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Fractional order thermoelastic thick circular plate with two temperatures in frequency domain

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Abstract

The present investigation is concerned with thermomechanical interactions in the fractional theory of thermoelasticity for a homogeneous isotropic thick circular plate in the light of two-temperature thermoelasticity theory in frequency domain. The upper and lower surfaces of the thick plate are traction free and subjected to an axisymmetric heat supply. The solution is found by using Hankel transform technique and a direct approach without the use of potential functions. The analytical expressions of displacement components, stresses, conductive temperature, temperature change and cubic dilatation are computed in transformed domain. Numerical inversion technique has been applied to obtain the results in the physical domain. Numerically simulated results are depicted graphically. The effect of fractional parameter has been shown by taking different values on the components of stress, cubic dilatation and displacement.

Keywords: Fractional parameter; two-temperature; isotropic; thick circular plate; frequency domain; Hankel transform

MSC 2010 No.: 74A10, 74A15, 74B05, 74D05, 74E10, 74F05

1. Introduction

The use of fractional order derivatives and integrals leads to the formulation of certain physical problems which is more economical and useful than the classical approach. There exists many material and physical situations like amorphous media, colloids, glassy and porous materials, manmade and biological materials/polymers, transient loading etc., where the classical thermoelasticity based on Fourier type heat conduction breaks down. In such cases, one needs to use a generalized thermoelasticity theory based on an anomalous heat conduction model involving time fractional (non- integer order) derivatives.

Povstenko (2005) proposed a quasi-static uncoupled theory of thermoelasticity based on the heat conduction equation with a time-fractional derivative of order α. Because the heat conduction equation in the case $1 \leq \alpha \leq 2$ interpolates the parabolic equation (α = 1) and the wave equation (α = 2), this theory interpolates a classical thermoelasticity and a thermoelasticity without energy dissipation. He also obtained the stresses corresponding to the fundamental solutions of a Cauchy problem for the fractional heat conduction equation for one-dimensional and two-dimensional cases.

Povstenko (2009) investigated the nonlocal generalizations of the Fourier law and heat conduction by using time and space fractional derivatives. Youssef (2010) proposed a new model of thermoelasticity theory in the context of a new consideration of heat conduction with fractional order and proved the uniqueness theorem. Jiang and Xu (2010) obtained a fractional heat conduction equation with a time fractional derivative in the general orthogonal curvilinear coordinate and also in other orthogonal coordinate system. Povstenko (2010) investigated the fractional radial heat conduction in an infinite medium with a cylindrical cavity and associated thermal stresses.

Ezzat (2011a) constructed a new model of the magneto-thermoelasticity theory in the context of a new consideration of heat conduction with fractional derivative. Ezzat (2011b) studied the problem of state space approach to thermoelectric fluid with fractional order heat transfer. The Laplace transform and state-space techniques were used to solve a one-dimensional application for a conducting half space of thermoelectric elastic material. Povstenko (2011) investigated the generalized Cattaneo-type equations with time fractional derivatives and formulated the theory of thermal stresses.

Biswas and Sen (2011) proposed a scheme for optimal control and a pseudo state space representation for a particular type of fractional dynamical equation. Ezzat and Ezzat (2016) constructed fractional thermoelasticity applications for porous asphaltic materials. Several researchers **(**Ezzat and Bary (2016), Marin and Baleanu (2016), Kumar, Sharma and Lata (2016a, 2016b, 2016c, 2017)) presented modelling of fractional magneto-thermoelasticity for perfect conducting materials. Xiong and Niu (2017) established fractional order generalized thermoelastic diffusion theory for anisotropic and linearly thermoelastic diffusive media.

In this investigation, the thermal interactions for the fractional order heat conduction in a thick circular plate is studied in the light of two temperature thermoelasticity theory in frequency domain. The components of displacements, stresses, conductive temperature, temperature change and cubic dilatation are computed numerically. Numerically computed results are depicted graphically. The effect of fractional order parameter is shown on the various components.

2. Basic Equations

Following Kumar et al. (2016) and Ezzat (2011a), the basic equations of motion, heat conduction in the fractional theory of thermoelasticity for a homogeneous isotropic thermoelastic solid with two temperature in the absence of body forces, heat sources are

