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Transient analysis of a Markovian Single Vacation Feedback Queue with an Interrupted Closedown Time and Control of Admission During Vacation

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Abstract

This paper analyzes the transient behavior of an *M/M/*1 queueing model with single vacation, feedback, interrupted closedown time and control of admission during vacation. The time-dependent system size probabilities for the proposed model are obtained using generating function in the closed form. Further, the system performance measures like mean and variance of system size are also obtained for the time-dependent case. Finally, numerical illustrations are presented to understand the effect for various system parameters.

Keywords: The *M*/*M*/1 queue; Single vacation; Feedback; Interrupted closedown time; Control of admission during vacation; Transient probabilities

MSC 2010 No.: 60K25, 90B22, 68M20

1. Introduction

During the past few decades, many researchers carried out works related to queues with server on vacations. In a vacation queue, the server stops serving the customers completely during the entire

vacation period. In a single vacation policy, after the completion of vacation, the server stays idle and waits for the arrivals if no customer is waiting in the queue (refer to Altman and Yechiali (2006), Doshi (1986), Sudhesh and Azhagappan (2016), Sudhesh et al. (2017), Tian et al. (2008)). If there is at least one customer waiting in the queue at the vacation completion instant, the server begins the service.

Closedown the system when it becomes empty performs major role in various real time situations as it helps to minimize the expenses of a system. Only few research works are carried out in literature related to Markovian queueing models with closedown times under transient case (refer Kumar et al. (2015)). After the completion of service, customers either leave the system or may rejoin (feedback) the queue for another service (refer to Choi and Kim (2003), Takacs (1963)). The arriving customers are either permitted to join the queue or rejected during the vacations (refer to Artalejo et al. (2005), Choudhury and Madan (2007)).

Doshi (1986) treats the situations that the server on secondary work as vacation where the server never provides any service to the primary customers waiting in the queue. Altman and Yechiali (2006) presented a comprehensive analysis of the impatient behavior of single server queues for both multiple and single vacation cases and obtained various closed form results. Sudhesh and Azhagappan (2016) investigated the impatience of customers in an $M/M/1$ vacation queueing model where the server waits dormant in the system for certain period after returning from vacation.

Sudhesh et al. (2017) investigated a Markovian single server queue in which the server starts working vacation when the system becomes empty and the impatience of customers is due to the vacation of the server. They obtained the time-dependent system size distributions, average and variance of size of the system. Azhagappan et al. (2017) derived the steady state solution for an $M/M/1$ queueing model where the system undergoes disastrous breakdown and the customers have balking behavior. Using matrix geometric method, the steady-state system size and virtual time distributions of the $M/M/1$ queueing model with single working vacations were obtained by Tian et al. (2008). They also computed the average busy period and stochastic decomposition structures of this model. Kumar et al. (2015) considered a Markovian queueing model where the server undergoes closedown and then maintenance work whenever the system becomes empty. They derived the transient solutions and some system measures such as asymptotic behavior of various system state probabilities, average system size, average workload, etc. Deepa and Azhagappan (2018) derived the probability generating function of queue size under steady state for an $M^{[X]}/G(a,b)/1$ queueing model with second optional service, closedown time and multiple vacations.

Takacs (1963) introduced the queue with feedback customers where the customers either join the queue for another service or leave the system after their service completion. He obtained the steady state queue size distribution and distribution function of sojourn time of a customer in the system. Choi and Kim (2003) analyzed a two phase vacation queueing model with feedback customers where the first phase is a batch service followed by a single service in the second phase. They obtained the steady state system size probabilities of that model.

Artalejo et al. (2005) considered a discrete-time single server batch arrival queueing model with retrial customers and a restriction to admit the arriving customers. They obtained the steady state system size distributions and a stochastic decomposition property for their model. Choudhury and Madan (2007) analyzed a batch arrival vacation queueing model with setup times and control over the arrivals. They derived the steady state distribution at a random and departure epoch.

This model has wide application in various real time situations such as closing the counter for tea break by a cashier in a bank with single cash counter, stopping the drilling or cutting works performed by a CNC machine before the commencement of maintenance period, etc. The rest of the paper is organized as follows. In section 2, the transient system size probabilities of the *M/M/*1 queueing model with single vacation, feedback, interrupted closedown time and control of admission during vacation are derived. In section 3, the time-dependent mean and variance of system size are obtained. In section 4, numerical illustrations are provided. In section 5, this research work is concluded with future scope.

