. The method proposed in this paper can apply simply, and numerical results showed the efficiency and high performance of the proposed method. Finally, we wish to extend our method to obtain the exact reliability for multistate consecutive k-out-of-r-from n: F system.

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Table 2. Exact reliability for $n = 12$, $r = 5$, and $p = 0.95$.

k	1	2	3	4	5
$R_{n,r,k,p}$	.54036	.92387	.99447	.99981	1.00000

Table 3. Exact reliability for $n = 13$, $r = 9$, and $p = 0.9$.

k	1	2	3	4	5	6	7	8	9
$R_{n,r,k,p}$	.25419	.65273	.89827	.98059	.99754	.99979	.99999	1.00000	1.00000

Table 4. Exact reliability for $n = 11$, $k = 3$, and $p = 0.85$.

r	3	4	5	6	7	8	9	10	11
$R_{n,r,k,p}$	.97381	.93959	.90542	.87429	.84643	.82209	.80197	.78709	.77881

Table 5. Exact reliability for $n = 14$, $k = 4$, and $p = 0.9$.

r	5	6	7	8	9	10	11	12	13	14
$R_{n,r,k,p}$	.99670	.99317	.98868	.98352	.97799	.97236	.96692	.96204	.95817	.95587

Table 6. Exact reliability for $n = 13$, $r = 6$, and $k = 3$.

p	0.25	0.5	0.75	0.8	0.9	0.95	0.99
$R_{n,r,k,p}$	.00038	.06079	.56372	.71270	.94208	.99093	.99991

Table 7. Exact reliability for $n = 12$, $r = 9$, and $k = 3$.

p	0.25	0.5	0.75	0.8	0.9	0.95	0.99
$R_{n,r,k,p}$	.00015	.03345	.45403	.61579	.90953	.98455	.99984

Table 8. Exact reliability for $n = 11$, $r = 5$, and $k = 2$.

p	0.25	0.5	0.75	0.8	0.9	0.95	0.99
$R_{n,r,k,p}$	.00006	.01660	.29721	.43621	.77915	.93128	.99673

Table 9. Exact reliability for $r = 8$, $k = 4$, and $p = 0.9$.

n	8	9	10	11	12	13	14	15
$R_{n,r,k,p}$	.99498	.99291	.99096	.98907	.98721	.98536	.98352	.98168

Table 10. Exact reliability for $r = 10$, $k = 4$, and $p = 0.9$.

n	10	11	12	13	14	15	16	17
$R_{n,r,k,p}$	.98720	.98319	.97944	.97585	.97236	.96892	.96553	.96215

Table 11. Exact reliability for $r = 6$, $k = 3$, and $p = 0.85$.

n	6	7	8	9	10	11	12	13	14
$R_{n,r,k,p}$	.95266	.93504	.91901	.90377	.88893	.87429	.85974	.84537	.83124

Table 12. Exact reliability for equal component probabilities for different n, r, k and p.

n	r	k	p_1	R_{n,r,k,p_1}	p_2	R_{n,r,k,p_2}	p_3	R_{n,r,k,p_3}
6	4	3	0.8	0.94310	0.9	0.99152	0.95	0.99885
6	3	2	0.8	0.75366	0.9	0.92510	0.95	0.97944
8	4	3	0.8	0.91488	0.9	0.98684	0.95	0.99818
8	6	3	0.8	0.83952	0.9	0.97142	0.95	0.99576
10	5	4	0.8	0.97376	0.9	0.99796	0.95	0.99986
11	7	4	0.8	0.91960	0.9	0.99229	0.95	0.99941
11	8	3	0.8	0.68115	0.9	0.93053	0.95	0.98861
12	4	2	0.8	0.45097	0.9	0.79227	0.95	0.93710
12	8	4	0.8	0.87930	0.9	0.98721	0.95	0.99897
13	9	6	0.8	0.99149	0.9	0.99979	0.95	1.00000
13	5	3	0.8	0.77414	0.9	0.95866	0.95	0.99385
14	4	2	0.8	0.38844	0.9	0.75771	0.95	0.92531
14	12	3	0.8	0.47159	0.9	0.85234	0.95	0.97237
15	5	2	0.8	0.30731	0.9	0.69873	0.95	0.90200
15	11	6	0.8	0.97320	0.9	0.99920	0.95	0.99998

Table 13. Exact reliability for unequal component probabilities for different n, r, k and p vector.

