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Similarity analysis of three dimensional nanofluid flow by deductive group theoretic method

[1]Hemangini Shukla, [2]Hema C. Surati and [3]M.G. Timol

[1]Department of Mathematics
Government Engg. College, Gandhinagar-382028
Gujarat, India
hsshukla94@gmail.com

[2]Department of Mathematics
Sarvajanik College of Engg. & Technology
Surat-395001, Gujarat, India
hema.surati@scet.ac.in

[3]Department of Mathematics
Veer Narmad South Gujarat University, Magdalla Road
Surat-395007, Gujarat, India
mgtimol@gmail.com

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Abstract

The objective of this paper is to obtain similarity solution of three-dimensional nanofluid flow over flat surface stretched continuously in two lateral directions. Two independent variables from governing equations are reduced by applying deductive two parameter group theoretical method. Partial differential equations with boundary conditions are converted into ordinary differential equations with appropriate boundary conditions. Obtained equations are solved for temperature and velocity. The effect of nanoparticles volume fraction on temperature and velocity profile is investigated.

Keywords: Deductive group theoretic method; Nano particles; Nanofluid flow; Similarity solution; Volume fraction

MSC 2010 No: 35Q35, 76M60

Nomenclature:

a , b - positive constants
G- group
u ,v, w - velocity component in X, Y and Z directions respectively
x, y, z-Cartesian co-ordinates
u_w, v_w-flat surface velocity in X and Y directions
T- Temperature of fluid
η - independent similarity variable
T_w- surface temperature
T_∞-ambient temperature
μ_f, μ_{nf}- effective viscosity of the fluid and nanofluid respectively
$\rho_f, \rho_s, \rho_{nf}$- effective density of the fluid, solid particles and nanofluid respectively
$\alpha_f, \alpha_s, \alpha_{nf}$- thermal diffusivity of the fluid, solid particles and nanofluid respectively
a_1, a_2 -group parameter
H, I, J, L, M, N, O-functions of parameters a_1, a_2
g_1, g_2, g_3, g_4-dependent similarity functions
$\alpha_1, \alpha_2, \dots, \alpha_{14}, \beta_1, \beta_2, \dots, \beta_{14}, \lambda_{ij}$ real constants
Pr- Prandtl number
$\vartheta_f, \vartheta_{nf}$-viscocity of fluid and nanofluid
$\emptyset$-solid volume fraction of the nanofluid,
k_f, k_s, k_{nf} - the thermal conductivity of the fluid, solid particles and nanofluid respectively
$(\rho c_p)_f, (\rho c_p)_s, (\rho c_p)_{nf}$- the heat capacitance of the fluid, solid particles and nanofluid respectively
θ_w- wall temperature of the fluid
P^S,Q^S- real-valued and differentiable in their real argument '(a_1, a_2)'.

1. Introduction:

The word nanofluid is introduced by Choi (1995) for suspension of base fluid and nanoparticles of nanometer size. Generally, water, ethylene glycol, oil is taken as base fluid and metals Cu, Ag, Au, Metallic oxides like Al_2o_3, CuO, Nitrides like AlN, SiN, Carbides like SiC, TiC, semiconductors like Tio2, SiC, and different types of carbon nanotubes like SWCNT, DWCNT, MWCNT are used as nanoparticles. There are many applications of nanofluid in different areas such as in automobiles as coolants, brake fluid and as gear lubrication, also in industrial cooling, in solar devices, in cancer drug etc. (2012). So, it is also called new generation heat transfer fluid. Because of wide applications of nanofluid many researchers recently worked on it.

Hayat et al. (2016) investigated the effect of thermophoresis and Brownian motion on Powell-Eyring nanofluid model over linearly stretching sheet using the series solution method. Similarity analysis was done for second-grade nanofluid over the exponentially stretching sheet and derived ordinary differential equations are solved using the homotopy analysis method by Hayat et al. (2015). Xuan and Li (2000) discussed the method of preparing nanofluid. They measured the thermal conductivity of nanofluid for various properties of nanoparticles. Nadeem et al. (2014) investigated nanofluid flow over

exponentially stretching sheet using different types of nanoparticles. Zhao et al. (2015) studied the effect of nanoparticle volume fraction on various parameters for three-dimensional nanofluid flow over a stretching sheet.

