	Available at http://pvamu.edu/aam Appl. Appl. Math. **ISSN: 1932-9466** **AAM Special Issue No. 4 (March 2019), pp. 31 - 43**	**Applications and Applied Mathematics:** An International Journal **(AAM)**

MHD Boundary Layer Slip Flow over a Flat Plate with Soret and Dufour Effects

[1]B. Shashidar Reddy and [2]K. Saritha

[1]Department of Science and Humanities
Sreenidhi Institute of Science and Technology
(Affiliated to JNTUH, Hyderabad)
Ghatkesar, Hyderabad - 501301
Telangana State, India
bsreddy_shashi@yahoo.com

[2]Department of Humanities and Science
Teegala Krishna Reddy Engineering College
(Affiliated to JNTUH, Hyderabad)
Meerpet, Ranga Reddy - 500097
Telangana State, India
kallu.saritha@gmail.com

Received: August 4, 2018; Accepted: October 28, 2018

Abstract

The present paper studies the effects of Soret and Dufour on MHD boundary layer slip flow over a flat plate. The governing partial differential equations are converted to a set of nonlinear ordinary differential equations by using similarity transformations. Then, these equations are solved numerically by implicit Finite Difference Scheme. The numerical solutions for Velocity, Temperature and Concentration profiles for the related essential physical parameters are visualized through graphs and discussed. Results show that the velocity rises whereas the temperature and concentration reduces with the respective slip parameters. The increase in Soret number or decrease in Dufour number reduces the temperature and enhances the concentration of the fluid.

Keywords: Power-law fluid, Prandtl number, Lewis number, Slip parameter, Finite Difference Method, Soret and Dufour number

MSC 2010 No.: 76A05, 65M06

1. Introduction

The boundary layer flow past a flat plate was the first example considered by Blasius, to illustrate the application of Prandtl's boundary layer theory. The important concept of boundary layer was applied to power-law fluids by Schowalter (2004). Acrivos (1960) investigated the boundary layer flows for such fluids in 1960. The effect of magnetic field on electrically conducting fluid flows was studied by various authors such as Damseh et al. (2006), Anderson et al. (1992), Cortell (2005), Howell et al. (1997). The effect of suction/blowing was studied by Mahapatra (2012) on MHD power-law fluid flow over an infinite porous flat plate. Oahimire and Olajuwon (2014) investigated the effects of radiation, absorption and thermo-diffusion on MHD heat and mass transfer flow of a Micro-polar fluid in the presence of heat source.

Quasi-linearization approach to MHD effects on boundary layer flow of power-law fluids past a semi infinite plate with thermal dispersion was studied by Kishan and Shashidar (2011). Jadhav (2013) examined laminar boundary layer flow of a non-Newtonian power-law fluid past a porous flat plate. Kishan and Shashidar (2013) studied MHD effects on non-Newtonian power-law fluid past a continuously moving porous flat plate.

Some researchers like Hayat and Hendi (2012) have not presented Dufour and Soret effects on heat and mass transfer according to Fourier's and Flick's laws. However, Devi and Devi (2011) have shown that, when density differences exist in the flow regime, these effects cannot be neglected. Afify (2009) has shown that when heat and mass transfer occurred in a moving fluid, the energy flux can be generated by a composition gradient namely, the Dufour or Diffusion Thermo effect and the mass fluxes developed by the temperature gradient namely, the Soret or Thermal-Diffusion effect. Rashidi et al. (2015) studied the effect of Soret and Dufour on heat and mass transfer for MHD visco elastic fluid flow over a vertical stretching sheet. Pal and Chartterjee (2013) studied the MHD mixed convection with the combined action of Soret and Dufour on heat and mass transfer of a power-law fluid over an inclined plate in a porous medium. Non-Newtonian Prandtl fluid over stretching permeable surface was discussed by Jain and Timol (2016).

Martin and Boyd (2006), Bhattacharyya et al. (2011) incorporated velocity and thermal slip conditions in their studies of laminar flow across flat plates to further refine our understanding of boundary layer flow. Hirschhorn et al. (2016) studied MHD boundary layer slip flow and heat transfer of power-law fluid over a flat plate. Ibrahim and Shanker (2013) investigated MHD boundary layer flow and heat transfer of a Nano-fluid past a permeable stretching sheet with Velocity, Thermal and Solutal slip boundary conditions. Heat and mass transfer in MHD Micro-Polar fluid in the presence of Diffusion Thermo and Chemical reaction was investigated by Kiran Kumar et al. (2016).

Recently, Saritha et al. (2016) examined heat and mass transfer of laminar boundary layer flow of non-Newtonian power-law fluid past a porous flat plate with Soret and Dufour effects. Motivated by these studies and the related applications, the current analysis aims to deliberate the effects of Soret and Dufour on heat and mass transfer MHD boundary layer flow of non-

Newtonian power-law fluid past a porous flat plate with Velocity, Thermal and Concentration slip boundary conditions.

