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Numerical Solution of MHD Bioconvection in a Porous Square Cavity due to Oxytactic Microorganisms

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Abstract

The present paper investigates the magnetohydrodynamic (MHD) bioconvection flow in a porous square cavity filled with oxytactic microorganism. The bioconvection flow and heat transfer in porous media is formulated using Darcy model of Boussinesq approximation. Finite element method based on Galerkin weighted residual scheme is used to solve the governing partial differential equations. The computational numerical results are illustrated in the form of streamlines, isotherms, isoconcentrations of oxygen and microorganisms, average Nusselt number and average Sherwood number. In the present study the effects of key parameters such as bioconvection Rayleigh number (Rb), Rayleigh number (Ra), Peclet number (Pe) magnetic field parameter (M) and Lewis number (Le) are presented and analyzed.

Keywords: Bioconvection; MHD; porous media; square cavity; finite element method; oxytactic microorganisms

MSC 2010 No.: 76S05, 76Z99, 74S05, 76R10, 76R50, 80A20

1. Introduction

Bioconvection is a significant convection mode of fluid due to density gradients provoked by hydrodynamic propulsion, i.e. swimming, of motile microorganisms. The accumulated microbiological organisms swim typically in the direction of an imposed or naturally present stimulus. The density of the microorganism is assumed to be greater than that of the surrounding fluid. In this paper the microorganisms are considered to be the oxytactic bacteria, as in Kessler (1989), Hillesdon et al. (1996), Pedley et al. (1988) and Metcalfe and Pedley (1998). These bacteria are oxygen consumers and thus they tend to swim from lower to higher oxygen concentrations.

However in many practical applications bioconvection must be controlled or suppressed Kuznetsov (2011). The feature of motile microorganisms to swim in a particular direction is utilized in the processes of purifying the cultures, separating of non-swimmers, slow swimmers and fast swimmers. Bioconvection is an effective method in the applications including the separation of dead cells and living cells if bioconvection takes place in a porous media Pedley (2010), Xu and Pop (2010), Aziz et al. (2012).

Porous media can be used to control bioconvection. Kessler (1986) reported experimentally that bioconvection can be suppressed using a porous medium such as surgical cotton. These examples exhibit the significance of investigating bioconvection in porous media and understanding the effect of porous media on bioconvection. The emerging volume of work in the area of convective heat transfer in porous media are amply illustrated in the books by Nield and Bejan (2013), Ingham and Pop (2005), Vafai (2010) and Bejan et al. (2004). Natural convective heat transfer in a porous square cavity is studied using finite element method by Balla and Kishan (2015). Kuznetsov (2005) and Sheremet and Pop (2014) investigated the onset thermo-bioconvection due to gyrotactic and oxitactic microorganisms.

The study of magnetic field has been a focus of current research owing to its wide importance and relevance ranging from many physical natural phenomenon concerning with geophysics, metallurgy, aerodynamic extrusion of plastic sheets and also in many engineering fields such as petroleum engineering, chemical engineering, composite or ceramic engineering, biochemical engineering and heat exchangers. The applications of magnetic field include the areas of engineering and physical sciences such as crystal growth, metal casting and liquid metal cooling blankets for fusion reactors. Zhang et al. (2015) analyzed MHD flow and heat transfer in porous media over a surface with variable surface heat flux. Ellahi and Hussain (2015) presented the effect of MHD on peristaltic flow in a rectangular duct. MHD convection in porous square cavity is studied by Balla et al. (2017), Balla et al. (2016), Balla et al. (2018) and Haritha et al. (2018). The combined effects of bioconvection and magnetic field on a free convection nanofluid flow over a stretching sheet containing gyrotactic microorganisms is investigated by Noreen and Zafar (2016). Alsaedi et al. (2017) investigated Magnetohydrodynamic bioconvective flow, heat, mass and motile microorganisms transfer due to gyrotactic microorganisms. Giri et al. (2017) investigated the effect of Stefan blowing on the hydro-magnetic bioconvection of a water-based nanofluid flow containing gyrotactic microorganisms through a permeable surface.

The aim of this paper is to extend the work of Kuznetsov (2005) and Sheremet and Pop (2014)

to the case of thermobioconvection in a suspension containing oxytactic microorganisms in the presence of external magnetic field. To the best of authors′ knowledge based on the literature review, this study is novel to analyze bioconvection caused by oxytactic microorganisms under the influence of external magnetic field. This study can be applied to examine the problems related to highly efficient fuel cells and hot springs in which thermophilic microorganisms are utilized.