2. Model Description

Consider an *M/M/1* queueing model with single vacation, feedback, interrupted closedown time and control of admission during vacation. The assumptions to derive the transient system size probabilities are given as follows:

- Arrival of customers follows a Poisson process with rate "λ" and the service time follows an exponential distribution with rate "μ".
- After the completion of service, customers may either leave the system with a probability "q" or rejoin the queue for another service with a probability "1-q".
- Whenever the system becomes empty, the single server starts an exponentially distributed closedown work at a rate "η". If any arrival occurs before the completion of closedown period, the closedown work is interrupted and the server begins an exhaustive busy period.
- After the completion of closedown period, the server resumes vacation at a rate "γ" which follows an exponential distribution. If no customer waits in the queue at moment of completion of vacation, the server stays idle and waits for the arrivals. Otherwise, the server resumes an exhaustive busy period.
- The arriving customers are either permitted to join the queue with a probability "σ" or rejected with a probability "1- σ" during vacation.
- Assume that inter-arrival time, service time, closedown time and vacation time are all independent. The service discipline is first-come first-served (FCFS).

Let $\{X(\mathrm{t}), \mathrm{t} \geq 0\}$ be the number of customers in the system and $J(t)$ be the status of the server at time t, which is defined as follows:

$$J(t) = \begin{cases} 0, & \textit{if the server is on vacation state}, \\ 1, & \textit{if the server is in busy state at time } t. \end{cases}$$

Then, $\{J(\mathrm{t}), X(\mathrm{t}), \mathrm{t} \geq 0\}$ is a continuous time Markov chain on the state space $S = \{C\} \cup \{j, n : j = 0 \, or \, 1;\ n = 0, 1, 2, ...\}$. Let

$$P_{j,n}(\mathrm{t}) = P\{J(\mathrm{t}) = j, \mathrm{X}(\mathrm{t}) = n : j = 0,1;\ n = 0,1,2,...\},$$

$$P_C(\mathrm{t}) = P\{Closedown\ period\}.$$

The probabilities $P_{j,n}(\mathrm{t}),\ j = 0,1;\ n = 0,1,2,...,$ satisfy the forward Kolmogorov equations as

$$P_C'(t) = -(\lambda + \eta)P_C(t) + q\mu P_{1,1}(t), \tag{2.1}$$

$$P_{1,0}'(t) = -\lambda P_{1,0}(t) + \gamma P_{0,0}(t), \tag{2.2}$$

$$P_{1,1}'(t) = -(\lambda + q\mu)P_{1,1}(t) + \lambda P_{1,0}(t) + \lambda P_C(t) + q\mu P_{1,2}(t) + \gamma P_{0,1}(t),\ n \geq 1, \tag{2.3}$$

$$P_{1,n}'(t) = -(\lambda + q\mu)P_{1,n}(t) + \lambda P_{1,n-1}(t) + q\mu P_{1,n+1}(t) + \gamma P_{0,n}(t),\ n \geq 2, \tag{2.4}$$

$$P_{0,0}'(t) = -(\sigma\lambda + \gamma)P_{0,0}(t) + \eta P_C(t), \tag{2.5}$$

$$P_{0,n}'(t) = -(\sigma\lambda + \gamma)P_{0,n}(t) + \sigma\lambda P_{0,n-1}(t),\ n \geq 1, \tag{2.6}$$

with $P_{0,0}(0) = 1$.

2.1. Transient solution

In this section, the time-dependent system size probabilities are derived for the model under consideration.

Theorem 1.

The probabilities $P_{1,n}(t)$, for $n = 1,\ 2,\ 3,\ ...$ are obtained from (2.3) and (2.4) in terms of modified Bessel function as

$$P_{1,n}(t) = \gamma \int_0^t \sum_{m=1}^{\infty} P_{0,m}(u) e^{-(\lambda+q\mu)(t-u)} \beta^{n-m} \left[I_{n-m}\left(\alpha\left(t-u\right)\right) - I_{n+m}\left(\alpha\left(t-u\right)\right) \right] du$$

$$+\lambda \int_0^t \left(P_{1,0}(u) + P_C(u)\right) e^{-(\lambda+q\mu)(t-u)} \beta^{n-1} \left[I_{n-1}\left(\alpha\left(t-u\right)\right) - I_{n+1}\left(\alpha\left(t-u\right)\right) \right] du, \tag{2.7}$$

where $P_{1,0}(t)$ is obtained from (2.2) as

$$P_{1,0}(t) = \gamma \int_0^t P_{0,0}(u) e^{-\lambda(t-u)} du, \tag{2.8}$$

$I_n(t)$ is the modified Bessel function of the first kind of order n,

$$\alpha = 2\sqrt{\lambda\mu} \text{ and } \beta = \sqrt{\lambda/\mu}.$$

Proof:

Define

$$G(z,t) = \sum_{n=1}^{\infty} P_{1,n}(t) z^n .$$

From (2.3) and (2.4), we get

$$\frac{\partial G(z,t)}{\partial t} = \left[-(\lambda + q\mu) + \lambda z + \frac{q\mu}{z} \right] G(z,t) + \gamma \sum_{n=1}^{\infty} P_{0,n}(t) z^n + \lambda z \left(P_{1,0}(u) + P_C(u) \right) - q\mu P_{1,1}(t).$$

Solving the above partial differential equation, we obtain

$$\begin{aligned} G(z,t) = \gamma \int_0^t \left[\sum_{m=1}^{\infty} P_{0,m}(u) z^m \right] e^{-(\lambda+q\mu)(t-u)} e^{\left(\lambda z + \frac{q\mu}{z}\right)(t-u)} du \\ + \lambda z \int_0^t \left(P_{1,0}(u) + P_C(u) \right) e^{-(\lambda+q\mu)(t-u)} e^{\left(\lambda z + \frac{q\mu}{z}\right)(t-u)} du \\ - q\mu \int_0^t P_{1,1}(u) e^{-(\lambda+q\mu)(t-u)} e^{\left(\lambda z + \frac{q\mu}{z}\right)(t-u)} du . \end{aligned} \tag{2.9}$$

Let us assume that

$$e^{\left(\lambda z + \frac{q\mu}{z}\right)t} = \sum_{n=-\infty}^{\infty} (\beta z)^n I_n(\alpha t). \tag{2.10}$$

Using (2.10) in (2.9) and comparing the coefficients of z^n on both sides, for $n \geq 1$, we get

$$\begin{aligned} P_{1,n}(t) = \gamma \int_0^t \sum_{m=1}^{\infty} P_{0,m}(u) e^{-(\lambda+q\mu)(t-u)} \beta^{n-m} I_{n-m}\left(\alpha(t-u)\right) du \\ + \lambda \int_0^t \left(P_{1,0}(u) + P_C(u) \right) e^{-(\lambda+q\mu)(t-u)} \beta^{n-1} I_{n-1}\left(\alpha(t-u)\right) du \\ - q\mu \int_0^t P_{1,1}(u) e^{-(\lambda+q\mu)(t-u)} \beta^n I_n\left(\alpha(t-u)\right) du. \end{aligned} \tag{2.11}$$

The above holds for $n \leq -1$. For n = 1, 2, 3, …, we have

$$0=\gamma\int_0^t\sum_{m=1}^{\infty}P_{0,m}(u)e^{-(\lambda+q\mu)(t-u)}\beta^{-n-m}I_{n+m}\left(\alpha\left(t-u\right)\right)du$$
$$+\lambda\int_0^t\left(P_{1,0}(u)+P_C(u)\right)e^{-(\lambda+q\mu)(t-u)}\beta^{-n-1}I_{n+1}\left(\alpha\left(t-u\right)\right)du$$
$$-q\mu\int_0^t P_{1,1}(u)e^{-(\lambda+q\mu)(t-u)}\beta^{-n}I_n\left(\alpha\left(t-u\right)\right)du. \tag{2.12}$$

From (2.11) and (2.12), we obtain the expression (2.7), for n = 1, 2, 3, …. Taking Laplace transforms of (2.2), we get

$$\overset{)}{P}_{1,0}(s)=\frac{\gamma}{s+\lambda}\overset{)}{P}_{0,0}(s). \tag{2.13}$$

On Laplace inversion (2.13), we get (2.8).

Theorem 2.