2. Mathematical Analysis

The flow of non-Newtonian power-law fluid past semi infinite porous flat plate is considered. x-axis is chosen in the direction of the flow and y-axis is considered perpendicular to it. The magnetic field strength B_0 is assumed along the direction of y-axis. We assumed that $T_w(x) = T_\infty + bx$ and $C_w(x) = C_\infty + cx$, where b and c are constants such that the uniform wall temperature T_w and concentration C_w are higher than those of their full stream values T_∞ , C_∞. By invoking all the boundary layer approximations, the governing equations for the flow in this investigation can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{K}{\rho}\frac{\partial}{\partial y}\left[-\frac{\partial u}{\partial y}\right]^n-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2}. \tag{4}$$

In the foregoing equations, u and v are the velocity components along the x and y-axes, n is the power-law index, K is the power-law fluid parameter, ρ is density, μ is the magnetic permeability, σ is the electrical conductivity of the fluid, $\alpha = k/\rho c_p$ is the thermal diffusivity, k is the thermal conductivity, c_p is the specific heat at a constant pressure, k_T is the thermal diffusion ratio, c_s is the concentration susceptibility, D_m is the coefficient of mass diffusivity, T is the temperature, C is the fluid concentration and T_m is the mean fluid temperature.

The boundary conditions associated with the present problem are as follows

$$u = L_1\left(\frac{\partial u}{\partial y}\right),\ \ v=0,\ \ T=T_w+D_1\left(\frac{\partial T}{\partial y}\right),\ \ C=C_w+P_1\left(\frac{\partial C}{\partial y}\right) at\ y=0, \tag{5a}$$

$$u\to U_\infty,\ \ T\to T_\infty,\ C\to C_\infty\ \ as\ \ y\to\infty, \tag{5b}$$

where $L_1 = L\sqrt{Re_x}$ is the velocity slip factor with L being the initial value at the leading edge, $D_1 = D\sqrt{Re_x}$ is the thermal slip factor with D being the initial value at the leading edge and $P_1 = P\sqrt{Re_x}$ is the concentration slip factor with P being the initial value at the leading edge. Here T_w and C_w are the temperature and concentration of the flat plate, T_∞ and C_∞ are the free stream temperature and concentration, and $Re_x = \frac{U_\infty^{2-n}x^n\rho}{K}$ is the local Reynolds number.

3. Method of Solution

To facilitate the analysis, we follow the previous studies Jadhav (2013) and Rashidi et al. (2015) and use the similarity variables

$$\Psi(\eta) = (\gamma x U^{2-n})^{\frac{1}{n+1}} f(\eta)\,, \tag{6a}$$

$$\eta = y\left[\frac{U^{2-n}}{\gamma x}\right]^{\frac{1}{n+1}}, \tag{6b}$$

$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \tag{6c}$$

$$\varphi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}, \tag{6d}$$

where Ψ is the stream function defined in the usual way and f is the reduced stream function for the flow. Then, the velocity components are defined using the similarity variables as

$$u = \frac{\partial\Psi}{\partial x} = Uf'(\eta), \tag{7a}$$

$$v = -\frac{\partial\Psi}{\partial y} = \frac{1}{n+1}\left(\gamma\frac{U^{2n-1}}{x^n}\right)^{\frac{1}{n+1}}(\eta f' - f). \tag{7b}$$

Introducing equation (6) and equation (7), the continuity equation is satisfied and the momentum, energy and concentration equations are transformed into a set of ordinary differential equations as follows:

$$n(-f'')^{n-1}f''' + \frac{1}{n+1}ff'' - Mf' = 0\,, \tag{8}$$

$$\theta'' + Pr\left(\frac{1}{n+1}f\theta' - f'\theta\right) + Du\emptyset'' = 0\,, \tag{9}$$

$$\frac{1}{Le}\emptyset'' + Pr\left(\frac{1}{n+1}\emptyset' f - f'\emptyset\right) + Sr\theta'' = 0\,, \tag{10}$$

Here, primes denote differentiation with respect to η and

$M = \frac{\sigma {B_0}^2 x}{\rho U}$ is the Magnetic parameter,

$Re_x = \frac{U^{2-n}x^n}{\nu}$ is the Reynolds number,

$Pr = \frac{Ux}{\alpha}{Re_x}^{-\frac{1}{n+1}}$ is the Prandtl number,

$Le = \frac{\alpha}{D_m}$ is the Lewis number,

$Du = \frac{D_m k_T}{c_s c_p}\cdot\frac{(C_w-C_\infty)}{(T_w-T_\infty)\alpha}$ is the Dufour number and

$Sr = \frac{D_m k_T}{T_m \alpha}\cdot\frac{(T_w-T_\infty)}{(C_w-C_\infty)}$ is the Soret number.

