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Numerical studies for MHD flow and gradient heat transport past a stretching sheet with radiation and heat production via DTM

Khadijah M. Abualnaja

College of Science
Department of Mathematics and Statistics
Taif University
Taif, KSA
dujam@windowslive.com

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Abstract

This paper presents a numerically study for the effect of the internal heat generation, magnetic field and thermal radiation effects on the flow and gradient heat transfer of a Newtonian fluid over a stretching sheet. By using a similarity transformation, the governing PDEs can be transformed into a coupled non-linear system of ODEs with variable coefficients. Numerical solutions for these equations subject to appropriate boundary conditions are obtained by using the differential transformation method (DTM). The effects of various physical parameters such as viscosity parameter, the suction parameter, the radiation parameter, internal heat generation or absorption parameter and the Prandtl number on velocity and temperature are discussed by using graphical approach.

Keywords: MHD Newtonian fluid, Exponentially stretching sheet, DTM

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1. Introduction

The study of flow and heat transfer of a Newtonian fluid due to stretching sheet is a vital problem in classical fluid mechanics due to its vast applications in many manufacturing processes in industry, such as extraction of polymer sheet, wire drawing, paper production, glass-fiber production, hot rolling, solidification of liquid crystals, petroleum production, continuous cooling and fibers spinning, exotic lubricants and suspension solutions. Much work on the boundary-layer Newtonian fluids has been carried out both experimentally and theoretically. Crane [Crane (1970)] was the first one who studied the stretching problem taking into account the fluid flow over a linearly stretched surface. Numerous studies [Gupta and Gupta (1977)-Cortell (2007)] have been conducted later to extend the pioneering work of Crane [Crane (1970)]. Using the homotopy analysis method, series solutions were obtained by [Hayat, et al. (2004)] for the stretching sheet problem with mixed convection. El-Aziz [El-Aziz (2009)] focused on the effect of thermal radiation in his studies for the flow and heat transfer over an unsteady stretching sheet. Due to the fact that the rate of cooling influences the quality of the product with desired characteristics, [Akyildiz and Siginer (2010)] investigated the thermal boundary layer flow by considering the non-linear stretching surface. Recently, [Mukhopadhyay (2013)] investigated the numerical solution for the thermally stratified medium subject to suction effects on the flow and heat transfer over an exponentially stretching sheet.

The DTM is a semi-numerical-analytic-technique that formalizes the Taylor series in a totally different manner. It was first introduced by Zhou in a study about electrical circuits [Zhou (1986)]. [Borhanifar and Abazari (2010a), (2010b)] used it for solving the linear and non-linear problems. In this paper, we extended DTM for obtaining the numerical solutions of the introduced problem. The DTM plays an important rule in recent researches in this field. This technique reduces the problem to a system of recurrence equations. It has been shown that this procedure is a powerful tool for solving various kinds of problems [Bildik and Konuralp (2006)-Arikoglu and Ozkol (2008)].

2. Formulation of the problem

Consider the two-dimensional boundary layer flow of an incompressible Newtonian fluid towards an exponentially stretching sheet. The origin is located at a slit, through which the sheet (see Figure 1) is drawn through the fluid medium [Mukhopadhyay (2013)]. The $x-$axis is chosen along the sheet and $y-$axis is taken normal to it. The stretching surface has the velocity $U = U_0 e^{\frac{x}{L}}$, where U_0 is the reference velocity and L is the reference length. Also, the sheet is assumed to be porous and a variable suction velocity v_w is taken into consideration. The surface of the sheet is held at a constant heat flux q_w and the ambient fluid temperature is T_∞. After using the usual boundary layer approximations, our governing equations can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{q'''}{\rho c_p}, \tag{3}$$

where u and v are the velocity components in the x and y directions, respectively. ρ and κ are the fluid density and the thermal conductivity, respectively. σ is the electrical conductivity, B_0 is the