2. Mathematical formulation

A two dimensional bioconvectional flow of a nanofluid in a porous square cavity filled with oxytactic microorganisms is considered. Let L be the length of the cavity. Assume that the top and bottom walls are adiabatic and the temperatures at left and right walls are T_H and T_C respectively such that $T_H > T_C$. The vector of acceleration due to gravity acts in the opposite direction to the $y-$axis. The suspension of oxytactic microorganisms is based on the model of Hillesdon et al. (1996). The suspension is assumed to be dilute and inertia terms are ignored due to the very slow motion of bioconvection flow Pedley et al. (1988). A uniform magnetic field is applied in the vertical direction across bottom wall of the cavity. Considering the Boussinesq approximation, the steady state governing equations in Cartesian form can be written as follows:

Continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

Momentum equations are

$$\frac{\mu}{K} u = -\frac{\partial p}{\partial x} - \sigma B_0^2 u, \tag{2}$$

$$\frac{\mu}{K} v = -\frac{\partial p}{\partial y} - \left[\gamma \Delta\rho n - \rho_f \beta (T - T_C)\right] g. \tag{3}$$

Energy equation is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right). \tag{4}$$

Oxygen conservation equation is

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - \delta n. \tag{5}$$

Cell conservation equation is

$$\frac{\partial}{\partial x}\left[un + \tilde{u}n - D_n \frac{\partial n}{\partial x}\right] + \frac{\partial}{\partial y}\left[vn + \tilde{v}n - D_n \frac{\partial n}{\partial y}\right] = 0, \tag{6}$$

where u and v are components of velocity in x and $y-$directions, T is the temperature of fluid, C is the concentration of oxygen, n is the number density of motile microorganisms, α_m is the effective thermal diffusivity of the porous medium, is the permeability of the porous medium, p is the pressure, β is the volume expansion coefficient of water at constant pressure, μ is the dynamic viscosity of the suspension, ρ_f is the density of the fluid, γ is the mean volume of a microorganism, D_B is the diffusivity of oxygen, D_n is the diffusivity of the microorganisms, $-\delta n$ describes the

consumption of oxygen by the microorganism, C_0 is the concentration of oxygen at free surface, C_{min} is the minimum concentration of oxygen essential for microorganisms to be active.

The difference between density of cells and that of fluid is given by, $\Delta\rho = \rho_{cell} - \rho_f$.

The difference between oxygen concentrations is given by, $\Delta C = C_0 - C_{\min}$.

The mean directional swimming velocities of a microorganism $\tilde{u}$ and $\tilde{v}$ are given by $\tilde{u} = \left(\frac{bW_C}{\Delta C}\right)\frac{\partial C}{\partial x}$ and $\tilde{v} = \left(\frac{bW_C}{\Delta C}\right)\frac{\partial C}{\partial y}$, where b is the chemotaxis constant and W_C is the maximum speed that cell swims.

Considering dimensional stream function ψ, defined by $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$, and using the following non-dimensional variables,

$$X = \frac{x}{H},\ Y = \frac{y}{H},\ \Psi = \frac{\psi}{\alpha_m},\ \theta = \frac{T - T_C}{\Delta T},\ \phi = \frac{C - C_{\min}}{\Delta C},\ N = \frac{n}{n_0},$$

where n_0 is the average density of the microorganism, the following dimensionless partial differential equations are obtained,

$$\frac{\partial^2\Psi}{\partial X^2} + \frac{\partial^2\Psi}{\partial Y^2} = -Ra\left(\frac{\partial\theta}{\partial X} - Rb\frac{\partial N}{\partial X}\right) - M\frac{\partial^2\Psi}{\partial Y^2}, \tag{7}$$

$$\frac{\partial\Psi}{\partial Y}\frac{\partial\theta}{\partial X} - \frac{\partial\Psi}{\partial X}\frac{\partial\theta}{\partial Y} = \frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}, \tag{8}$$

$$Le\left(\frac{\partial\Psi}{\partial Y}\frac{\partial\phi}{\partial X} - \frac{\partial\Psi}{\partial X}\frac{\partial\phi}{\partial Y}\right) = \frac{\partial^2\phi}{\partial X^2} + \frac{\partial^2\phi}{\partial Y^2} - \sigma_1 N, \tag{9}$$

$$Le.\chi\left(\frac{\partial\Psi}{\partial Y}\frac{\partial N}{\partial X} - \frac{\partial\Psi}{\partial X}\frac{\partial N}{\partial Y}\right) + Pe\left[\frac{\partial}{\partial X}\left(N\frac{\partial\phi}{\partial X}\right) + \frac{\partial}{\partial Y}\left(N\frac{\partial\phi}{\partial Y}\right)\right] = \frac{\partial^2 N}{\partial X^2} + \frac{\partial^2 N}{\partial Y^2}, \tag{10}$$

where the parameters of bioconvection Rayleigh number Rb, Rayleigh number of porous medium Ra, Peclet number Pe, Lewis number Le, constant σ_1 and magnetic field M are defined as

$$Rb = \frac{\gamma\Delta\rho n_0}{\rho_f\beta\Delta T},\ Ra = \frac{gK\beta\Delta TH}{\nu\alpha_m},\ Pe = \frac{bW_C}{D_n},\ Le = \frac{\alpha_m}{D_C}, \sigma_1 = \frac{\delta n_0 H^2}{D_C\Delta C},\ \chi = \frac{D_C}{D_n},\ M = \frac{K\sigma B_0^2}{\mu}.$$