The probabilities $P_C(t),\ P_{0,0}(t),\ P_{0,n}(t), n\geq 1$ are obtained from (2.1), (2.5) and (2.6) respectively as

$$P_C(t)=q\mu e^{-(\lambda+\eta)t}*P_{1,1}(t), \tag{2.14}$$
$$P_{0,0}(t)=q\mu\eta e^{-(\lambda+\gamma)t}*e^{-(\lambda+\eta)t}*P_{1,1}(t)+e^{-(\lambda+\gamma)t}, \tag{2.15}$$
$$P_{0,n}(t)=\lambda^n e^{-(\lambda+\gamma)t}\frac{t^{n-1}}{(n-1)!}*q\mu\eta e^{-(\lambda+\gamma)t}*e^{-(\lambda+\eta)t}*P_{1,1}(t)+e^{-(\lambda+\gamma)t}, \tag{2.16}$$

where

$$P_{1,1}(t)=\gamma\sum_{m=1}^{\infty}\lambda^m e^{-(\lambda+\gamma)\mathrm{t}}\frac{t^m}{m!}*\beta^{1-m}e^{-(\lambda+q\mu)\mathrm{t}}\left[I_{m-1}\left(\alpha t\right)-I_{m+1}\left(\alpha t\right)\right]$$
$$+\lambda\gamma e^{-\lambda t}*e^{-(\lambda+\gamma)\mathrm{t}}\left[I_0\left(\alpha t\right)-I_2\left(\alpha t\right)\right]\sum_{k=0}^{\infty}\sum_{r=0}^{k}\binom{k}{r}\left(\eta\gamma q\mu\right)^{k-r}$$
$$\times\left\{\sum_{m=1}^{\infty}\lambda^m\beta^{1-m}e^{-(\lambda+\gamma)\mathrm{t}}\frac{t^m}{m!}*e^{-(\lambda+\eta)\mathrm{t}}*e^{-(\lambda+q\mu)\mathrm{t}}\left[I_{m-1}\left(\alpha t\right)-I_{m+1}\left(\alpha t\right)\right]\right\}^{*(\mathrm{k}-\mathrm{r})}$$
$$\times(\lambda q\mu)^r\sum_{j=0}^{r}\binom{r}{j}(\gamma\eta)^j e^{-\lambda t}\frac{t^{j-i}}{(\mathrm{j}-1)!}*e^{-(\lambda+\gamma)\mathrm{t}}\frac{t^{j-i}}{(\mathrm{j}-1)!}$$
$$*e^{-(\lambda+\eta)\mathrm{t}}\frac{t^{r-1}}{(\mathrm{r}-1)!}*e^{-(\lambda+q\mu)\mathrm{t}}\left[I_{r-1}\left(\alpha t\right)-I_{r+1}\left(\alpha t\right)\right], \tag{2.17}$$

where ‘ $*$ ’ denotes the convolution and ‘ $*k$ ’ denotes the ‘ k ’-fold convolution.

Proof:

Taking Laplace transform on (2.1), we get

$$\overset{)}{P}_C(s) = \frac{q\mu}{s+\lambda+\eta}\overset{)}{P}_{1,1}(s). \qquad . \tag{2.18}$$

which on inversion leads to (2.14). Laplace transform of (2.5) and using (2.18) gives

$$\overset{)}{P}_{0,0}(s) = \frac{\eta q\mu}{(s+\lambda+\gamma)(s+\lambda+\eta)}\overset{)}{P}_{1,1}(s) + \frac{1}{(s+\lambda+\gamma)}. \tag{2.19}$$

On Laplace inversion of (2.19), we get (2.15). Laplace transform of (2.6) and using (2.18) yields,

$$\overset{)}{P}_{0,n}(s) = \left(\frac{\lambda}{s+\lambda+\gamma}\right)^n \left(\frac{\eta q\mu}{(s+\lambda+\gamma)(s+\lambda+\eta)}\overset{)}{P}_{1,1}(s) + \frac{1}{(s+\lambda+\gamma)}\right). \tag{2.20}$$