Most of the analysis is done on a nanofluid flow by assuming similarity variable. Here we deduced similarity variables systematically. Here we reduced two independent variables in governing equation by using deductive two parameter group theoretical method. The purpose of the method is to obtain proper groups for a given system of equations. Moran and Gajjoli (1968) had given remarkable contribution in similarity analysis. In the method, they developed deductive group formalism for similarity analysis and also included auxiliary conditions for boundary layer problems. Recently, Similarity solution is derived for Sisko fluid using dimensional analysis method by Surati et al. (2016).

Al-salihi et al. (2013) derived more than twenty similarity equations using the deductive two-parameter method for the unsteady MHD power-law fluid model. El-Hawary et al. (2014) considered the non-Newtonian Hiemenz flow and used the deductive group-theoretic method to reduce two independent variables. Darji and Timol (2014) solved the problem of unsteady natural convection flow of Sisko fluid using the two-parameter deductive group-theoretic method. The deductive group-theoretic method for quasi-three-dimensional power-law fluid is applied by Jain and Timol (2015). They solved the problem numerically using method of MSABC. Powell-Eyring and Prandtl-Eyring fluid models are analysed to find effects of the different physical parameter on velocity and temperature using the one-parameter group theoretic method by Shukla et al. (2017).

In this paper, we derived a complete set of similarity variables and then using these similarity variables we had converted set of partial differential equations given in governing equations into ordinary differential equations.

2. Governing Equation (2015)

Present research is based on the following assumption.

- Flow is steady, laminar and three dimensional.
- Nano fluid model is homogeneous.
- Nano fluid is incompressible.
- The shape of nanoparticles is spherical.
- The flat surface is stretching continuously in both x-and y-directions with the velocities $u_w = ax$ and $v_w = by$, respectively.
- The surface temperature is T_w and the ambient temperature is T_∞,where T_w and T_∞ are two constants, with$T_w > T_\infty$.
- The nanoparticles considered here are Cu, and the base fluid is water.

Using the mathematical model for nanofluids proposed by Choi (1995), the governing equations are given by (refer to (2015))

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0. \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial z^2}. \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 v}{\partial z^2}. \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha_{nf}\frac{\partial^2 T}{\partial z^2}. \tag{4}$$

Boundary conditions:

$$u(x,y,0)=u_w=ax, v(x,y,0)=v_w=by, w(x,y,0)=0. \tag{5}$$

$$T(x,y,0)=T_w, u(x,y,\infty)=0, v(x,y,\infty)=0, T(x,y,\infty)=T_\infty. \tag{6}$$

Take,

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}. \tag{7}$$

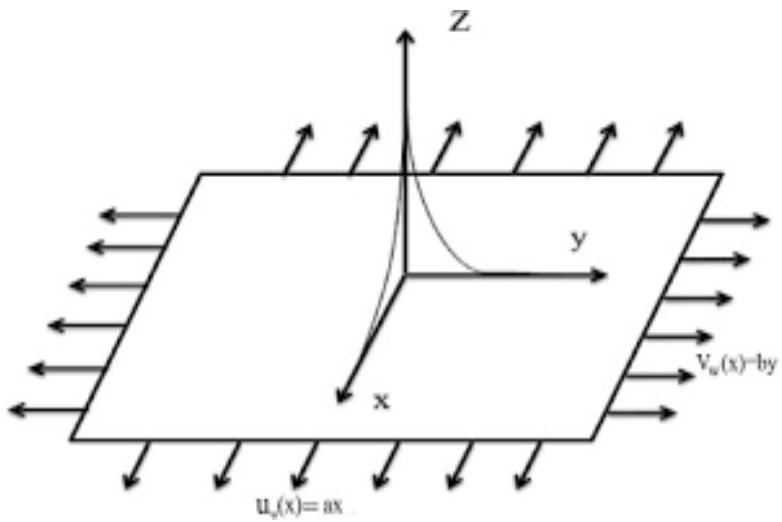

Figure 1. Physical model and coordinate system

μ_{nf}, ρ_{nf}, and α_{nf} are the effective viscosity of the nanofluid, the effective density of the nanofluid, and the effective thermal diffusivity of the nanofluid, respectively, which are defined as