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u})+\mu\nabla^2\boldsymbol{u}-\beta_1\nabla\mathrm{T}=\rho\ddot{\boldsymbol{u}}, \tag{1}$$

$$k\nabla^2T=\frac{\partial}{\partial t}\left(1+\frac{(\tau_0)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)(\rho C_E T+\beta_1 T_0\nabla.\boldsymbol{u}), \tag{2}$$

$$T=(1-a\nabla^2)\varphi,$$

and the constitutive relations are

$$t_{ij}=2\mu e_{ij}+\delta_{ij}(\lambda e_{kk}-\beta_1 T). \tag{3}$$

Following Caputo (1967), the fractional derivative of order $\alpha\in(0,1]$ of the absolutely continuous function $f(t)$ is

$$\frac{d^\alpha}{dt^\alpha}f(t)=I^{1-\alpha}f'(t), \tag{4}$$

and the fractional integral

$$I^\alpha f(t)=\int_0^t\frac{(t-\tau)^{\alpha-1}}{\Gamma(\alpha)}f(\tau)d\tau\,,\quad \alpha>0, \tag{5}$$

where I^α is the fractional integral of the function $f(t)$ of order α defined by Miller and Ross (1993). λ , μ are Lame's constants, ρ is the density assumed to be independent of time, u_i are components of displacement vector $\boldsymbol{u}$, k is the coefficient of thermal conductivity, C_E is the specific heat at constant strain, T is the absolute temperature of the medium, φ is the conductive temperature, t_{ij} and e_{ij} are the components of stress and strain respectively, e_{kk} is dilatation, $\beta_1=(3\lambda+2\mu)\alpha_t$, α_t is the coefficient of thermal linear expansion, α is the fractional order parameter and τ_0 is the relaxation time, a is the two temperature parameter.

3. Formulation and solution of the problem

Consider a thick circular plate of thickness *2b* occupying the space *D* defined by $0\le r\le\infty$, $-b\le z\le b$. Let the plate be subjected to an axisymmetric heat supply depending on the radial and axial directions of the cylindrical co-ordinate system. The initial temperature in the thick plate is given by a constant temperature T_0, and the heat flux $g_0F(r,z)$ is prescribed on the upper and lower boundary surfaces. Under these conditions, the thermoelastic quantities in a thick circular plate are required to be determined. We take a cylindrical polar co-ordinate system (r,θ,z) with symmetry about z –axis. As the problem considered is plane axisymmetric, the field component $u_\theta=0$, and u_r, u_z, T, φ and e are independent of θ and restrict our analysis to the two dimensional problem with

$$\boldsymbol{u} = (u_r,\ 0,\ u_z). \tag{6}$$

Equations (1) - (2) with the aid of (6) take the form

$$(\lambda+\mu)\frac{\partial e}{\partial r} + \mu\left(\nabla^2 - \frac{1}{r^2}\right)u_r - \beta_1\frac{\partial T}{\partial r} = \rho\frac{\partial^2 u_r}{\partial t^2}, \tag{7}$$

$$(\lambda+\mu)\frac{\partial e}{\partial z} + \mu\nabla^2 u_z - \beta_1\frac{\partial T}{\partial z} = \rho\frac{\partial^2 u_z}{\partial t^2}, \tag{8}$$

$$k\nabla^2 T = \left(1 + \frac{(\tau_0)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)([\rho C_E\frac{\partial}{\partial t}(1 - a\nabla^2)\varphi + \beta_1 T_0\frac{\partial}{\partial t}div\, u]. \tag{9}$$

Equations (7) - (9) are supplemented by the constitutive relations

$$t_{rr} = 2\mu e_{rr} + \lambda e - \beta_1(1 - a\nabla^2)\varphi, \tag{10}$$

$$t_{\theta\theta} = 2\mu e_{\theta\theta} + \lambda e - \beta_1(1 - a\nabla^2)\varphi, \tag{11}$$

$$t_{zz} = 2\mu e_{zz} + \lambda e - \beta_1(1 - a\nabla^2)\varphi, \tag{12}$$

$$t_{rz} = \mu e_{rz},\ t_{r\theta} = 0,\ t_{z\theta} = 0, \tag{13}$$

where

$$e = e_{ii} = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z},\ e_{rr} = \frac{\partial u_r}{\partial r},\ \ e_{\theta\theta} = \frac{u_r}{r},\ e_{zz} = \frac{\partial u_z}{\partial z},\ e_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right),$$

$$e_{rr} = \frac{\partial u_r}{\partial r},\ \ e_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta},\ e_{zz} = \frac{\partial u_z}{\partial z},\ e_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right), e_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right),$$

$$e_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right),\ \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}.$$

To facilitate the solution, the following dimensionless quantities are introduced

$$r' = \frac{\omega_1^*}{c_1}r,\ z' = \frac{\omega_1^*}{c_1}z,\ (u_r', u_z') = \frac{\omega_1^*}{c_1}(u_r, u_z),\ t' = \omega_1^*,\ \omega_1^* = \frac{\rho C_E c_1^2}{K},\ c_1^2 = \frac{\lambda+2\mu}{\rho},$$

$$(t_{rr}', t_{\theta\theta}', t_{zz}', t_{rz}') = \frac{1}{\beta_1 T_0}(t_{rr}, t_{\theta\theta}, t_{zz}, t_{rz}),\ (T', \varphi') = \frac{\beta_1}{\rho c_1^2}(T, \varphi). \tag{14}$$

Assuming the harmonic behaviour as

$$(u_r, u_z, \varphi, T, e)(r, z, t) = (u_r, u_z, \varphi, T, e)(r, z)e^{i\omega t}, \tag{15}$$

where ω is the angular frequency.