The dimensionless boundary conditions of Equations (7)-(10) are given by

$$\begin{gathered}
\Psi = 0,\ \theta = 1,\ \phi = 1,\ N = 1 \text{ at } X = 0,\\
\Psi = 0,\ \theta = 0,\ \phi = 1,\ N = 1 \text{ at } X = 1,\\
\Psi = 0,\ \frac{\partial\theta}{\partial Y} = 0,\ \phi = 1,\ PeN\frac{\partial\phi}{\partial Y} = \frac{\partial N}{\partial Y} \text{ at } Y = 0,\\
\Psi = 0,\ \frac{\partial\theta}{\partial Y} = 0,\ \frac{\partial\phi}{\partial Y} = 0,\ \frac{\partial N}{\partial Y} = 0 \text{ at } Y = 1.
\end{gathered} \tag{11}$$

The local Nusselt number on the vertical walls and average Nusselt number are given by

$$Nu_Y = -\left(\frac{\partial\theta}{\partial X}\right)_{X=0,1},\ Nu = \int_0^1 Nu_Y dY.$$

Table 1. Comparison of the Nusselt numbers at the hot wall

Author	Ra=10	Ra=100	Ra=1000	Ra=10000
Sheremet and Pop (2014)	1.079	3.115	13.667	48.823
Manole and Lage (1992)	-	3.118	13.637	48.117
Baytas and Pop (1999)	1.079	3.16	14.06	48.33
Revnic et al. (2009)	-	-	13.664	-
Present results	1.08	3.129	13.7353	48.5451

3. Method of solution

The boundary value problem given by Equations (7)-(11) is solved by using finite element method of Galerkin weighted residual approach. In this method the unknown functions are restored by approximate trial functions to obtain the residuals. The weight functions are now multiplied to residuals and their integrals over a typical element are taken to be zero.

The convergence of solutions is marked when $\left|\varphi^{r+1} - \varphi^{r}\right| \leq 10^{-5}$, where r denotes number of iterations and φ denotes Ψ, θ, ϕ and N. In order to fix the mesh size, a mesh convergence test is performed for the sizes 21×21, 41×41, 61×61, 81×81 and 91×91. The mesh convergence test reveals that a grid of size 81×81 is sufficient to study the bioconvection phenomena. The current problem has been validated in the absence of microorganisms. The calculated Nusselt numbers at the hot wall are compared with those of Sheremet and Pop (2014), Manole and Lage (1992), Baytas and Pop (1999) and Revnic et al. (2009), and presented in Table 1. The present results are found to be in excellent agreement with the available results.

4. Results and discussions

The present boundary value problem governed by Equations (7)-(11) is solved numerically for the investigation of various parameters: Rayleigh number Ra, bioconvection Rayleigh number Rb, Lewis number Le, Peclet number Pe and magnetic field parameter M. In the present study, the following ranges of values have been considered for key parameters: Ra=(10-150), Rb=(10-150), Le=(1-10), Pe=(0.1-0.9), M=(0-5). Figure 1 illustrates the effect of Rayleigh number on streamlines, isotherms, oxygen isoconcentrations and microorganism isoconcentrations. The streamlines form a single cell occupying the entire cavity with anticlockwise rotation when $Ra = 10$. As Ra increases the flow strength enhances and the single cell splits into two cells when $Ra = 150$. The isotherms are parallel to the heated walls when $Ra = 10$. As Ra increases the isotherms are stratified horizontally at the center of the cavity indicating the convection mode of heat transfer. It is observed that the consumption of oxygen and microorganism is at the top of the cavity. The profiles of isoconcentrations of oxygen and microorganisms are pronounced by increasing the Rayleigh number. A vortex is formed in the isoconcentrations of oxygen and microorganisms which indicate the higher density of oxygen and motile organisms at center of the cavity.

Figure 2 depicts the effect of bioconvection Rayleigh number on streamlines, isotherms and isoconcentrations of oxygen and microorganisms. The streamlines form a single cell with anticlockwise