n	r	k	p	$R_{n,r,k,p}$
10	7	2	$p = \{0.8, 0.9, 0.9, 0.95, 0.9, 0.8, 0.7, 0.8, 0.9, 0.9\}$	0.59146
12	6	4	$p = \{0.8, 0.95, 0.9, 0.95, 0.9, 0.8, 0.8, 0.8, 0.95, 0.95, 0.95, 0.95\}$	0.99164
6	4	2	$p = \{0.85, 0.8, 0.95, 0.9, 0.95, 0.9\}$	0.90001
9	3	2	$p = \{0.95, 0.9, 0.85, 0.8, 0.8, 0.95, 0.95, 0.95, 0.8\}$	0.84227
13	6	3	$\begin{cases} p_1 = p_2 = p_5 = p_6 = p_9 = p_{10} = p_{13} = 0.9 \\ p_3 = p_4 = p_7 = p_8 = p_{11} = p_{12} = 0.8 \end{cases}$	0.84920
14	11	8	$\begin{cases} p_1 = p_2 = p_5 = p_6 = p_9 = p_{10} = p_{13} = p_{14} = 0.9 \\ p_3 = p_4 = p_7 = p_8 = p_{11} = p_{12} = 0.8 \end{cases}$	0.99999
12	10	7	$\begin{cases} p_1 = p_2 = p_5 = p_6 = p_9 = p_{10} = 0.9 \\ p_3 = p_4 = p_7 = p_8 = p_{11} = p_{12} = 0.8 \end{cases}$	0.99975
11	6	3	$\begin{cases} p_1 = p_2 = p_5 = p_6 = p_9 = p_{10} = 0.9 \\ p_3 = p_4 = p_7 = p_8 = p_{11} = 0.8 \end{cases}$	0.88227
10	5	3	$p_i = 0.95 \quad i \neq 8 \quad \& \ p_8 = 0.8$	0.99205
14	12	8	$p_i = 0.95 \quad i \neq 8 \quad \& \ p_8 = 0.8$	1.00000
13	4	3	$p_i = 0.95 \quad i \neq 8 \quad \& \ p_8 = 0.8$	0.99365
12	7	5	$p_i = 0.95 \quad i \neq 8 \quad \& \ p_8 = 0.8$	0.99993
9	7	4	p_i $is\ random\ \&\ 0.8 \leq p_i \leq 1$	0.98675
12	6	3	p_i $is\ random\ \&\ 0.8 \leq p_i \leq 1$	0.90991
14	8	5	p_i $is\ random\ \&\ 0.8 \leq p_i \leq 1$	0.99826
15	12	3	p_i $is\ random\ \&\ 0.8 \leq p_i \leq 1$	0.85637

Table 14. Exact reliability for unequal component probabilities for different n, r, k and p vector.

n	r	k	p	$R_{n,r,k,p}$
10	8	5	$p_i = 0.9 \quad i \neq 3, 6 \quad \& \ p_3 = 0.75 \ \& \ p_6 = 0.99$	0.99914
12	9	4	$p_i = 0.9 \quad i \neq 3, 6 \quad \& \ p_3 = 0.75 \ \& \ p_6 = 0.99$	0.98299
13	9	5	$p_i = 0.9 \quad i \neq 3, 6 \quad \& \ p_3 = 0.75 \ \& \ p_6 = 0.99$	0.99785
14	9	6	$p_i = 0.9 \quad i \neq 3, 6 \quad \& \ p_3 = 0.75 \ \& \ p_6 = 0.99$	0.99983
15	11	4	$p_{odd} = 0.95 \ \& \ p_{even} = 0.9$	0.98658
14	10	4	$p_{odd} = 0.95 \ \& \ p_{even} = 0.9$	0.99017
12	8	6	$p_{odd} = 0.95 \ \& \ p_{even} = 0.9$	1.00000
11	6	4	$p_{odd} = 0.95 \ \& \ p_{even} = 0.9$	0.99859
15	11	8	$p_1 = 0.99 \ \& \ p_i = p_{i-1} - 0.01 \quad \forall i = 2, 3, ..., n$	1.00000
14	8	4	$p_1 = 0.99 \ \& \ p_i = p_{i-1} - 0.01 \quad \forall i = 2, 3, ..., n$	0.99166
13	9	5	$p_1 = 0.99 \ \& \ p_i = p_{i-1} - 0.01 \quad \forall i = 2, 3, ..., n$	0.99936
12	5	3	$p_1 = 0.99 \ \& \ p_i = p_{i-1} - 0.01 \quad \forall i = 2, 3, ..., n$	0.98447

Table 15. Comparison exact reliability for equal component probabilities for different k, r, and n, with the results obtained by Sfakianakis et. al., 1992 * and Habib and Szantai, 2000 **.