$$\mu_{nf}=\frac{\mu_f}{(1-\emptyset)^{2.5}}, \rho_{nf}=(1-\emptyset)\rho_f+\emptyset\rho_s, \alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}},$$
$$(\rho c_p)_{nf}=(1-\emptyset)(\rho c_p)_f+\emptyset(\rho c_p)_s, \frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\emptyset(k_f-k_s)}{(k_s+2k_f)+\emptyset(k_f-k_s)}, \tag{8}$$

where $\emptyset$ is the solid volume fraction of the nanofluid, ρ is the density, k is the thermal conductivity, ρc_p is the heat capacitance. Subscript nf is for nanofluid, s for solid particles, f for base fluid. Formula for μ_{nf} is given by Brinkman (1952) and k_{nf} is by Maxwell-Garnett's Model (2005).

3. Deductive two parameter group theoretic method

The method used in this paper is the Deductive group-theoretic method. Under this General group of transformation, the three independent variables will be reduced by two and the boundary value type partial differential equations (1)-(4) which has three independent variables x, y and z transform into boundary value type ordinary differential equations in only one-independent variable, which is called similarity equation. Following is the group of transformations of two parameters (a_1, a_2) in the form of

$$G{:}\bar{s} = P^s(a_1, a_2)s + Q^s(a_1, a_2), \tag{9}$$

where s stands for x, y, z, u, v, w, θ. $P^S$and $Q^S$are real-valued and at least differentiable in their real argument (a_1, a_2).

3.1. The invariance analysis

Derivatives of the transformation are obtained from G using chain rule.

$$\bar{s}_{\bar{i}} = \frac{P^s}{P^i} s_i,\ \bar{s}_{\bar{i}\bar{j}} = \frac{P^s}{P^i P^j} s_{ij}, \tag{10}$$

where i and j stands for x, y, z and s stands for u, v, w, θ.

Equations (1) to (4) remain invariant under group of transformations defined by G in equation (9) and derivatives in equation (10)

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} + \frac{\partial \bar{w}}{\partial \bar{z}} = H(a_1, a_2)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right). \tag{11}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{u}}{\partial \bar{z}} - \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 \bar{u}}{\partial \bar{z}^2} = I(a_1, a_2)\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial z^2}\right) + J(a_1, a_2). \tag{12}$$

$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{v}}{\partial \bar{z}} - \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 \bar{v}}{\partial \bar{z}^2} = L(a_1, a_2)\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} - \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 v}{\partial z^2}\right) + M(a_1, a_2). \tag{13}$$

$$\bar{u}\frac{\partial \bar{\theta}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{\theta}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{\theta}}{\partial \bar{z}} - \alpha_{nf}\frac{\partial^2 \bar{\theta}}{\partial \bar{z}^2} = N(a_1, a_2)\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} - \alpha_{nf}\frac{\partial^2 \theta}{\partial z^2}\right) + O(a_1, a_2). \tag{14}$$

The invariance of above equations implies that

$$J(a_1, a_2) = M(a_1, a_2) = O(a_1, a_2) = 0. \tag{15a}$$

This is satisfied if we take

$$Q^u = Q^v = Q^w = 0. \tag{15b}$$

and

$$H(a_1, a_2) = \frac{P^u}{P^x} = \frac{P^v}{P^y} = \frac{P^w}{P^z}. \tag{16}$$

$$I(a_1, a_2) = \frac{(P^u)^2}{P^x} = \frac{P^u P^v}{P^y} = \frac{P^u P^w}{P^z} = \frac{P^u}{(P^z)^2}\,. \tag{17}$$

$$L(a_1, a_2) = \frac{(P^v)^2}{P^y} = \frac{P^u P^v}{P^x} = \frac{P^v P^w}{P^z} = \frac{P^v}{(P^z)^2}\,. \tag{18}$$

$$N(a_1, a_2) = \frac{P^u P^\theta}{P^x} = \frac{P^v P^\theta}{P^y} = \frac{P^w P^\theta}{(P^z)^2} = \frac{P^\theta}{(P^z)^2}\,. \tag{19}$$