We define the Hankel transform as

$$\bar{f}(\xi, z, t) = \int_0^\infty f(r, z, t) r J_n(r\xi) dr. \tag{16}$$

Using these dimensionless quantities defined by (14) in equations (7) - (9) and suppressing the primes, and applying (15) - (16) on the resulting equations, we obtain after simplification

$$(\nabla^2+\omega^2)\bar{e}-\nabla^2\bar{\varphi}+\delta_1\nabla^4\bar{\varphi}=0, \tag{17}$$

$$\tau_q^1\zeta_2\bar{e}+\tau_q^1\zeta_1\bar{\varphi}-(\tau_q^1\delta_1-K)\nabla^2\bar{\varphi}=0, \tag{18}$$

where

$$\tau_q^1=\left(1+(\omega_1^{*\alpha})\frac{(\tau_0)^\alpha}{\alpha!}(i\omega)^\alpha\right),\quad \nabla^2=-\xi^2+\frac{d^2}{dz^2}. \tag{19}$$

Solving the equations (17)-(18), we obtain

$$\left(\frac{d^4}{dz^4}+P\frac{d^2}{dz^2}+Q\right)(\bar{e},\bar{\varphi})=0, \tag{20}$$

which can be written in the form

$$(D^2-q_1^2)(D^2-q_2^2)(\bar{e},\bar{\varphi})=0,$$

where

$$P=P^1-2\xi^2,$$
$$Q=\xi^4-P^1\xi^2+Q^1,$$
$$P^1=(-\omega^2\zeta_3+\tau_q^1\zeta_2+\tau_q^1\zeta_1)/R,$$
$$Q^1=-\omega^2\tau_q^1\zeta_1/R,\ R=\tau_q^1\zeta_2 a+\zeta_3,$$
$$\mu^1=\frac{2\mu}{\beta_1T_0},\quad \lambda^1=\frac{\lambda}{\beta_1T_0},\quad \beta_1^1=\frac{1}{\beta_1},\quad \zeta_1=\frac{\rho C_E c_1^2}{\omega_1},\quad \zeta_2=i\frac{\beta_1^2T_0}{\rho\omega_1}\omega.$$

The solution of (20) is assumed of the form

$$\bar{e}=\sum_{i=1}^2 A_i\cosh(q_iz), \tag{21}$$

$$\bar{\varphi}=\sum_{i=1}^2 d_iA_i\cosh(q_iz), \tag{22}$$

where $q_i=\sqrt{\xi^2+k_i^2}$, are the roots of the equation (20),

$d_i=\frac{\tau_q^1\zeta_2}{\tau_q^1\zeta_1+\zeta_3\xi^2-\zeta_3q_i^2}$ are the coupling constants, $\zeta_3=\tau_q^1a-K$.

4. Boundary Conditions

We consider a cubical thermal source and mechanical force of unit magnitude along with vanishing of shear stress at $z = \pm d$. These Boundary conditions can be written as

$$\frac{\partial \varphi}{\partial z} = \pm g_0 F(r,z), \tag{23}$$

$$t_{zz} = f(r,t), \tag{24}$$

$$t_{rz} = 0. \tag{25}$$

Here, we have considered the function $F(r,z)$ decaying exponentially as one moves away from the centre of the plate in the radial direction and increases symmetrically along the axial direction given by

$$F(r,z) = z^2 e^{-\delta r}, \delta > 0.$$

Thus, on applying Hankel Transform, we get

$$\bar{F}(\xi, z) = \frac{z^2 \delta}{(\xi^2+\delta^2)^{3/2}} .$$

As an application, we consider a specific type of source function of the type

$$f(r,t) = H(\propto - r) e^{i\omega t} ,$$

where $H(\propto - r)$ is the Dirac delta function. Applying Hankel Transform , we obtain

$$\bar{f}(\xi,\omega) = \frac{\propto J_1(\xi \propto)}{\xi} e^{i\omega t}.$$

Using equations (10) - (13), (14), (15) - (16) and the boundary conditions (23) - (25), we achieve the displacement and stress components, conductive temperature, temperature change and cubic dilatations as

$$\overline{u_r} = \frac{g_0 \bar{F}(\xi,d)}{\Delta}\left\{\frac{\eta_1 m_1}{q_1}\Lambda_1\vartheta_1 - \frac{\eta_2 m_2}{q_2}\Lambda_2\vartheta_2 + \Lambda_3\vartheta\right\} - \frac{\bar{f}(\xi,\omega)}{\Delta}\left\{\frac{\eta_1 m_1}{q_1}\Lambda_4\vartheta_1 - \frac{\eta_2 m_2}{q_2}\Lambda_5\vartheta_2 + \Lambda_6\vartheta\right\}, \tag{26}$$