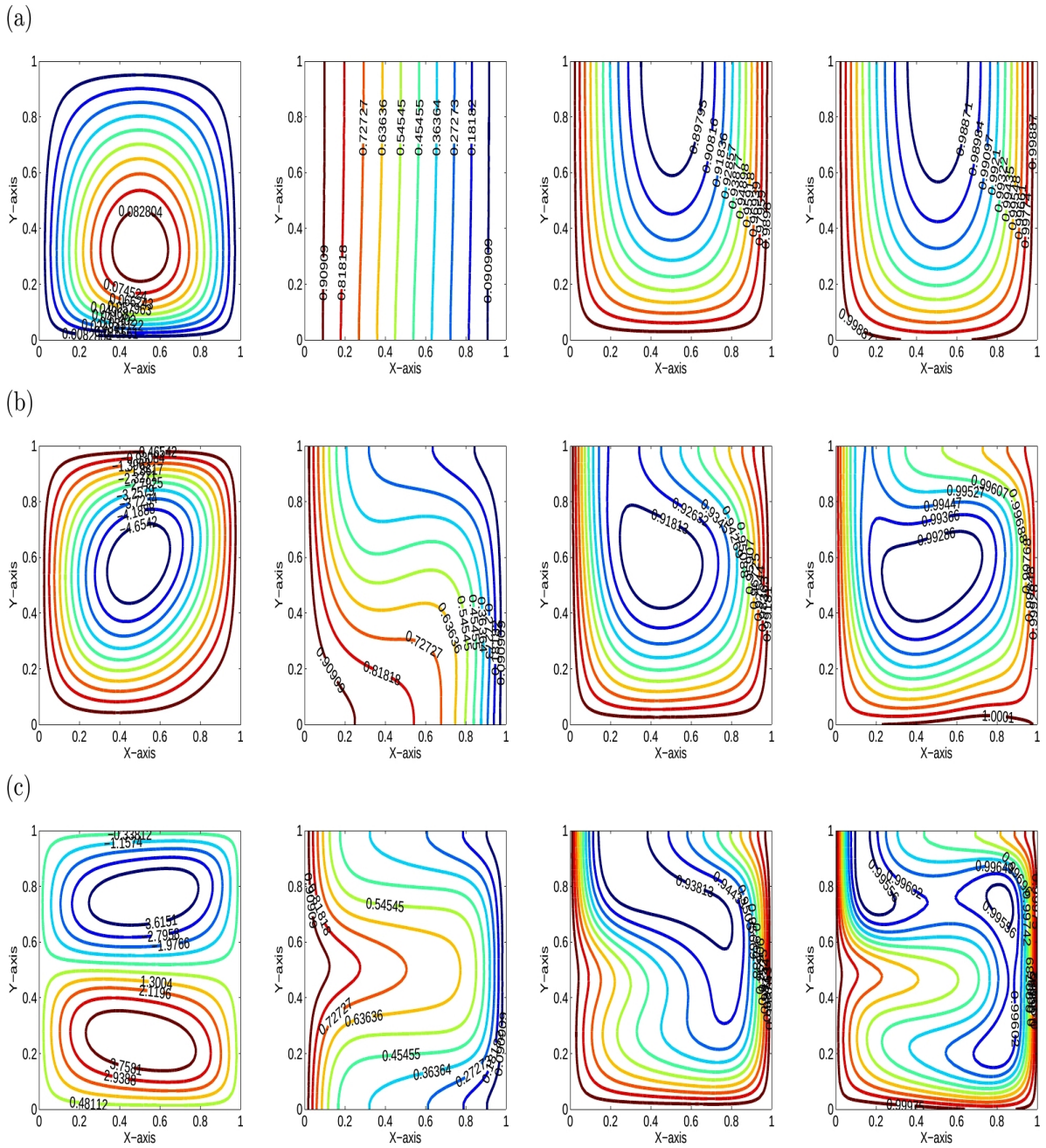

Figure 1. Streamlines, Isotherms, oxygen isoconcentrations and microorganisms isoconcentrations for (a) Ra=10 (b) Ra=100 (c) Ra=150 and Rb=10, Pe=0.1, Le=1, M=1.

rotation when $Rb = 10$. As Rb increases the flow strength enhances and the cell shifts towards the bottom adiabatic wall slightly. The isotherms are slightly affected by increasing the bioconvection Rayleigh number. The isoconcentration fields of oxygen and microorganisms are enhanced by increasing the bioconvection Rayleigh number.

Influence of Peclet number on the streamlines, isotherms and isoconcentrations of oxygen and