Laplace inversion of (2.20) gives (2.16). Substitute $n=1$ in (2.7) and taking Laplace transforms, we get

$$\overset{)}{P}_{1,1}(s) = \frac{\gamma}{\lambda}\sum_{m=1}^{\infty}\overset{)}{P}_{0,\mathrm{m}}(s)\beta^{2-m}\left(\frac{p-\sqrt{p^2-\alpha^2}}{\alpha}\right)^m + \beta\left(\overset{)}{P}_{1,0}(s)+\overset{)}{P}_C(s)\right)\left(\frac{p-\sqrt{p^2-\alpha^2}}{\alpha}\right),$$

where $p = s+\lambda+q\mu.$. Using (2.13), (2.20), (2.18), (2.19) in the above equation and rearranging, we get

$$\overset{)}{P}_{1,1}(s) = \frac{\gamma}{\lambda}\sum_{m=1}^{\infty}\frac{\lambda^m}{\left(s+\sigma\lambda+\gamma\right)^{m+1}}\beta^{2-m}\left(\frac{p-\sqrt{p^2-\alpha^2}}{\alpha}\right)^m + \frac{\beta\gamma}{\left(s+\lambda\right)(s+\lambda+\gamma)}\left(\frac{p-\sqrt{p^2-\alpha^2}}{\alpha}\right)$$

$$\times\sum_{k=0}^{\infty}\sum_{r=0}^{k}\binom{k}{r}(\gamma\eta q\mu)^{k-r}\left\{\frac{\lambda^{m-1}\beta^{2-m}}{\left(s+\sigma\lambda+\gamma\right)^{m+1}(s+\lambda+\eta)}\left(\frac{p-\sqrt{p^2-\alpha^2}}{\alpha}\right)^m\right\}^{k-r}$$

$$\times(\lambda q\mu)^r\sum_{j=0}^{r}\binom{r}{j}\frac{\left(\gamma\eta\right)^j\beta^r}{\lambda^r\left(s+\sigma\lambda+\gamma\right)^j\left(s+\lambda\right)^j(s+\lambda+\eta)^r}\left(\frac{p-\sqrt{p^2-\alpha^2}}{\alpha}\right)^r. \tag{2.21}$$

Laplace inversion of (2.21) yields (2.17).

2.2. Special case

When $\eta=0, \sigma=1, q=1, P_C(\mathrm{t})=0,$ then (2.7) becomes

$$P_{1,n}(t)=\gamma\int_0^t\sum_{m=1}^{\infty}P_{0,m}(u)e^{-(\lambda+\mu)(t-u)}\beta^{n-m}\left[I_{n-m}\left(\alpha\left(t-u\right)\right)-I_{n+m}\left(\alpha\left(t-u\right)\right)\right]du$$
$$+\lambda\int_0^t P_{1,0}(u)e^{-(\lambda+\mu)(t-u)}\beta^{n-1}\left[I_{n-1}\left(\alpha\left(t-u\right)\right)-I_{n+1}\left(\alpha\left(t-u\right)\right)\right]du,$$

which coincides with (3.7) in Sudhesh et al. (2017) if $\mu_0=0,\xi=0.$

3. Performance Measures

In this section, the time-dependent mean and variance of the number of customers in the system at time t are derived.

(I) The mean system size, $E(\mathrm{X(t)})$ is

$$E\left(X(t)\right)=-q\mu\sum_{n=1}^{\infty}\int_0^t P_{1,n}(u)du+\sigma\lambda\sum_{n=1}^{\infty}\int_0^t P_{0,n-1}(u)du+\lambda\sum_{n=1}^{\infty}\int_0^t P_{1,n-1}(u)du+\lambda\int_0^t P_C(u)\,\mathrm{du}.$$

(II) The variance of system size, $V(\mathrm{X(t)})$ is given as

$$V\left(X(t)\right)=E\left(X^2(t)\right)-\left[E\left(X(t)\right)\right]^2,$$

where

$$E\left(X^2(t)\right)=2\lambda\sum_{n=1}^{\infty}\int_0^t E\left(X(\mathrm{u})\right)du+q\mu\sum_{n=1}^{\infty}\int_0^t(1-2\,\mathrm{n})P_{1,n}(u)du+\sigma\lambda\sum_{n=1}^{\infty}\int_0^t P_{0,n-1}(u)du$$
$$+\lambda\sum_{n=1}^{\infty}\int_0^t P_{1,n-1}(u)du+\lambda\int_0^t P_C(u)\,\mathrm{du}.$$

4. Numerical Illustrations

For the proposed queueing model, the time-dependent system size probabilities are plotted for $\lambda=1,\mu=1.5,\eta=0.5,\gamma=0.5,\sigma=0.5,q=0.5$ in Figure 1 and Figure 2. Assume that there is no customer initially in the system. It is observed that the probability curves in Figure 1 (except $P_{0,0}(\mathrm{t})$) and Figure 2 increase initially and attain steady-state as time t increases. Figure 3 shows that the increase in vacation rate γ leads to decrease in the average number of customers in the system. The variance also decreases with the increment of vacation rate γ which is shown in Figure 4.