Boundary conditions given by equation (5) are transformed into

$$f(\eta) = 0,\ f'(\eta) = A_1 f''(\eta),\ \theta(\eta) = 1 + B_1\theta'(\eta),\ \emptyset(\eta) = 1 + C_1\emptyset'(\eta)\ \ at\ \ \eta = 0,$$

$$f'(\eta) \to 1, \quad \theta(\eta) \to 0, \quad \emptyset(\eta) \to 0 \quad as \quad \eta \to \infty\,, \tag{11}$$

where A_1, B_1 and C_1 are respectively the velocity, temperature and concentration slip parameters, which are further defined as

$$A_1 = L\frac{U_\infty \rho}{K}, \quad B_1 = D\frac{U_\infty \rho}{K} \text{ and } C_1 = P\frac{U_\infty \rho}{K}\,. \tag{12}$$

The physical quantities of engineering interest in this problem are the local Nusselt number and local Sherwood number, which are defined respectively by

$$Nu_x = \frac{q_w x}{k(T_w - T_\infty)} = -\theta'(0)Re_x^{1/n+1} \quad \text{and} \quad Sh_x = \frac{J_w x}{D_m(C_w - C_\infty)} = -\emptyset'(0)Re_x^{1/n+1},$$

where the rate of heat transfer q_w and rate of mass transfer J_w are defined as

$$q_w = -k\left[\frac{\partial T}{\partial y}\right]_{y=0} \text{ and } J_w = -D_m\left[\frac{\partial C}{\partial y}\right]_{y=0}.$$

4. Numerical Method

The combined effects of various physical parameters will have large impact on heat and mass characteristics. The transformed governing equation (8), equation (9) and equation (10) are coupled and highly non-linear. The non-linearity of the basic equations and additional mathematical difficulties associated with the solution part has led us to use the numerical method. Hence, the solutions of these equations with the boundary conditions equation (11) are solved numerically using implicit finite difference scheme.

The numerical solutions can be obtained in the following steps:

- Linearize equation (8) using Quasi Linearization method (1965),
- Write the difference equations using implicit finite difference scheme,
- Linearize the algebraic equations by Newton's method, and express them in matrix-vector form, and
- Solve the linear system by Gauss Seidal Iteration method.

Since the equations governing the flow are nonlinear, iteration procedure is followed. To carry out the computational procedure, first the momentum equation is solved which gives the values of f necessary for obtaining the solution of coupled energy equation and concentration equations under the given boundary conditions by Thomas algorithm. The numerical solutions of f are considered as $(n+1)^{th}$ order iterative solutions and F are the nth order iterative solutions. To prove convergence of finite difference scheme, the computation is carried out for slightly changed value of h by running same program. No significant change was observed in the value. The convergence criterion used in this study is that the maximum change between the current and the previous iteration values in all the dependent variables satisfy 10^{-5}.

5. Results and Discussion

In this section, by applying the numerical values to different flow parameters, the effects on velocity, temperature and concentration fields are discussed. Graphical illustration of the results is very useful and practical to discuss the effect of different parameters. All the numerical solutions are found for Newtonian and non-Newtonian fluids. In non-Newtonian fluids two cases were considered i.e., Pseudo-Plastic Fluids (n = 0.5) and Dilatant Fluids (n = 1.5). Hence, the graphs are shown for three cases, i.e. Newtonian Fluids (n = 1.0), Pseudo-Plastic Fluids (n = 0.5) and Dilatant Fluids (n = 1.5).

The influence of varying the velocity slip parameter A_1 on the fluid velocity $f'(\eta)$ for Power-law fluids is shown in Figure 1. As the velocity slip parameter increases, the fluid velocity also increases for a given distance from the plate. Figure 2 depicts the variation of fluid temperature with temperature slip parameter B_1 for Power-law fluids. It is evident from the illustrations that the temperature of the power-law fluid $\theta(\eta)$ decreases with the increase in the slip parameter for a given distance from the plate. The thickness of the thermal boundary layer decreases due to the fluid at the surface of the flat plate having a temperature lower than that of the flat plate. Figure 3 shows the behavior of Concentration slip parameter C_1 on the concentration profiles $\phi(\eta)$ for Power-law fluids. It is clear from the graphs that the fluid concentration decreases with an increase in the slip parameter near the plate. Hence, the concentration boundary layer decreases at the surface of the plate.

Figure 4 demonstrates the effect of variation of the Prandtl number Pr on the temperature $\theta(\eta)$. It is revealed from the plots that with an increase in the values of Prandtl number, the fluid temperature reduces. It is due to the fact that by increasing the Prandtl number, the thermal diffusivity of the fluid reduces and hence the temperature also decreases. Figure 5 illustrates the influence of concentration profiles for power-law fluids with different values of Lewis number Le. It shows that effect of Lewis number is to decrease the concentration of the fluid. It is due to the fact that Lewis number (diffusion ratio) is the ratio of Schmidt number and Prandtl number.

The influence of Soret and Dufour on the Temperature and Concentration of the fluids is demonstrated in the Figure 6 and Figure 7, respectively. The Soret effect is a mass flux due to temperature gradient which appears in concentration equation whereas the Dufour effect is heat flux due to concentration gradient which appears in energy equation. We have considered the effects of Soret and Dufour such that their product remains constant. It is clear from the graphs that the increase in Soret number or decrease in Dufour number reduces the fluid temperature and enhances the concentration of the fluid. We notice that this behavior is a direct consequence of Soret effect which produces a mass flux from lower to higher solute concentration driven by the temperature gradient. Hence, increase in Soret number cools the fluid and reduces the temperature.

6. Conclusion

In this paper we considered the flow of non-Newtonian power-law fluid past semi infinite porous flat plate taking into account Soret and Dufour effects. The governing partial differential equations are converted to set of non-linear ordinary differential equations by using similarity

transformations. Then, these equations are solved numerically by implicit Finite Difference Scheme. From the above investigation the following conclusions may be drawn:

- Velocity at the surface of the plate increases with the increase in the velocity slip parameter.
- Thickness of the boundary layer decreases with the increase in the Prandtl number and the temperature slip parameter.
- Concentration boundary layer thickness decreases with the increase in the Lewis number and the concentration slip parameter.
- The effect of Soret number is to reduce the temperature and enhance the concentration.