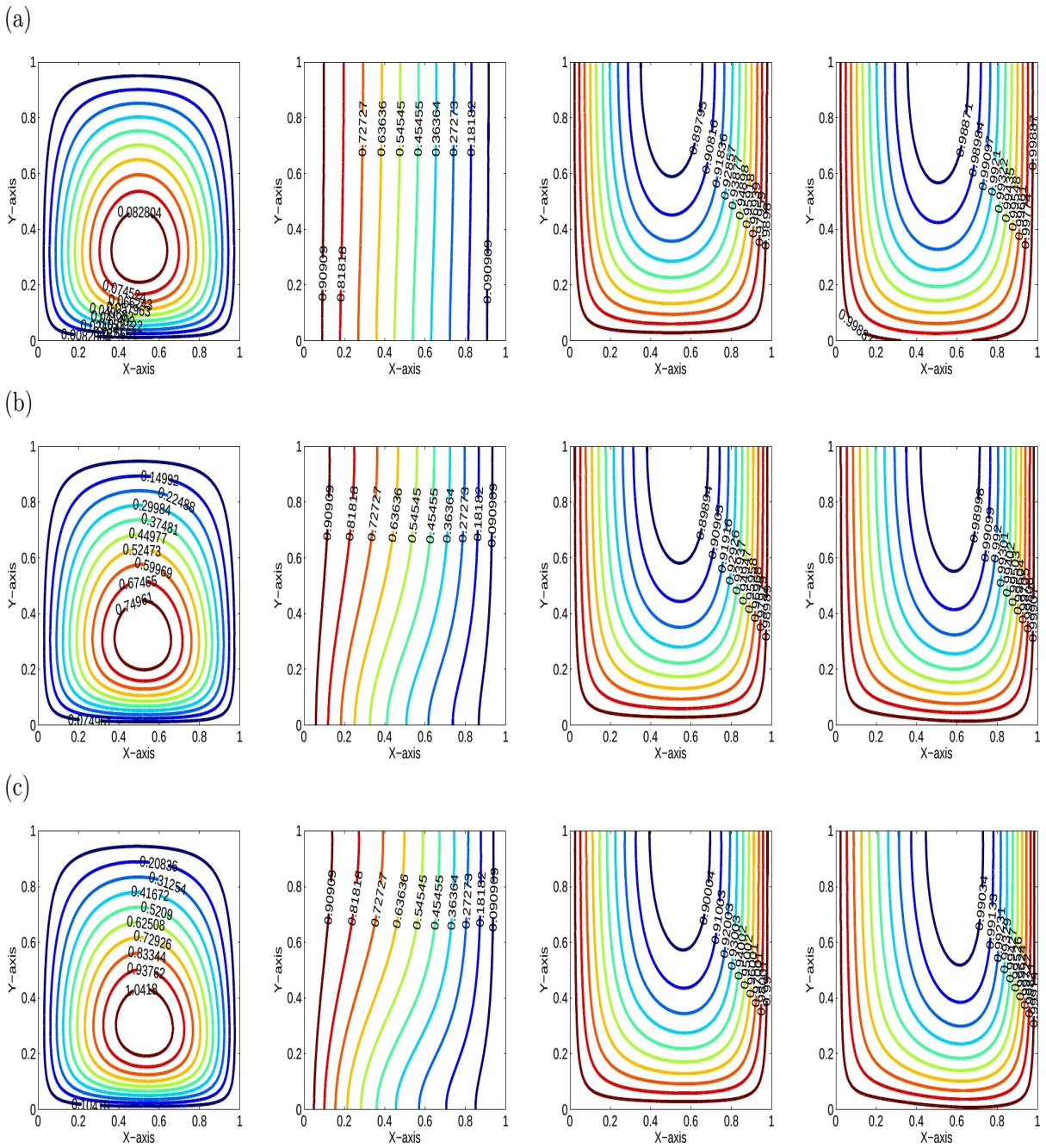

Figure 2. Streamlines, isotherms, oxygen isoconcentrations and microorganisms isoconcentrations for (a) Rb=10 (b) Rb=100 (c) Rb=150 and Ra=10, Pe=0.1, Le=1, M=1.

microorganisms is shown in figure 3. Increasing the Peclet number causes to significant changes in the profiles of streamlines and concentrations, while the profile of isotherms change insignificantly. A flow in the reverse direction is appeared in the top right corner of the cavity, indicating the influence of swimming of oxytactic bacteria. Dominance of the velocity due to diffusive swimming of microorganisms leads to a clockwise rotating cell. It is also observed that despite the Peclet number the oxygen consumption of the bacteria appears in the top part of the cavity. A decrease in