So, from equation (15) to (19) with boundary conditions (5) to (7) we get following relations.

$$P^{\mathrm{u}} = P^x, P^{\mathrm{v}} = P^{\mathrm{y}}, P^{\mathrm{w}} = P^{\mathrm{z}} = P^{\theta} = 1. \tag{20}$$

$$\mathrm{Q}^{\mathrm{u}} = Q^{\mathrm{x}} = \mathrm{Q}^{\mathrm{v}} = Q^{\mathrm{y}} = \mathrm{Q}^{\mathrm{w}} = Q^{\mathrm{z}} = \mathrm{Q}^{\theta} = 0. \tag{21}$$

Thus, we obtained a two-parameter group transformation of the form

$$G: \begin{cases} \bar{x} = P^x x, \\ \bar{y} = P^y y, \\ \bar{z} = z, \\ \bar{u} = P^x u, \\ \bar{v} = P^y v, \\ \bar{w} = w, \\ \bar{\theta} = \theta. \end{cases} \tag{22}$$

3.2. The Complete set of absolute invariants

Now, we proceed in our analysis to obtain a complete set of absolute invariants so that the original problem will transformed into an ordinary differential equation in a similarity variable via group theoretic method. The application of a basic theorem in group theory; [see Moran and Gaggioli (1968)] States that: A function g_j is an absolute invariant of a one-parameter group if it satisfies the following first-order linear differential equation:

$$(\alpha_1 x_1 + \alpha_2)\frac{\partial g}{\partial x_1} + (\alpha_3 x_2 + \alpha_4)\frac{\partial g}{\partial x_2} + (\alpha_5 y_1 + \alpha_6)\frac{\partial g}{\partial y_1} + (\alpha_7 y_2 + \alpha_8)\frac{\partial g}{\partial y_2}$$
$$+(\alpha_9 y_3 + \alpha_{10})\frac{\partial g}{\partial y_3} + (\alpha_{11} y_4 + \alpha_{12})\frac{\partial g}{\partial y_4} + (\alpha_{13} y_5 + \alpha_{14})\frac{\partial g}{\partial y_5} = 0. \tag{23a}$$

$$(\beta_1 x_1 + \beta_2)\frac{\partial g}{\partial x_1} + (\beta_3 x_2 + \beta_4)\frac{\partial g}{\partial x_2} + (\beta_5 y_1 + \beta_6)\frac{\partial g}{\partial y_1} + (\beta_7 y_2\, \beta_8)\frac{\partial g}{\partial y_2}$$
$$+(\beta_9 y_3 + \beta_{10})\frac{\partial g}{\partial y_3} + (\beta_{11} y_4 + \beta_{12})\frac{\partial g}{\partial y_4} + (\beta_{13} y_5 + \beta_{14})\frac{\partial g}{\partial y_5} = 0, \tag{23b}$$

where

$$\alpha_i = \frac{\partial P^{s_i}}{\partial a_1}\Big|(a_1^0, a_2^0)\,, \alpha_{i+1} = \frac{\partial Q^{s_i}}{\partial a_1}\Big|(a_1^0, a_2^0)$$

and

$$\beta_i = \frac{\partial P^{s_i}}{\partial a_2}\Big|(a_1^0, a_2^0)\,, \beta_{i+1} = \frac{\partial Q^{s_i}}{\partial a_2}\Big|(a_1^0, a_2^0)\,, i = 1,3,5,7,9,11,13, \tag{24}$$

where (a_1^0, a_2^0) denotes the value of which yield the identity element of the group.