Acknowledgements

We thank the referees and the in particular the Editor in Chief, Dr. Aliakbar Montazer Haghighi, for their valuable suggestions which helped us in improving the quality of the paper.

REFERENCES

Acrivos, A. (1960). On laminar boundary layer flows with a rapid homogeneous chemical reaction, Chemical engineering science, Vol. 13, No. 2, pp. 57-62.

Afify, A. A. (2009). Similarity solution in MHD effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface considering suction or injection, Communications on Non-Linear Science and Numerical Simulation, Vol. 14, No. 5, pp. 2202-2214.

Anderson, H. I., Bech, K. H. and Dandapat, B.S. (1992). Magnetohydrodynamic flow of a power-law fluid over a stretching sheet, International Journal nonlinear mechanics, Vol. 72, pp. 929-936.

Bellman, R.E. and Kalaba, R.E. (1965). *Quasi-Linearization and Non-linear boundary value problems*, Elsevier, NewYork.

Bhattacharyya, K., Mukhopadhyay, S. and Layek, G. (2011). MHD boundary layer slip flow and heat transfer over a flat plate, Chinese Physics Letters, Vol. 28, No. 2, Article ID 024701.

Cortell, R. (2005). A note on magnetohydrodynamic flow of a power-law fluid over a stretching sheet. Applied Mathematics and Computation, Vol. 168, pp. 557-566.

Damseh, R., Duwairi, H. and Al-Odat M. (2006). Similarity analysis of magnetic field and thermal radiation effects on forced convection flow, Turkish Journal of Engineering and Environmental Sciences, Vol. 30, No. 2, pp. 83-89.

Devi, S. P. A. and Devi, R. U. (2011). Soret and Dufour effects on MHD effects on MHD slip flow with thermal radiation over a porous rotating infinite disk, Communications on Non-Linear Science and Numerical Simulation, Vol. 16, No. 4, pp. 1917-1930.

Hayat, T. and Hendi, F.A. (2012). Thermal-diffusion and diffusion thermo effects on MHD three-dimensional axisymmetric flow with hall andion-slip currents, Journal of American science, Vol. 8, pp. 284-294.

Hirschhorn, J., Madsen, M., Mastroberardino, A., Siddique, J.I. (2016), Magnetohydrodynamic boundary layer slip flow and heat transfer of power-law fluid over a flat plate, Journal of applied fluid mechanics, Vol. 9, No. 1, pp. 11-17.

Howell, T.G., Jeng, D. R. and Dewitt, K.J. (1997). Momentum and heat transfer on a continuous moving surface in a power-law fluid, International Journal of Heat and Mass transfer, Vol. 40, pp. 1853-1861.

Jadhav, B.P. (2013). Laminar Boundary layer flow of non-Newtronian power-law fluid past a porous flat plate, Journal of global research in Mathematical archives, Vol. 1, No. 10, pp. 46-55.

Jain, N. R. and Timol, M. G. (2016). Non-Newtonian Prandtl fluid over stretching permeable surface, Applications and applied Mathematics: An international journal, Vol. 11, No. 1, pp. 238-250.

Kiran Kumar, R. V. M. S. S., Raju, V. C. C., Durga Prasad, P. and Varma, S. V. K. (2016). Heat and Mass Transfer in MHD Micropolar Fluid in The Presence of Diffusion Thermo and Chemical Reaction, Applications and applied Mathematics: An international journal, Vol. 11, No. 2, pp. 704-721.

Kishan, N. and Shashidar Reddy, B. (2011). Quasi-linearization approach to MHD effects on boundary layer flow of power-law fluids past a semi infinite plate with thermal dispersion, International Journal of non-linear science, Vol. 11, No. 3, pp. 301-311.

Kishan, N. and Shashidar Reddy, B. (2013). MHD effects on non-Newtonian power-law fluid past a moving porous flat plate with heat flux and viscous dissipation, International journal of applied mechanics and engineering, Vol. 18, No. 2, pp. 425-445.

Mahapatra, T.R., Nandy, S.K. and Gupta, A.S. (2012). Heat transfer in the magneto Hydrodynamic flow of a power-law fluid past a porous flat plate with suction or blowing, International communication in heat and mass transfer, Vol. 39, No. 1, pp. 17-23.

Martin, M. and Boyd, I. (2006). Momentum and heat transfer in a laminar boundary layer slip flow, Journal of Thermophysics and heat transfer, Vol. 20, No. 4, pp. 710-719.

Oahimire, J. I. and Olajuwon, B. I. (2014). Effects of Radiation Absorption and Thermo-diffusion on MHD Heat and Mass Transfer Flow of a Micro-polar Fluid in the Presence of Heat Source, Applications and applied Mathematics: An international journal, Vol. 9, No. 2, pp. 763-779.