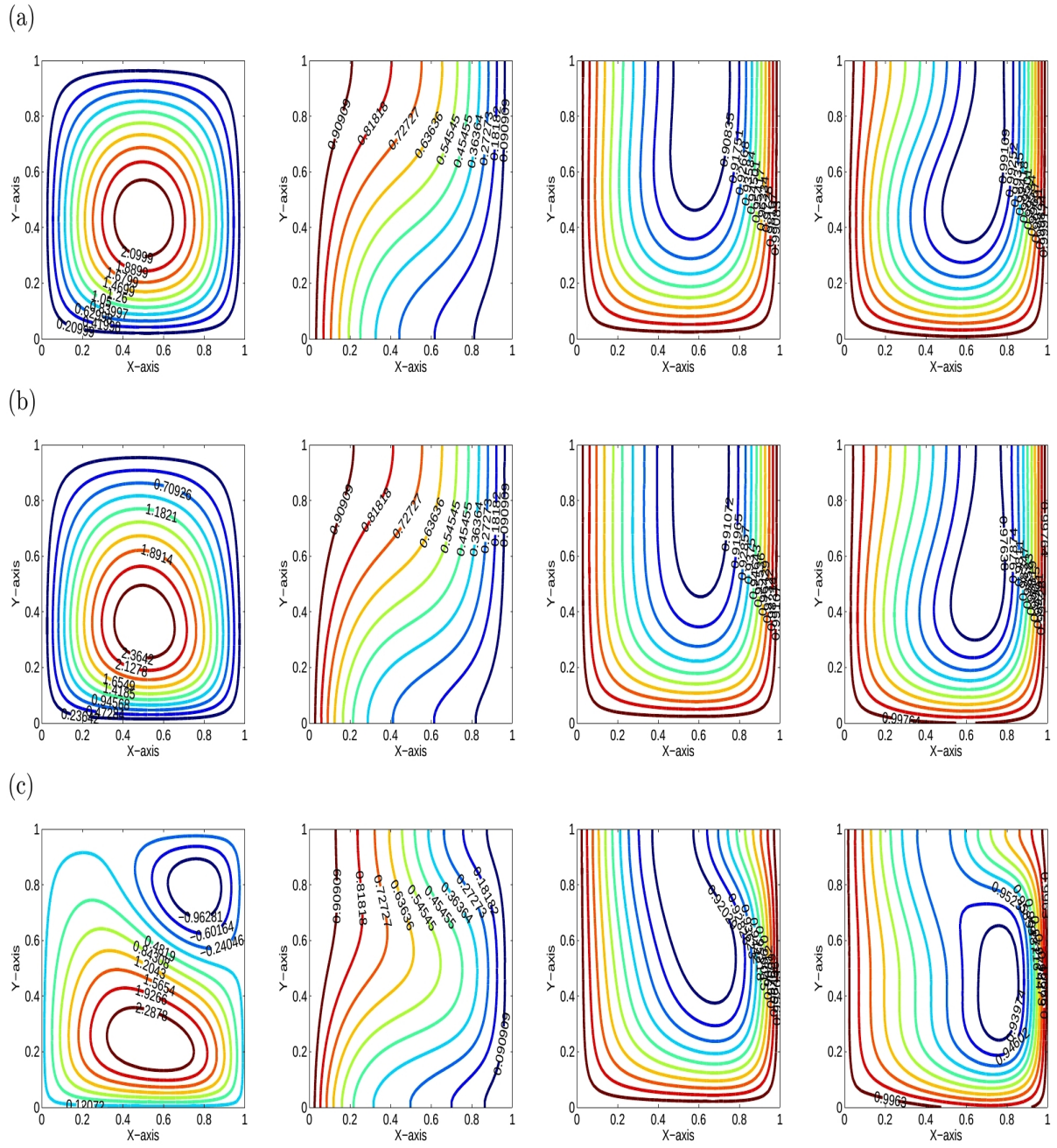

Figure 3. Streamlines, isotherms, oxygen isoconcentrations and microorganisms isoconcentrations for (a) Pe=0.1 (b) Pe=0.3 (c) Pe=0.9 and Ra=10, Rb=10, Le=1, M=1.

motile bacteria concentration is noticed with the enhancement in the Peclet number. Boosting the average directional swimming velocity of the microorganisms causes less oxygen consumption in the top part of the cavity.

Effect of magnetic field on the streamlines, isotherms and isoconcentrations of oxygen and microorganisms is shown in Figure 4. Enhancement in M strongly reduces the flow strength of