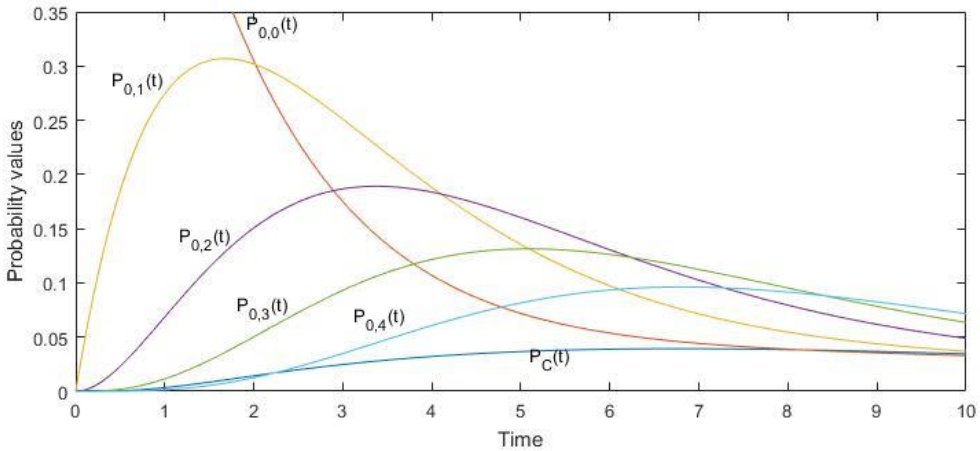

Figure 1. Transient probabilities for the vacation state of the server

Table 1 shows the variations of mean and variance values of system size for various values of the vacation rate. From Table 1, it is observed that the average and variance values decrease with the increment of γ values. Table 2 and Table 3 present the performances of mean and variance for different values of 'probability of customers leaving the system after the service completion' and 'the probability of arrivals joining the queue' respectively. From Table 2, it is evident that the average and variance fall down when 'q' increases. Table 3 depicts that the expected value and variance raise while σ increases.