By considering $x_1 = x, x_2 = y, y_1 = z, y_2 = \mathrm{u}, y_3 = \mathrm{v}, y_4 = \mathrm{w}, y_5 = \theta$.

3.2.1. Independent absolute invariant:

Now we obtained independent absolute invariants.

From first order differential equations in (23a), (23b) and using (24) we get

$$(\alpha_1 x)\frac{\partial \eta}{\partial x} + (\alpha_3 y)\frac{\partial \eta}{\partial y} + (\alpha_5 z)\frac{\partial \eta}{\partial z} = 0, \tag{25a}$$

and

$$(\beta_1 x)\frac{\partial \eta}{\partial x} + (\beta_3 y)\frac{\partial \eta}{\partial y} + (\beta_5 z)\frac{\partial \eta}{\partial z} = 0. \tag{25b}$$

Here, $\alpha_2 = \alpha_4 = \alpha_6 = 0$, since $Q^x = Q^y = Q^z = 0$.

By eliminating $\frac{\partial \eta}{\partial y}, \frac{\partial \eta}{\partial x}$ from equation (25a) and (25b) we get

$$\lambda_{13} x\frac{\partial \eta}{\partial x} + \lambda_{53} z\frac{\partial \eta}{\partial z} = 0, \tag{26a}$$

and

$$-\lambda_{13} y\frac{\partial \eta}{\partial y} + \lambda_{51} z\frac{\partial \eta}{\partial z} = 0, \tag{26b}$$

where $\lambda_{ij} = \alpha_i\beta_j - \alpha_j\beta_i$.

According to the basic theorem of group theory, an equation has one and only one solution if the coefficient matrix has a rank two. The matrix has rank two whenever at least one of its two by two submatrices has a non-vanishing determinant. So, we have the following case.

case (i): $\lambda_{53} \neq 0, \lambda_{13} \neq 0, \lambda_{51} = 0$

Using the definitions of α_i's and $\beta_i's$ in equation (23a), (23b) and from (20), (21) and (22) we got

$$\alpha_5 = \beta_5 = 0.$$
$$\lambda_{51} = \alpha_5\beta_1 - \alpha_1\beta_5 = 0.$$
$$\frac{\partial \eta}{\partial y} = 0. \tag{27}$$

η is a function of only x and z.

But $\alpha_5 = 0$.

$\eta = Az$, where A is an arbitrary constant.

case (ii): $\lambda_{53} = 0, \lambda_{51} \neq 0, \lambda_{13} \neq 0$

But we have $\lambda_{51} = 0$. ($\alpha_5 = \beta_5 = 0$)

Here the rank of the coefficient matrix is one So this case is not possible.

case (iii): $\lambda_{53} \neq 0, \lambda_{13} = 0, \lambda_{51} \neq 0$

Here also, rank is one so this case is not possible.

So, from all cases

$$\eta = Az. \tag{28}$$

Similarly, the absolute invariants of the dependent variables u, v, w, θ are obtained from the group transformation.

3.2.2. The absolute invariants of the dependent variables.

3.2.2.1. The absolute invariants of the dependent variable *u*.

$$(\alpha_1 x)\frac{\partial g_1}{\partial x} + (\alpha_3 y)\frac{\partial g_1}{\partial y} + (\alpha_7 u)\frac{\partial g_1}{\partial u} = 0,$$
$$(\beta_1 x)\frac{\partial g_1}{\partial x} + (\beta_3 y)\frac{\partial g_1}{\partial y} + (\beta_7 u)\frac{\partial g_1}{\partial u} = 0. \tag{29}$$

Eliminating $\frac{\partial g_1}{\partial x}, \frac{\partial g_1}{\partial y}$

$$(\lambda_{31} y)\frac{\partial g_1}{\partial y} + (\lambda_{71} u)\frac{\partial g_1}{\partial u} = 0,$$
$$(-\lambda_{31} x)\frac{\partial g_1}{\partial x} + (\lambda_{73} u)\frac{\partial g_1}{\partial u} = 0. \tag{30}$$

case (i): $\lambda_{31} \neq 0, \lambda_{71} = 0, \lambda_{73} \neq 0$ (Because $\alpha_1 = \alpha_7, \beta_1 = \beta_7$)

Using the definitions of α_i's and $\beta_i's$ from (21) and (22) we have for $\lambda_{71} = 0$

$\frac{\partial g_1}{\partial y} = 0$. (Because in equation (30) put $\lambda_{71} = 0$).