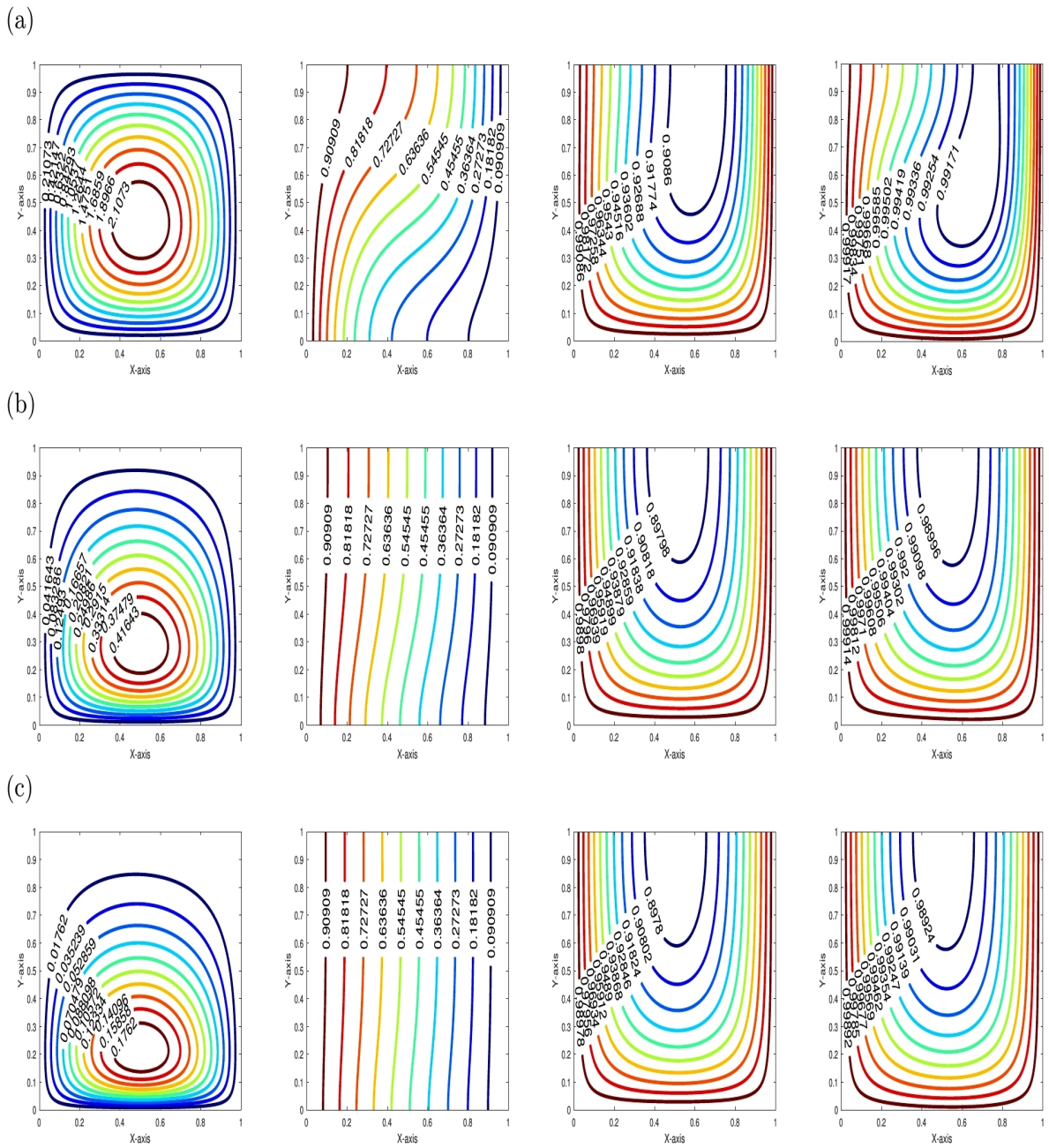

Figure 4. Streamlines, isotherms, oxygen isoconcentrations and microorganisms isoconcentrations for (a) M=0 (b) M=2 (c) M=5 and Ra=10, Rb=10, Pe=0.1, Le=1.

streamlines and shifts the flow cell to bottom. As the magnetic field increases the isotherms are parallel to the left and right walls. The isoconcentration fields of oxygen and microorganisms are diminished by increasing magnetic field.

Figure 5 presents the effect of Lewis number Le on the streamlines, isotherms and isoconcentrations of oxygen and microorganisms. Small flow cell is formed on the top right of the cavity.