n	r	k	p	$R_{n,r,k,p}$	$1-R_{n,r,k,p}$	R_{old}**	$1-R_{old}$*
10	7	2	0.25	0.00008	0.99992		$0.993 \leq R \leq 1.000$
10	7	5	0.5	0.62207	0.37793		$0.364 \leq R \leq 0.393$
10	7	5	0.75	0.96979	0.03021		$0.030 \leq R \leq 0.033$
15	7	5	0.75	0.94295	0.05705		$0.047 \leq R \leq 0.068$
15	7	5	0.75	0.942955		$0.942484 \leq R \leq 0.943516$	
15	10	4	0.75	0.60546	0.39454		$0.367 \leq R \leq 0.471$
15	10	4	0.75	0.605462		$0.603743 \leq R \leq 0.606257$	
15	10	5	0.75	0.83148	0.16852		$0.152 \leq R \leq 0.196$
15	10	6	0.75	0.94857	0.05143		$0.045 \leq R \leq 0.058$
15	10	7	0.5	0.66620	0.33380		$0.321 \leq R \leq 0.421$
15	12	9	0.25	0.23227	0.76773		$0.745 \leq R \leq 0.781$

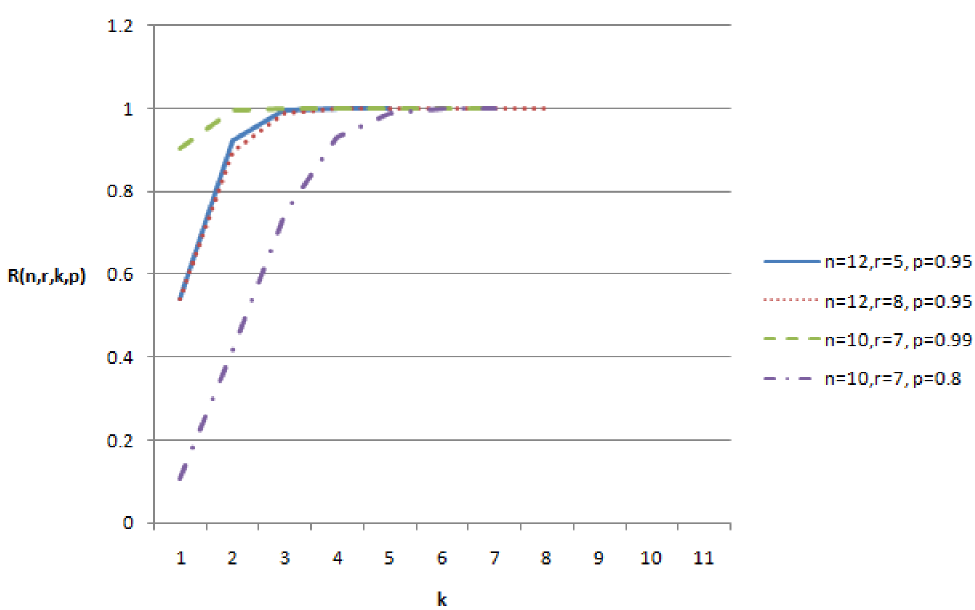

Figure 1. Reliability graph versus k

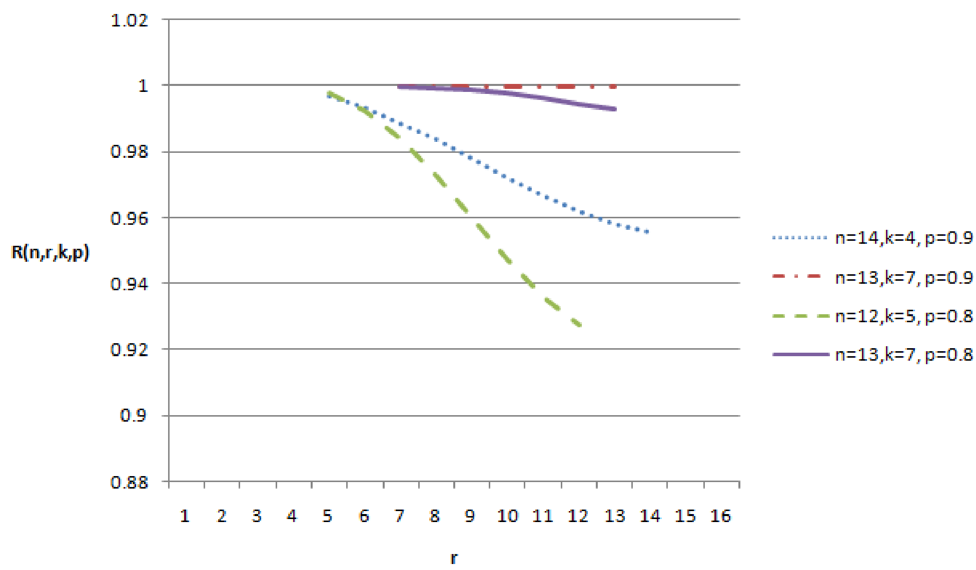

Figure 2. Reliability graph versus r

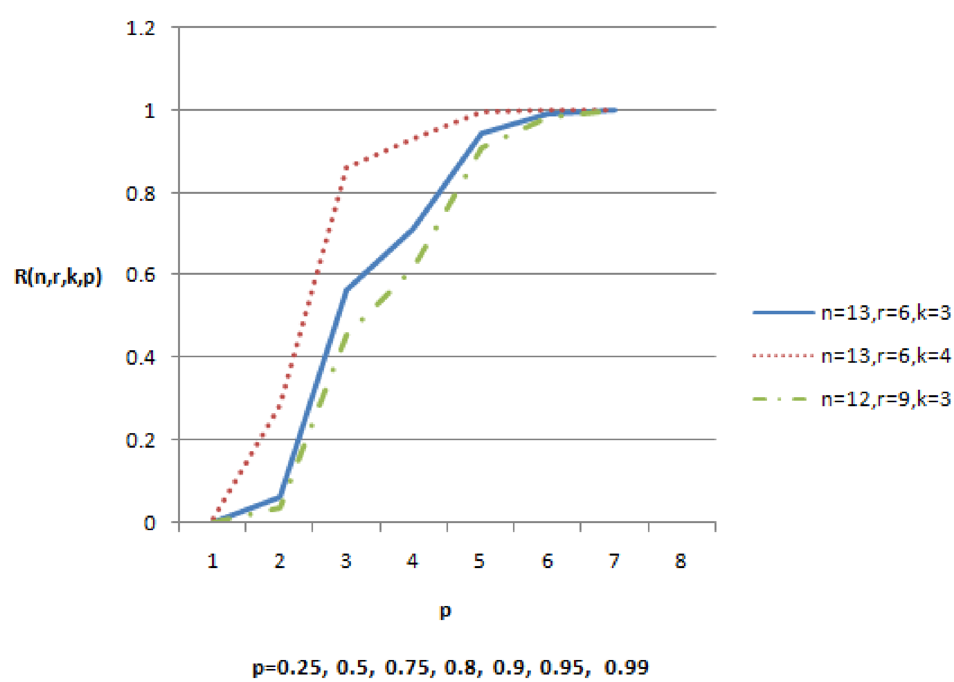

Figure 3. Reliability graph versus p

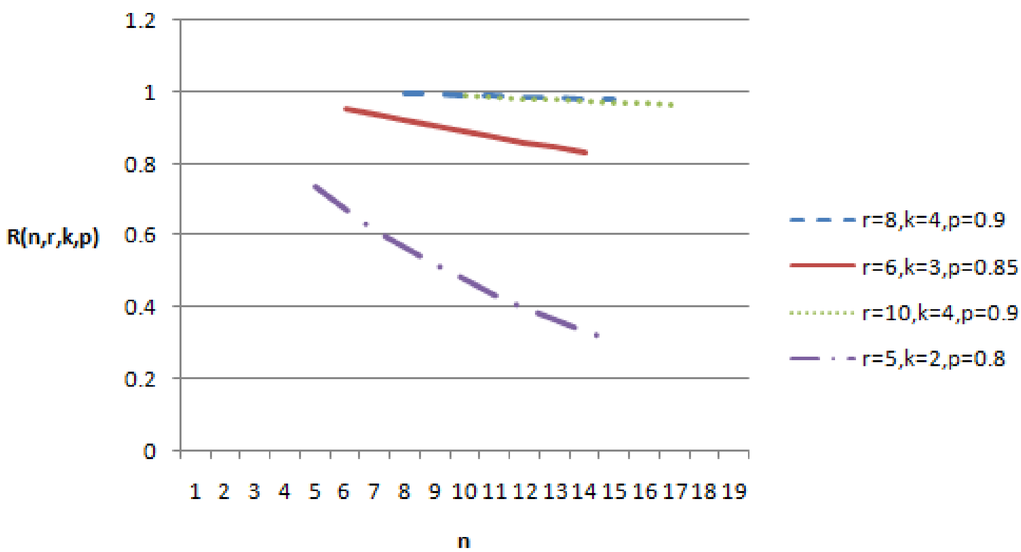

Figure 4. Reliability graph versus n