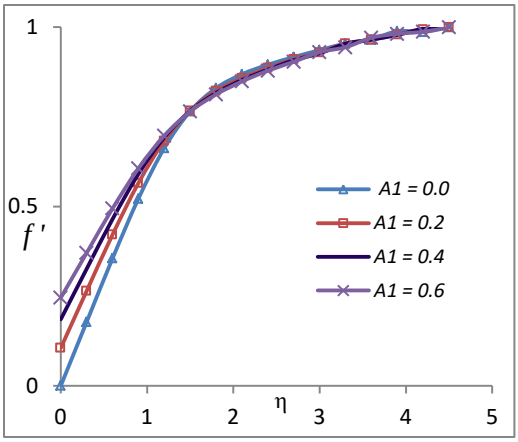
Pal, D. and Chatterjee, S. (2013). Soret and Dufour effects on MHD convective heat and mass transfer of power-law fluid over an inclined plate with variable thermal conductivity in a porous medium, Applied mathematics and computation, Vol. 219, pp. 7556-7574.

Rashidi, M. M., Ali, M., Rostami, B., Rostami, P. and Gong-Nan Xie, (2015). Heat and mass transfer for MHD viscoelastic fluid flow over a vertical stretching sheet with considering Soret and Dufour effects, Mathematical problems in Engineering, Vol. 2015, Article ID 861065, pp. 1-12.

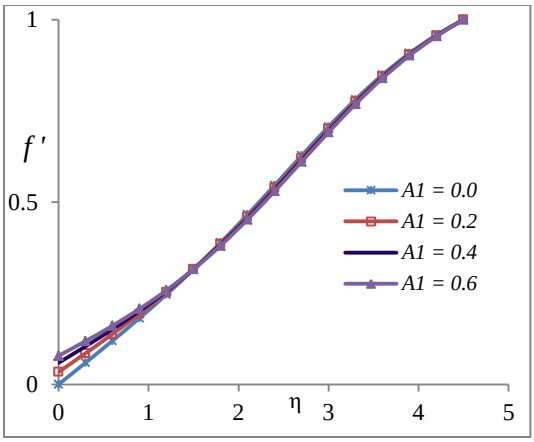
Saritha, K., Rajasekhar, M. N. and Shashidar Reddy, B. (2016). Heat and mass transfer of laminar boundary layer flow of non-Newtonian power-law fluid past a porous flat plate with Soret and Dufour effects, Physical Science International Journal, Vol. 11, No. 1, pp. 1-13.

Schowalter, W. R. (2004). The application of boundary layer theory to power-law pseudo plastic fluids: similar solutions, AIChE Journal, Vol. 6, No. 1, pp. 24-28.

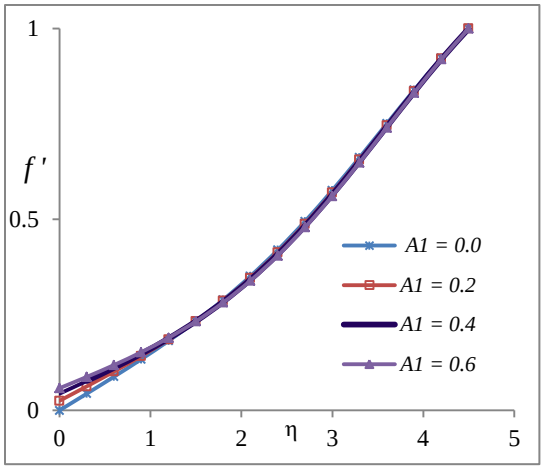
Wubshet Ibrahim and Bandari Shankar, (2013). MHD boundary layer flow and heat transfer of a nano-fluid past a permeable stretching sheet with velocity, thermal solutal slip boundary conditions, Computers and Fluids, Vol. 75, pp.1-10.



Pseudo-Plastic Fluids (n = 0.5)

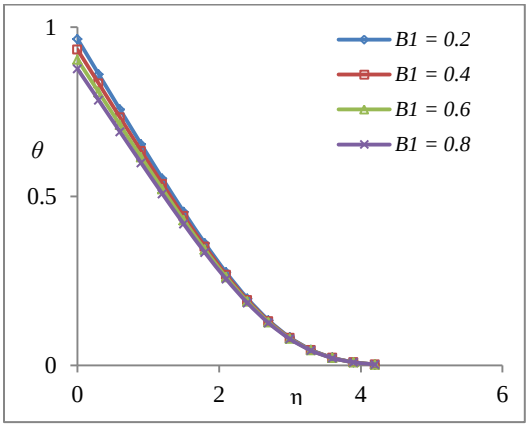


Newtonian Fluids (n = 1.0)

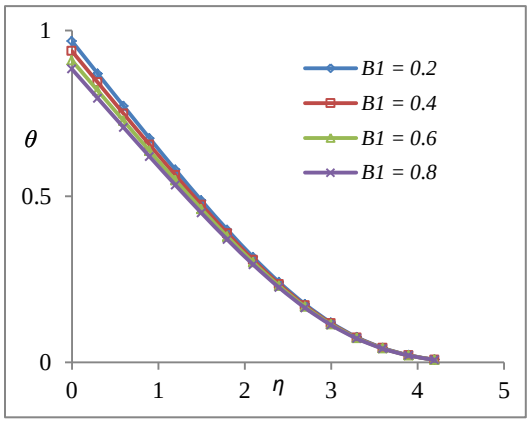


Dilatant Fluids (n = 1.5)