From equation (29) using separable variable method we get

$$g_1(\eta) = \frac{x}{u}.$$

case (ii): $\lambda_{31} = 0, \lambda_{71} \neq 0, \lambda_{73} \neq 0$

But we have $\lambda_{71} = 0$. (Because $\alpha_1 = \alpha_7, \beta_1 = \beta_7$). Here, rank of the coefficient matrix is one so this case is not possible.

case (iii): $\lambda_{31} \neq 0, \lambda_{71} \neq 0, \lambda_{73} = 0$

But we have $\lambda_{71} = 0$. (Because $\alpha_1 = \alpha_7, \beta_1 = \beta_7$). Here, also, rank is one so this case is not possible. Thus, we get

$$x = u\mathrm{g}_1(\eta). \tag{31}$$

Similarly, we get,$\mathrm{g}_2(\eta) = \frac{v}{y}$, $\mathrm{g}_3(\eta) = Bw$, $\mathrm{g}_4(\eta) = C\theta$.

As a special case, we choose

$$\mathrm{A}=\sqrt{\frac{a}{\vartheta_f}}\ B = -\sqrt{\frac{\vartheta_f}{a}}, C = 1. \tag{32}$$

Thus, we got following absolute invariants:

$$\begin{aligned}
&\mathrm{g}_1(\eta) = \frac{x}{u},\\
&\mathrm{g}_2(\eta) = \frac{v}{y},\\
&\mathrm{g}_3(\eta) = -\sqrt{\frac{\vartheta_f}{a}}w,\\
&\mathrm{g}_4(\eta) = \theta.
\end{aligned} \tag{33}$$

4. The reduction to an ordinary differential equation

Differentiating equations in (33) with respect to x, y, z and applying on equation (1) to (4), we get following

$$\mathrm{g}_3'(\eta) - (\mathrm{g}_1(\eta) + \mathrm{g}_2(\eta)) = 0. \tag{34}$$

$$a\varepsilon_1\mathrm{g}_1''(\eta) + \mathrm{g}_1'(\eta)\mathrm{g}_3(\eta) - (\mathrm{g}_1(\eta))^2 = 0. \tag{35}$$

$$a\varepsilon_1\mathrm{g}_2''(\eta) + \mathrm{g}_2'(\eta)\mathrm{g}_3(\eta) - (\mathrm{g}_2(\eta))^2 = 0. \tag{36}$$

$$\mathrm{g}_4'(\eta)\mathrm{g}_3(\eta) + \frac{a\varepsilon_2}{pr}\mathrm{g}_4''(\eta) = 0. \tag{37}$$

With boundary conditions:

$$\begin{aligned}
&\mathrm{g}_1(0) = a, \mathrm{g}_2(0) = b, \mathrm{g}_3(0) = 0, \mathrm{g}_4(0) = 1,\\
&\mathrm{g}_1(\infty) = 0, \mathrm{g}_2(\infty) = 0, \theta(\infty) = 0,
\end{aligned} \tag{38}$$

where

$$\varepsilon_1 = \frac{1}{(1-\emptyset)^{2.5}\{(1-\emptyset)+\emptyset\frac{(\rho c_p)_s}{(\rho c_p)_f}\}} = \frac{\vartheta_{nf}}{\vartheta_f}, \varepsilon_2 = \frac{\frac{k_{nf}}{k_f}}{\{(1-\emptyset)+\emptyset\frac{\rho_s}{\rho_f}\}}, \frac{\varepsilon_2}{pr} = \frac{\alpha_{nf}}{\vartheta_f} \ , \ pr = \frac{\vartheta_f}{\alpha_f}. \tag{39}$$

Thermal conductivity of nanofluid depends on parameters like particle volume fraction, particle material, particle size, particle shape, temperature, base fluid properties. We analysed thermal conductivity of nanofluid depends on parameter nanoparticle volume fraction.