$$\overline{u_z} = \frac{g_0\bar{F}(\xi,d)}{\Delta}\{m_1\Lambda_1(\gamma_1\theta_1 - \alpha_1\vartheta_1) - m_2\Lambda_2(\gamma_2\theta_2 - \alpha_2\vartheta_2) + \Lambda_3\vartheta\}$$
$$- \frac{\bar{f}(\xi,\omega)}{\Delta}\{\Lambda_4 m_1(\gamma_1\theta_1 - \alpha_1\vartheta_1) - \Lambda_5\, m_2\Lambda_2\gamma_2\theta_2 - \alpha_2\vartheta_2 + \Lambda_6\,\vartheta\}, \tag{27}$$

$$\overline{t_{zz}} = \frac{g_0\bar{F}(\xi,d)}{\Delta}\{\Lambda_1(\nu_1\vartheta_1 - 2\mu m_1\,\alpha_1\theta_1 - \Lambda_2(\nu_2\vartheta_2 - 2\mu m_2\,\alpha_2\theta_2 + \Lambda_3 2\mu q\theta\}$$
$$-\frac{\bar{f}(\xi,\omega)}{\Delta}\{\Lambda_4\,(\nu_1\vartheta_1 - 2\mu m_1\,\alpha_1\theta_1 - \Lambda_5\,(\nu_2\vartheta_2 - 2\mu m_2\,\alpha_2\theta_2 + \Lambda_6\,2\mu q\theta\}, \tag{28}$$

$$\overline{t_{rz}} = \frac{g_0\bar{F}(\xi,d)\mu}{2\Delta}\{\Lambda_1\varsigma_1 - \xi\alpha_1\vartheta_1) - \Lambda_2\varsigma_2 - \alpha_2\xi\vartheta_2) + \Lambda_3\,\xi q(\vartheta + \theta\}$$
$$-\frac{\bar{f}(\xi,\omega)\mu}{2\Delta}\{\Lambda_4\varsigma_1 - \xi\alpha_1\vartheta_1) - \Lambda_5\,\varsigma_2 - \alpha_2\xi\vartheta_2 + \Lambda_6\,\xi q(\vartheta + \theta)\}, \tag{29}$$

$$\bar{T} = \frac{g_0\bar{F}(\xi,d)}{\Delta}\{\chi_1 d_1\,\Lambda_1\,\vartheta_1 - \chi_2 d_2\Lambda_2\vartheta_2\} - \frac{\bar{f}(\xi,\omega)}{\Delta}\{\chi_1 d_1\Lambda_4\vartheta_1 - \chi_2 d_2\Lambda_5\vartheta_2\}, \tag{30}$$

$$\bar{\varphi} = \frac{g_0\bar{F}(\xi,d)}{\Delta}\{d_1\,\Lambda_1\,\vartheta_1 - d_2\Lambda_2\vartheta_2\} - \frac{\bar{f}(\xi,\omega)}{\Delta}\{d_1\Lambda_4\vartheta_1 - d_2\Lambda_5\vartheta_2\}, \tag{31}$$

$$\bar{e} = \frac{g_0\bar{F}(\xi,d)}{\Delta}\{\Lambda_1\,\vartheta_1 - \Lambda_2\vartheta_2\} - \frac{\bar{f}(\xi,\omega)}{\Delta}\{\Lambda_4\vartheta_1 - \Lambda_5\vartheta_2\}, \tag{32}$$

where

$$\gamma_i = \frac{\rho c_1^2}{\mu}(d_i + a\xi^2 d_i - 1), \quad \alpha_i = -\frac{\rho c_1^2}{\mu}a d_i q_i\,, \quad m_i = \frac{q_i}{q_i^2 - q^2},$$

$$\eta_i = -\frac{\xi\rho c_1^2}{\mu} + \xi d_i(1 + a\xi^2 - \;\; q_i^2), \quad q = \xi^2 - \frac{\rho c_1^2}{\mu}\omega^2,$$

$$\Lambda_1 = (\Delta_{22}\Delta_{33} - \Delta_{23}\Delta_{32})\,, \quad \Lambda_2 = \Delta_{21}\Delta_{33} - \Delta_{23}\Delta_{31},$$

$$\Lambda_3 = \Delta_{21}\Delta_{32} - \;\; \Delta_{22}\Delta_{31}, \;\; \Lambda_4 = \Delta_{12}\Delta_{33}, \;\; \Lambda_5 = \Delta_{11}\Delta_{33},$$