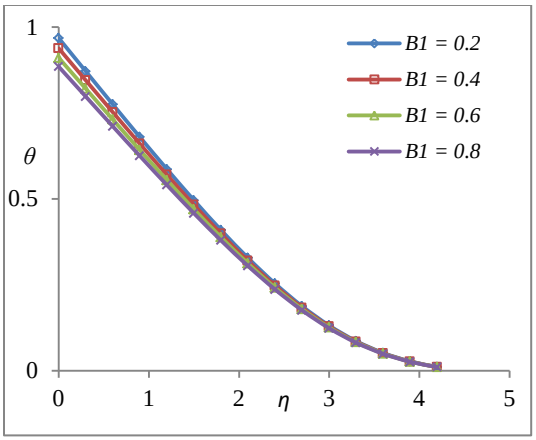
Figure 1. Velocity profiles for various values of Velocity Slip parameter A_1 with Pr = 1, Le = 1, Sr = 0.05, Du = 0.08, M = 0.1, B_1 = 0, C_1 = 0



Pseudo-Plastic Fluids (n = 0.5)

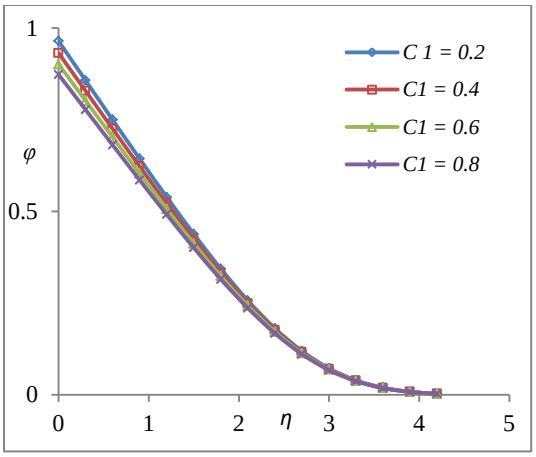


Newtonian Fluids (n = 1.0)

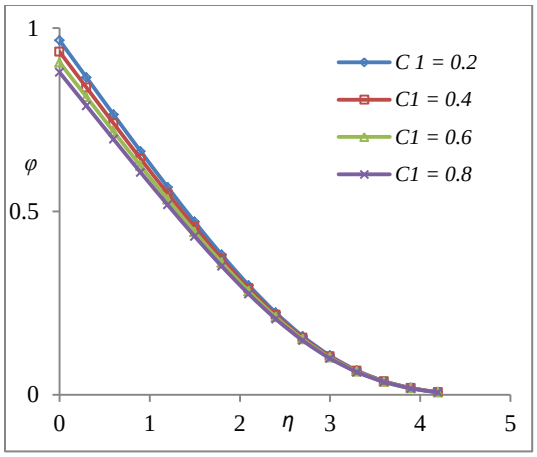


Dilatant Fluids (n = 1.5)

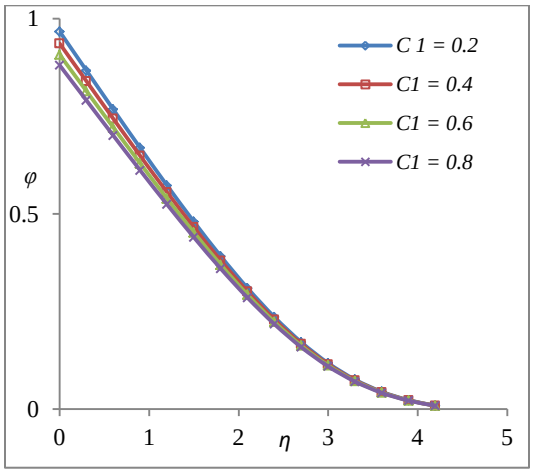
Figure 2. Temperature profiles for various values of Temperature Slip parameter B_1 with Pr = 1, Le = 1, Sr = 0.05, Du = 0.08, M = 0.1, A_1 = 0, C_1 = 0



Pseudo-Plastic Fluids (n = 0.5)

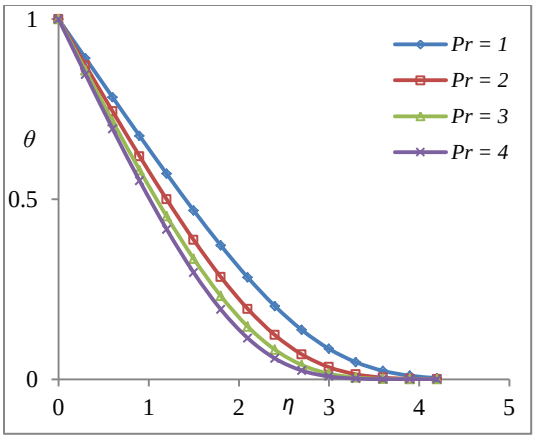


Newtonian Fluids (n = 1.0)