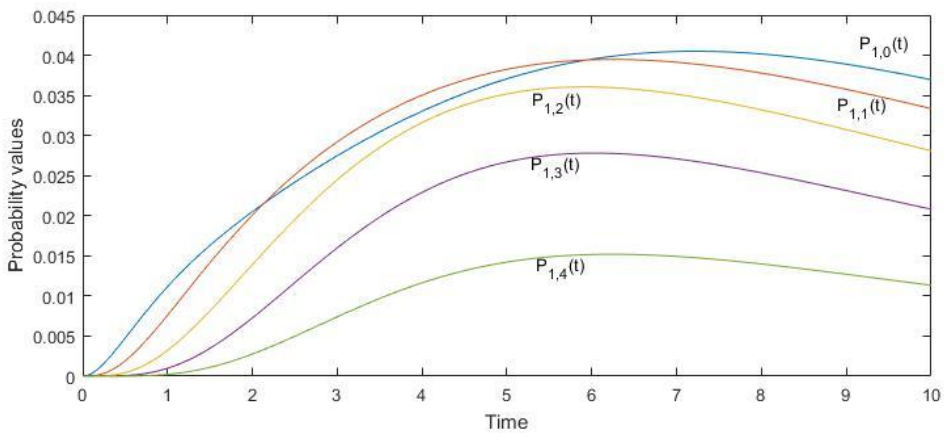

Figure 2. Transient probabilities for the busy state of the server

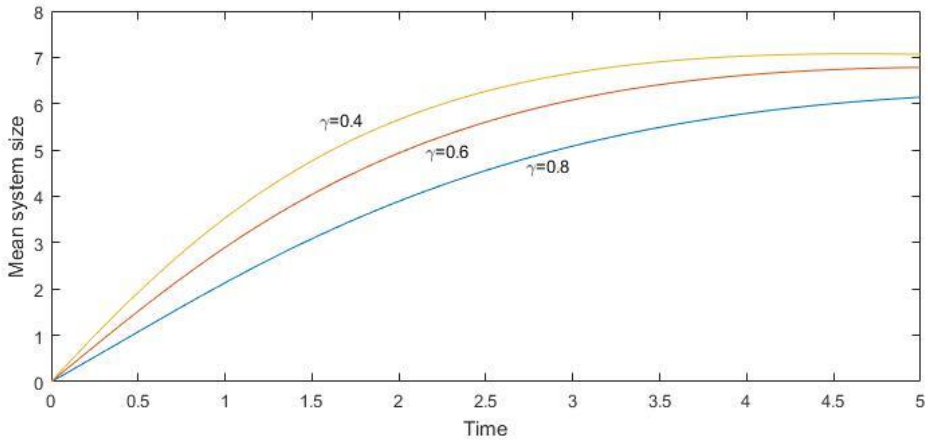

Figure 3. Mean system size with different values of vacation rate

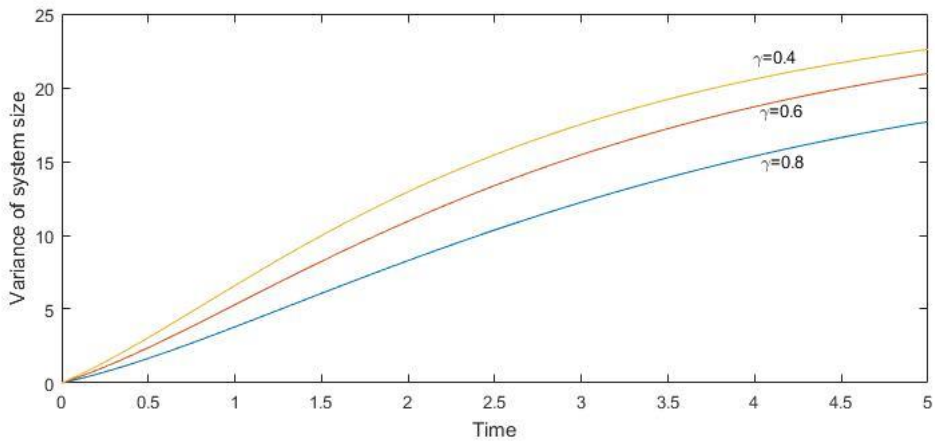

Figure 4. Variance of system size with different values of vacation rate

Table 1. Variations of mean and variance for different values of vacation rate at $t = 5, \lambda = 1, \mu = 1.5, \eta = 0.5 \;\; \sigma = 0.5, q = 0.5$

γ	*E[X(t)]*	*V[X(t)]*
0.1	7.6015	23.5218
0.2	7.4859	23.2107
0.3	7.3254	22.9246
0.4	7.0121	22.5623
0.5	6.9572	22.1028
0.6	6.8510	21.8752
0.7	6.4758	19.8561
0.8	6.1547	18.2317

Table 2. Variations of mean and variance for different values of probability that the customers leave the system at $t = 5, \lambda = 1, \mu = 1.5, \eta = 0.5, \sigma = 0.5, \gamma = 0.5$

q	*E[X(t)]*	*V[X(t)]*
0.1	5.1058	20.2547
0.2	4.8754	19.3282
0.3	4.5628	18.8751
0.4	4.3251	18.4181
0.5	4.1540	17.7452
0.6	3.9528	17.3212
0.7	3.7425	16.5228
0.8	3.6951	16.1052

Table 3. Variations of mean and variance for different values of probability that the customers join the queue at $t = 5, \lambda = 1, \mu = 1.5, \eta = 0.5, q = 0.5, \gamma = 0.5$

σ	*E[X(t)]*	*V[X(t)]*
0.1	4.3258	11.3285
0.2	4.8127	12.5268
0.3	5.4529	13.8691
0.4	5.9125	15.0217
0.5	6.2512	16.3204
0.6	6.7589	17.5248
0.7	7.2481	18.2510
0.8	7.8415	19.3223

5. Conclusion and future scope

This paper deals with the time-dependent analysis of an *M/M/1* queueing model with single vacation, feedback, interrupted closedown time and control of admission during vacation. Explicit expressions for the transient system size probabilities are derived using the method of generating function. The major contribution in this research work is the introduction of interrupted closedown time of the server. If any customer arrives before the completion of the closedown time, then the closedown work of the server is interrupted and the server begins a busy period. This is called as an interrupted closedown time of the server. Also the most significant transient system distributions are derived for the proposed model. This model is limited to a single vacation policy for a single server Markovian queueing model. Numerical illustrations help us to visualize the influence of various system indices. In future, this work may be extended into a multi server queueing model with the similar parameters.

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