$$\Lambda_6 = \Delta_{11}\Delta_{32} - \Delta_{12}\Delta_{31}, \;\; \nu_i = 2\mu\gamma_i m_i + \lambda - \;\; d_i(1 + \delta_1\xi^2 - \delta_1 q_i^2),$$

$$\chi_i = d_i(1 + \delta_1\xi^2 - \delta_1 q_i^2), \;\; \Delta = det(\Delta_{ij}), \quad \Delta_{1i} = d_i q_i\, sinh(q_i b),$$

$$\Delta_{13} = 0, \;\; \Delta_{2i} = cosh(q_i b)(2\mu\gamma_i m_i + \lambda - d_i(1 + \delta_1\xi^2 - \;\; \delta_1 q_i^2) - sinh(q_i b)(2\mu\alpha_i m_i),$$

$$\Delta_{23} = 2\mu q sinh(qb), \;\; \Delta_{3i} = sinh(q_i b)(\mu\eta_i m_i + \;\; m_i\xi\gamma_i) - \mu\alpha_i m_i cosh(q_i b),$$

$$\Delta_{33} = \mu q\xi(sinh(qb) + cosh(qb)\}, \quad \theta_i = sinh(q_i z), \quad \vartheta_i = cosh(q_i z), \;\; (\, i = 1{,}2),$$

$$\vartheta = cosh(qz), \quad \theta = sinh(qz), \quad (m_1(\eta_1 + \gamma_1\xi)\,\theta_1 = \varsigma_1, \;\; (m_2(\eta_2 + \gamma_2\xi)\,\theta_2 = \varsigma_2.$$

5. Particular cases

(i) If $a = 0$, in the Equations (26) - (32), we obtain the resulting expressions for thermal interactions in thick circular plate without two temperature in frequency domain.

(ii) If $\alpha = 0$ in the fractional heat equation and putting in Equations (26) - (32), the resulting expressions reduce for thermoelastic interactions in a thick circular plate with two temperatures in frequency domain.

6. Inversion of transform

To obtain the solution of the problem in physical domain, we must invert the transforms in Equations (26) - (32) These expressions are functions of ξ and z, and hence are of the form $\tilde{f}(\xi, z, \omega)$. To get the function $f(r, z, \omega)$ in the physical domain, we invert the Hankel transform using

$$f(r, z, \omega) = \int_0^\infty \xi\, \bar{f}\xi, z, \omega) J_n(\xi r) d\xi. \tag{33}$$

The last step is to calculate the integral in Equation (33). The method for evaluating this integral is described in Press et al. (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

7. Numerical results and discussion

The graphs have been plotted to study the effect of fractional parameter on the various quantities in the range $0 \leq r \leq 1$.

The mathematical model is prepared with copper material for purposes of numerical computation. The material constants for the problem are taken from Dhaliwal and Singh (1980)

$$\lambda = 7.76 \times 10^{10} Nm^{-2},\ \ \mu = 3.86 \times 10^{10} Nm^{-2},\ K = 386 JK^{-1}m^{-1}s^{-1},$$

$$\beta_1 = 5.518 \times 10^6\ \ Nm^{-2}deg^{-1},\ \ \ \rho = 8954\, Kgm^{-3},\ a = 1.2\ \times 10^4 m^2/s^2 k,$$

$$b = 0.9\ \times 10^6 m^5/kgs^2, \qquad D = 0.88\ \times 10^{-8} kgs/m^3,$$

$$\beta_2 = 61.38 \times 10^6\ \ Nm^{-2}deg^{-1},\ T_0 = 293K,\ \ C_E = 383.1\, Jkg^{-1}K^{-1}.$$

In the Figures 1-8,

(i) Small dashed line corresponds to the fractional parameter $\alpha = 0.2$.

(ii) Solid line with centre symbol circle corresponds to fractional parameter $\alpha = 0.5$.

(iii) Small dashed line with centre symbol diamond corresponds to fractional parameter $\alpha = 0.7$

Figure 1 exhibits variations of cubic dilatation e with distance r. We notice that corresponding to all the values α, the variations are steady near the loading surface, and then a sharp decrease is noticed in the range $0.4 \leq r \leq 0.5$, followed by continuous increase in the rest with a difference in the magnitude.

Figure 2 exhibits variations of temperature change T with displacement r. Here corresponding to $\alpha = 0.2$ and $\alpha = 0.5$, there is a hike in the range $0.4 \leq r \leq 0.6$ and descent in the rest whereas opposite behaviour is observed corresponding to $\alpha = 0.7$.

Figure 3 gives variation of conductive temperature φ with distance r. Here , we notice that corresponding to $\alpha = 0.5$, there is a sharp decrease in the range $0.3 \leq r \leq 0.5$ followed by increase in the rest. Corresponding to $\alpha = 0.2$ and $\alpha = 0.7$, the variations are similar with less magnitude.