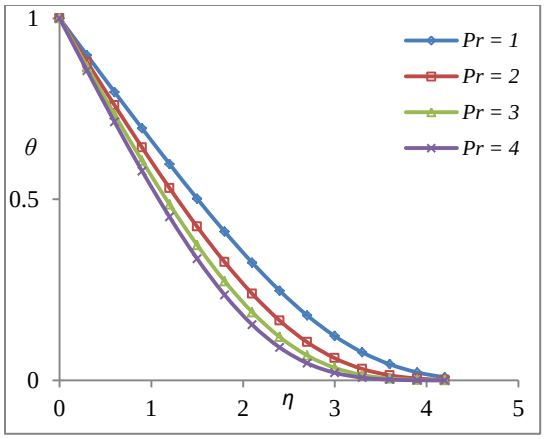


Dilatant Fluids (n = 1.5)

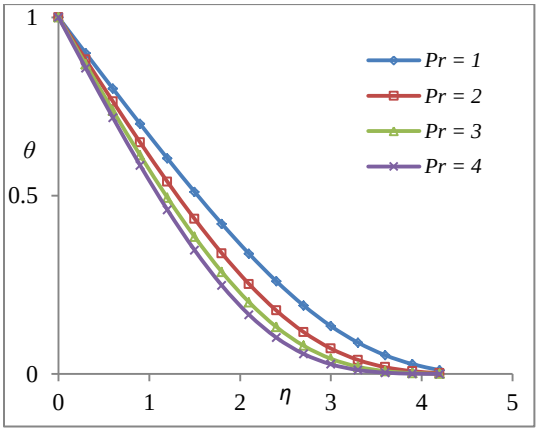
Figure 3. Concentration profiles for various values of Concentration Slip parameter C_1 with Pr = 1, Le = 1, Sr = 0.05, Du = 0.08, M = 0.1, A_1 = 0, B_1 = 0



Pseudo-Plastic Fluids (n = 0.5)

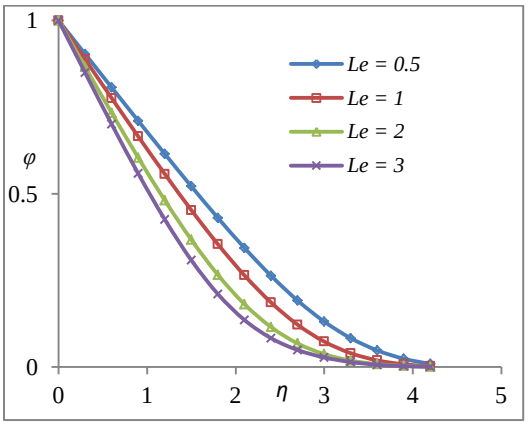


Newtonian Fluids (n = 1.0)

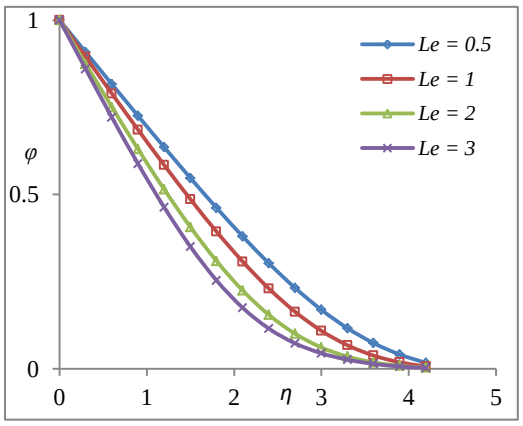


Dilatant Fluids (n = 1.5)

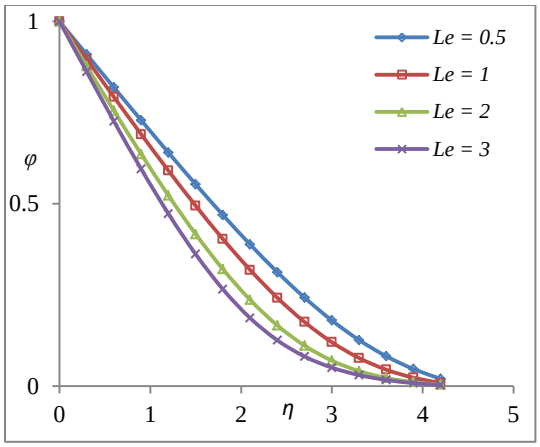
Figure 4. Temperature profiles for various values of Prandtl number Pr with Le = 1, Sr = 0.05, Du = 0.08, M = 0.1, A_1 = 0, B_1 = 0, C_1 = 0



Pseudo-Plastic Fluids (n = 0.5)

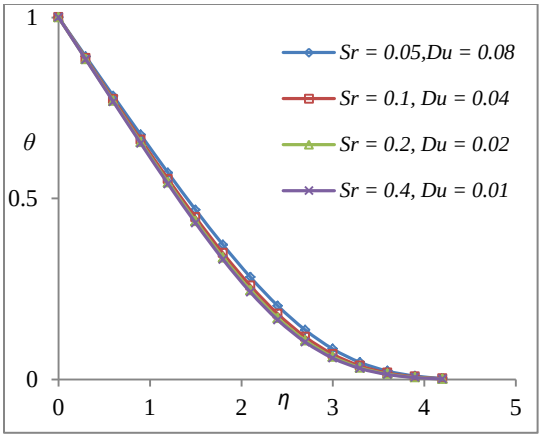


Newtonian Fluids (n = 1.0)