5. Numerical Solution:

Table 1. Values of physical properties.

Physical properties	Base fluid(water)	Nano particles (Cu)
$\boldsymbol{c_p}$ (J/kg K)	4179	385
$\boldsymbol{\rho}$ (kg/m^3)	997.1	8933
$\boldsymbol{k}$ (W/mK)	0.613	400

Values of parameters are as follows for following tables.

$a = 2, b = 1, pr = 1$
For $\emptyset = 0, \varepsilon_1 = 1, \varepsilon_2 = 1$
For $\emptyset = 0.1, \varepsilon_1 = 0.7246, \varepsilon_2 = 1.3553$
For $\emptyset = 0.2, \varepsilon_1 = 0.6740, \varepsilon_2 = 1.8089$

Table 2. Values of g_1 for different values of $\emptyset$

η	g_1-values (for $\emptyset = 0$)	g-values (for $\emptyset = 0.1$)	g_1-values (for $\emptyset = 0.2$)
0	2	2	2
0.25	1.60465	1.54682	1.53285
0.5	1.30012	1.21269	1.19207
0.75	1.06197	0.96143	0.93822
1	0.87314	0.76913	0.74557
2	0.41330	0.33177	0.31439
3	0.19082	0.14181	0.13183
4	0.07008	0.04870	0.04452
5	0	0	0

Table 3. Values of g_2 for different values of $\emptyset$

η	g_2-values (for $\emptyset = 0$)	g_2-values (for $\emptyset = 0.1$)	g_2-values (for $= 0.2$)
0	1	1	1
0.25	0.84131	0.81752	0.81170
0.5	0.70875	0.67002	0.66071
0.75	0.59781	0.55048	0.53931
1	0.50467	0.45320	0.44125
2	0.25504	0.20929	0.19926
3	0.12090	0.09181	0.08573
4	0.04475	0.03176	0.02917
5	0	0	0

Table 4. Values of g_4 for different values of $\emptyset$

η	g_4-values (for $\emptyset = 0$)	g_4-values (for $\emptyset = 0.1$)	g_4-values (for $\emptyset = 0.2$)
0	1	1	1
0.25	0.90239	0.91864	0.92745
0.5	0.80735	0.83883	0.85594
0.75	0.71676	0.76168	0.78621
1	0.63183	0.68793	0.71876
2	0.35681	0.43278	0.47638
3	0.17717	0.24069	0.27948
4	0.06650	0.10071	0.12311
5	0	0	0