Figure 4 displays variations of stress component t_{rr} with displacement r. Here opposite trends are noticed corresponding to the case $\alpha = 0.2$ with the cases $\alpha = 0.5$, $\alpha = 0.7$ with a difference in magnitude.

Figure 5 gives variations of stress component t_{rz} with distance r. It is evident that the behaviour of variations is opposite corresponding to $\alpha = 0.5$ and $\alpha = 0.7$. As we see that corresponding to $\alpha = 0.5$, the variations decrease smoothly throughout the range whereas corresponding to $\alpha = 0.7$, the variations increase smoothly. Corresponding to $= 0.2$, the trend is steady and smooth and small variations are seen near the loading surface.

Figure 6 gives variations of stress component t_{zz} with distance r. Here we notice that the trends of variations are opposite corresponding to the cases $\alpha = 0.5$ and $\alpha = 0.7$ as corresponding to $\alpha = 0.5$, the variations are increasing smoothly whereas corresponding to $\alpha = 0.7$ and $\alpha = 0.2$, variations are decreasing smoothly.

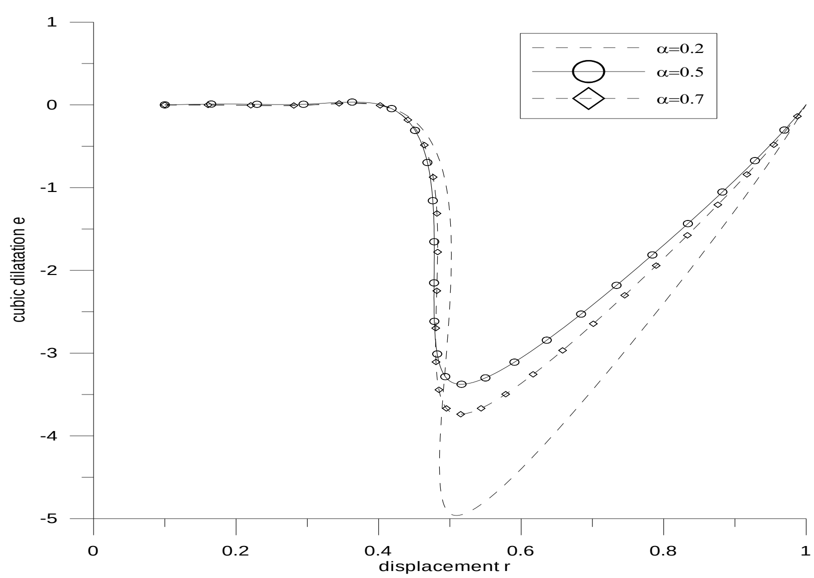

Figure 1. Cubic dilatation e with distance r

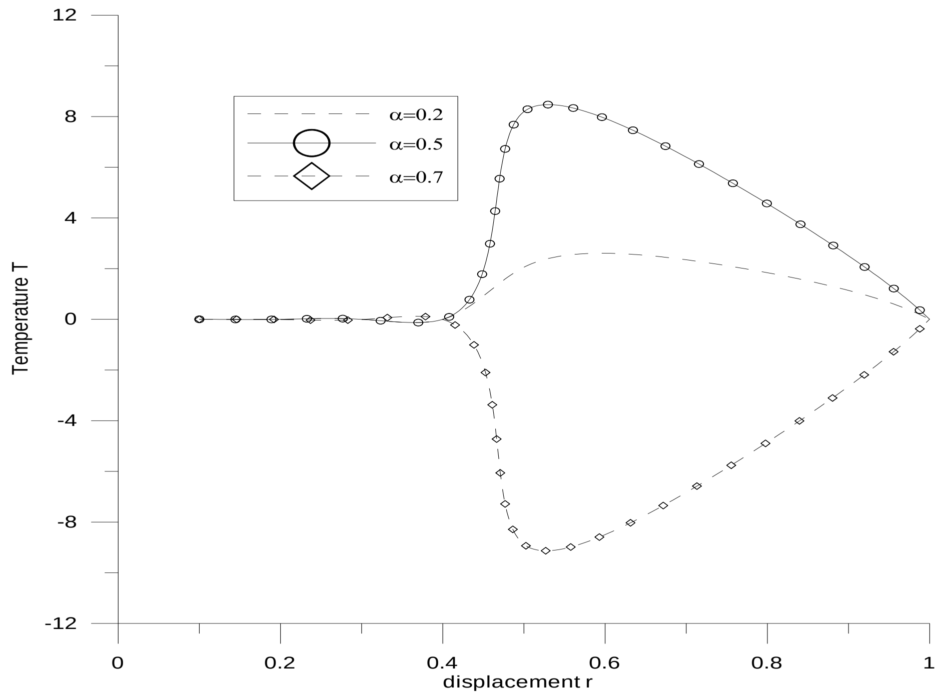

Figure 2. Temperature change T with distance r

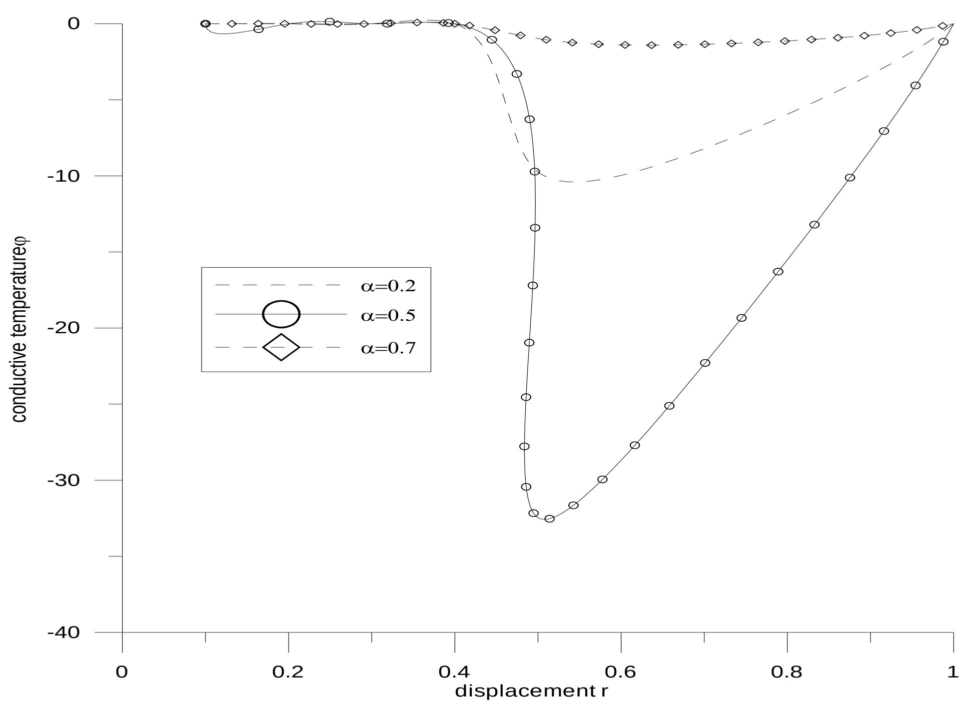

Figure 3. Conductive temperature φ with distance r

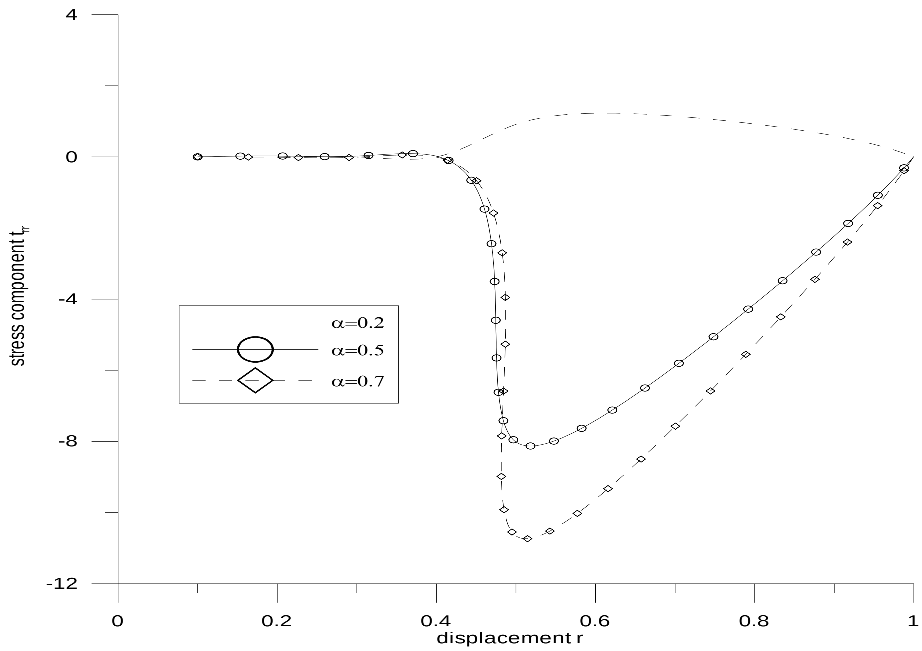

Figure 4. Stress component t_{rr} with distance *r*

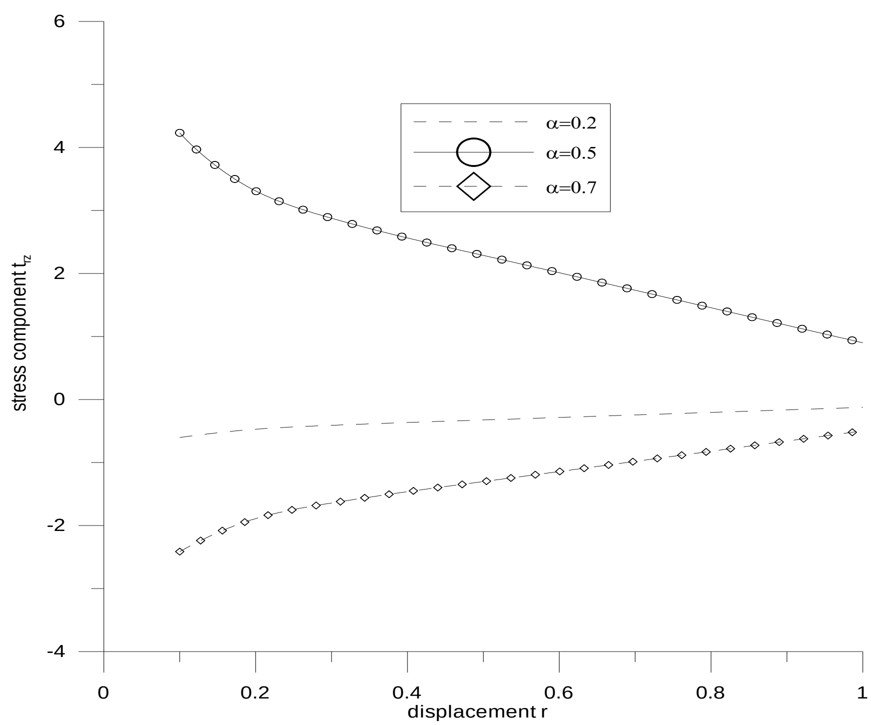

Figure 5. t_{rz} with distance *r*

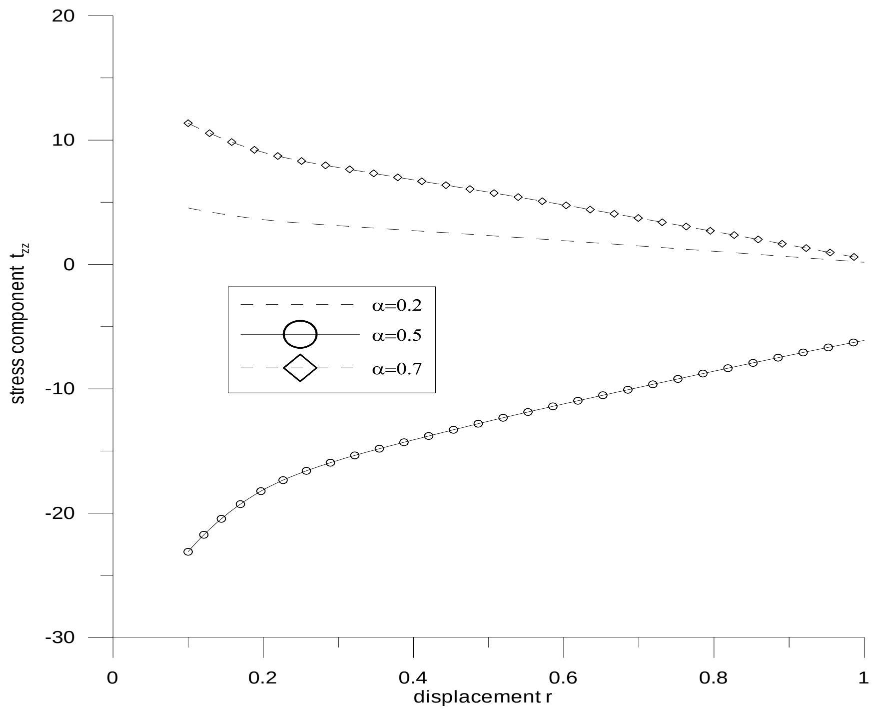

Figure 6. t_{zz} with distance r

8. Conclusion

From the present research, we conclude that

- There is a significant impact of fractional parameter on the various components. The fractional parameter somewhere changes the amplitude of variation whereas somewhere it causes the variations to move in the opposite manner.

- While examining the components of cubic dilatation, conductive temperature, temperature change T and the stress component t_{rr}, it is noticed that the pattern of variations consists of sharp descents and jumps.

- The components of displacements u_r , u_z and stress components t_{rz} and t_{zz} face smooth increase or decrease while studying the variations.

- In future such a mathematical model can be established for a transversely isotropic medium and the variations can be examined.

- This model is really useful to the people who are working in the field of thermodynamics and thermoelasticity.

- The result of the problem is useful in the two dimensional problem of dynamic response due to various thermal sources which has various geophysical and industrial applications.

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