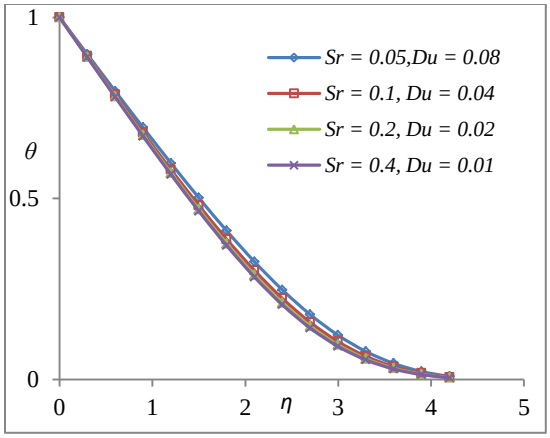


Dilatant Fluids (n = 1.5)

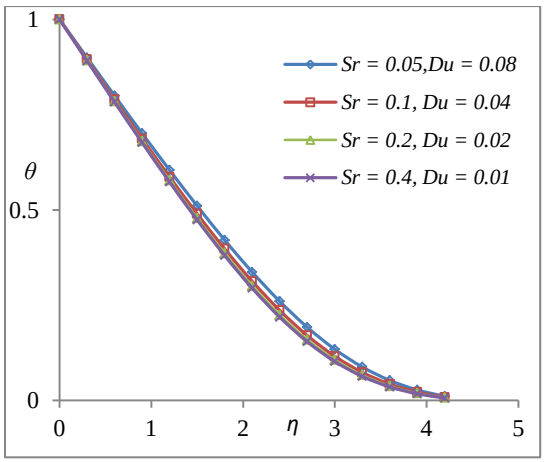
Figure 5. Concentration profiles for various values of Lewis number Le with Pr = 1, Sr = 0.05, Du = 0.08, M = 0.1, A_1 = 0, B_1 = 0, C_1 = 0



Pseudo-Plastic Fluids (n = 0.5)

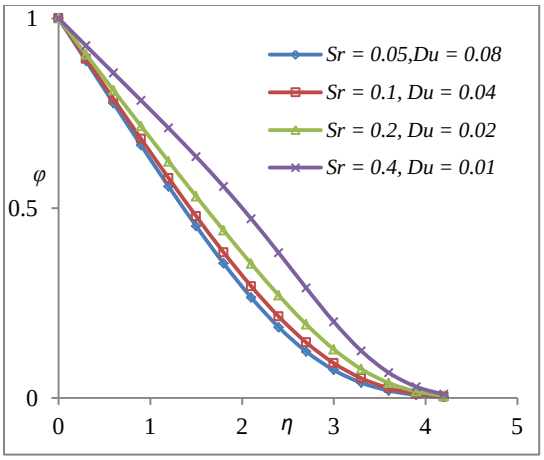


Newtonian Fluids (n = 1.0)

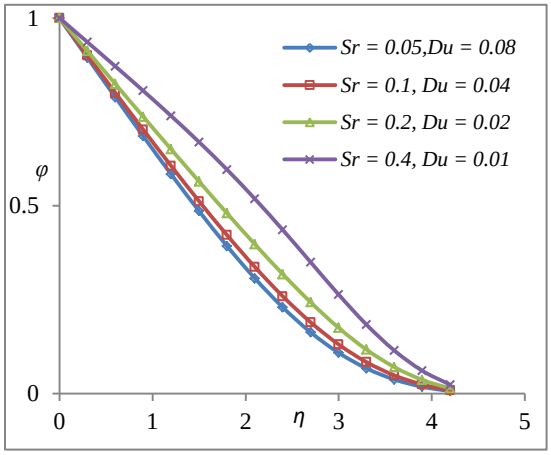


Dilatant Fluids (n = 1.5)

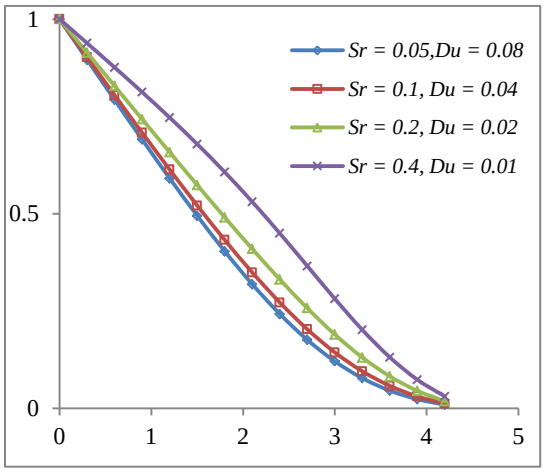
Figure 6. Temperature profiles for various values of Soret and Dufour number with Pr = 1, Le=1, M = 0.1, A_1 = 0, B_1 = 0, C_1 = 0



Pseudo-Plastic Fluids (n = 0.5)



Newtonian Fluids (n = 1.0)



Dilatant Fluids (n = 1.5)

Figure 7. Concentration profiles for various values of Soret and Dufour number with Pr = 1, Le=1, M = 0.1, A_1 = 0, B_1 = 0, C_1 = 0