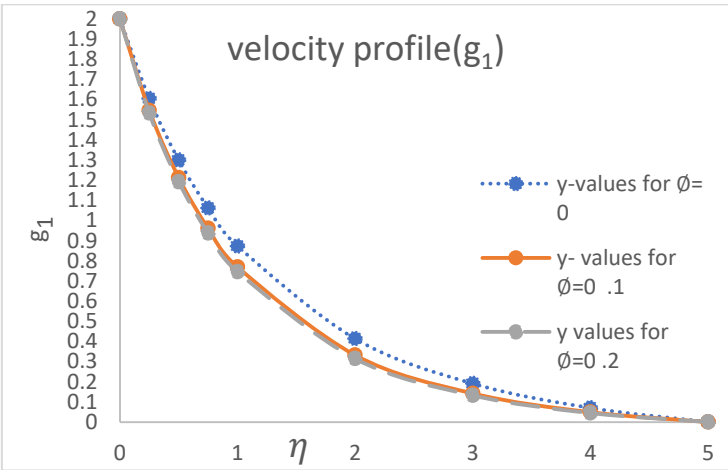

Figure 2. Effect of $\emptyset$ on velocity profile g_1

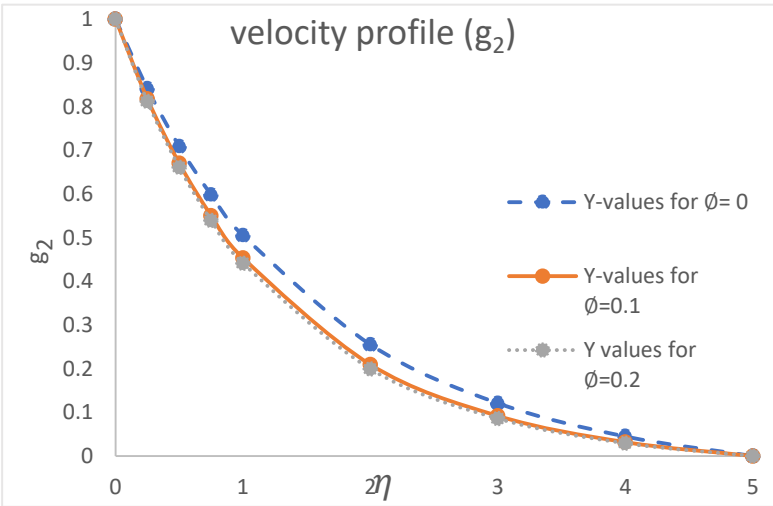

Figure 3. Effect of $\emptyset$ on velocity profile

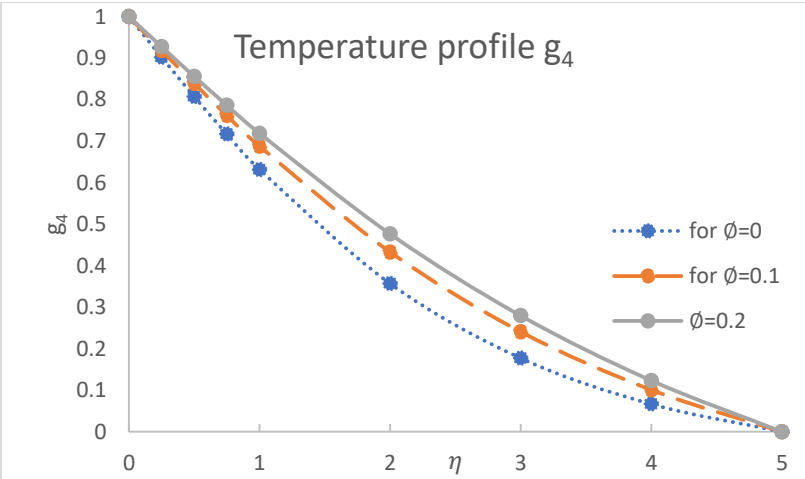

Figure 4. Effect of $\emptyset$ on Temperature profile g_4

6. Conclusion

Deductive two parameter group theory technique is applied to obtain similarity equations by transforming partial differential equations into ordinary differential equations. Using MAPLE ODE Solver, we solved the similarity equations. Here effects of the nanoparticles volume fraction $\emptyset$ on the fluid flow is investigated. We know that for pure fluid $\emptyset = 0$ and for nanofluid $\emptyset \geq 0$ .So, here we compared properties of pure fluid with nanofluid for different amount of nanoparticle volume fraction. We examined the influence of nanoparticle volume fraction $\emptyset$ on the velocity and temperature distributions.

Figures 2 and 3 and Tables 2 and 3 shows that for all values of $\emptyset$ the velocity profiles g_1 and g_2 decrease continuously as η increases. The descending trend becomes slower as $\emptyset$ increases. Figure 4 and Table 4 shows temperature profile for different values of $\emptyset$. Here also we had seen same nature for temperature profile g_4 as velocity profiles. Temperature profile g_4 decrease as η increases. Here, we seen opposite nature than velocity profile for different values of $\emptyset$. For temperature profile g increases as $\emptyset$ increases. Thus, we can say that nanofluids have higher thermal conductivity than pure fluid and g_4 increases as $\emptyset$ increases.

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