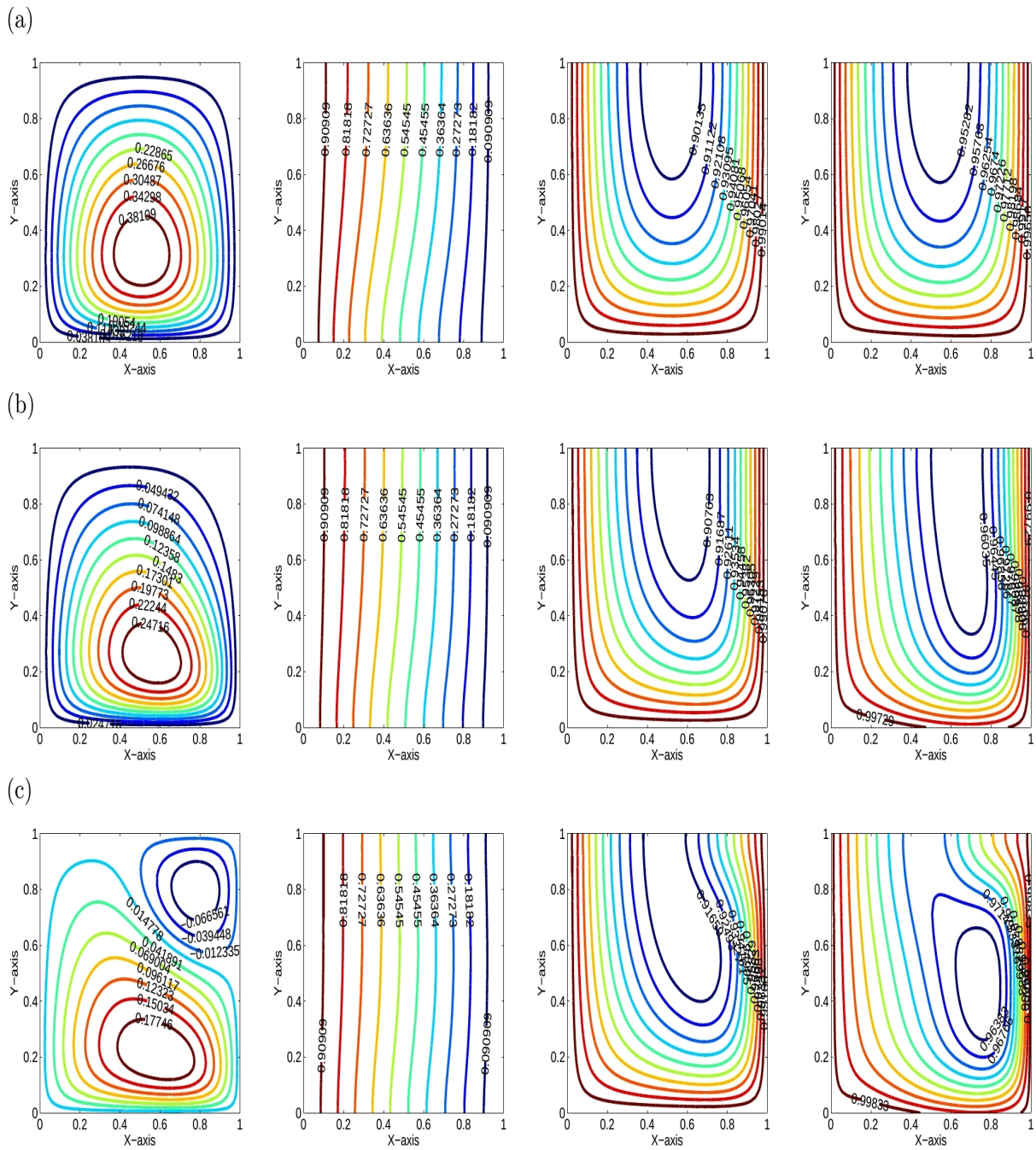

Figure 5. Streamlines, isotherms, oxygen isoconcentrations and microorganisms isoconcentrations for (a) Le=1 (b) Le=5 (c) Le=10 and Ra=10, Rb=10, Pe=0.1, M=1.

Region of high oxygen density and motile density elongates towards the bottom of the cavity. Le reduces the unicellular flow intensity. Insensible change in the isotherms is observed. High oxygen density flux and motile density flux is observed at the cold wall of the cavity.

Figure 6(a) shows the effect of Peclet number versus Lewis number on the average Nusselt number. From the figure, it is clear that the average Nusselt number increases with the increase in Peclet

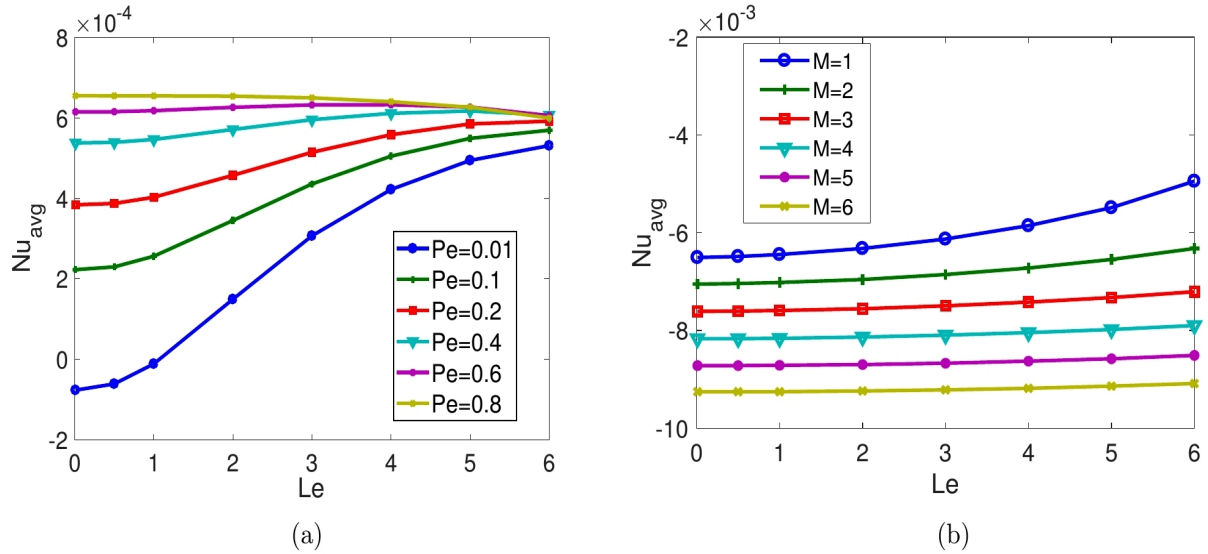

Figure 6. Effect of (a) Peclet number (b) Magnetic field parameter versus Lewis number on average Nusselt number.

number. The effect of Lewis number in more for low Peclet number and vice versa. Figure 6(b) depicts the effect of Magnetic field versus Lewis number on Nu_{avg}. The average Nusselt number increases with the decrease in the magnetic field. The effect of Lewis number on Nu_{avg} is significant for low magnetic field.

5. Conclusion

In this paper, the effect of magnetic field on the bioconvection heat transfer in a square cavity filled with oxytactic microorganisms is investigated. The basic partial differential equations are transformed into non-dimensional form and solved using finite element method. The streamlines, isotherms, oxygen isoconcentrations, microorganisms isoconcentrations and Nusselt number were calculated and presented graphically. The main conclusions can be drawn from the present study could be summarized as:

- The flow strength is pronounced with Rayleigh number, bioconvection Rayleigh number and Peclet number. Lewis number and magnetic field reduces the flow strength in the cavity.
- The temperature distribution is affected significantly with Rayleigh number, bioconvection Rayleigh number, magnetic field and Peclet number.
- The oxygen density distribution is pronounced with Rayleigh number, bioconvection number, Peclet number and Lewis number. Increase in Magnetic field reduces oxygen density distribution.
- The motile isoconcentrations profile enhanced with Rayleigh number, bioconvection number and Lewis number. Peclet number and magnetic field reduces the motile density distribution.
- The average Nusselt number is increased with Peclet number and Lewis number and decreased with the magnetic field.

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