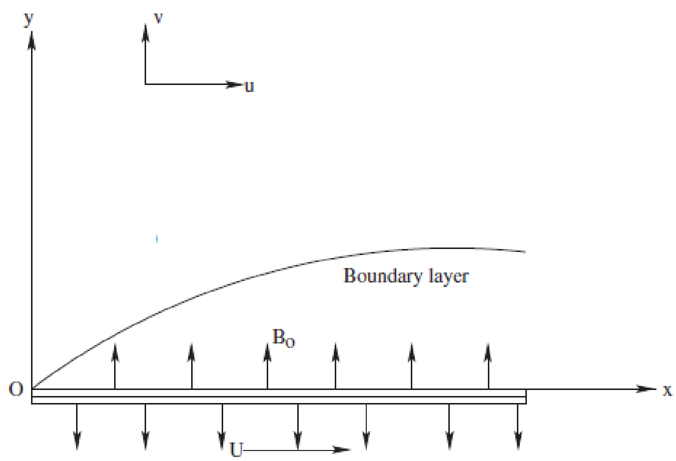

Figure 1. Flow geometry and coordinate system

magnetic field strength, T is the temperature of the fluid, ν is the fluid kinematic viscosity, c_p is the specific heat at constant pressure, q''' is the rate of internal heat generation and q_r is the radiative heat flux. Also, q_r is employed according to Rosseland approximation [Raptis (1998)] such that

$$q_r = -(4\sigma^*/3k^*)(T^4)_y, \tag{4}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Following [Raptis (1999)] and expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, we have

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \tag{5}$$

Also, the boundary conditions can be written as

$$u = U, \qquad v = -v_w, \qquad -\kappa(\frac{\partial T}{\partial y}) = q_w \qquad at \qquad y = 0, \tag{6}$$

$$u \to 0, \qquad T \to T_\infty \qquad as \qquad y \to \infty. \tag{7}$$

Here, assume that $v_w = v_0 e^{\frac{x}{2L}}$, where v_0 is a constant. The mathematical analysis of the problem is simplified by introducing the following dimensionless coordinates

$$\eta = y\sqrt{\frac{U}{2\nu L}}, \qquad u = Uf'(\eta), \qquad v = -\sqrt{\frac{\nu U}{2L}}\left(f(\eta) + \eta f'(\eta)\right), \tag{8}$$

$$\theta(\eta) = \frac{\kappa}{q_w}(T - T_\infty)\sqrt{\frac{Re}{2L^2}}, \tag{9}$$

where $\theta(\eta)$ is the dimensionless temperature, $f(\eta)$ is the dimensionless stream function and $Re = \frac{UL}{\nu}$ is the Reynolds number.

The internal heat generation or absorption q''' is modeled according to the following equation [Chamkha and Khaled (2001)]

$$q''' = \kappa \left(\frac{Re}{2L^2}\right) \left[a^* \left(T_w - T_\infty\right) e^{-\eta} + b^*(T - T_\infty)\right], \tag{10}$$

where T_w is the surface temperature.

After using the previous dimensionless coordinates, the boundary layer governing equations (1)-(3) can be written in the following non-dimensional form:

$$f^{'''} + ff^{''} - 2f^{'2} - Mf^{'} = 0, \tag{11}$$

$$(1 + R)\,\theta^{''} + Pr(f\theta^{'} + f^{'}\theta) + a^* e^{-\eta} + b^*\theta = 0. \tag{12}$$

The transformed boundary conditions are

$$f = f_w, \qquad f^{'} = 1, \qquad \theta' = -1 \qquad at \quad \eta = 0, \tag{13}$$

$$f' \to 0, \qquad \theta \to 0 \qquad as \qquad \eta \to \infty, \tag{14}$$

where $M = \frac{2\sigma B_0^2 L}{\rho U e^{\frac{x}{L}}}$ is the magnetic parameter, $Pr = \frac{\mu c_p}{\kappa}$ is the Prandtl number, $R = \frac{16\sigma^* T_\infty^3}{3k^*\kappa}$ is the radiation parameter and $f_w = v_0 \left(\frac{\mu U_0}{2\rho L}\right)^{\frac{-1}{2}}$ is the dimensionless suction velocity.

On the other hand, we must refer that there exist a physical quantities which is called the skin-friction coefficient which is proportional to $-f^{''}(0)$ and the Nusselt number which is proportional to $\frac{1}{\theta(0)}$.

3. Solution procedure using DTM

Our aim in this section is to use the differential transformation method to solve Equations (11)-(12) at the bounded domain $(0, \eta_\infty)$, $\eta_\infty = 6$ with the boundary conditions (13)-(14). Using DTM on Equations (11)-(12) and from Table 1 [Ozdemir and Kaya (2006)] we get

$$\begin{aligned}(k+1)(k+2)(k+3)F(k+3) = 2\sum_{r=0}^{k}(r+1)(k-r+1)F(r+1)F(k-r+1)\\ -\sum_{r=0}^{k}(k-r+1)(k-r+2)F(r)F(k-r+2) + M(k+1)F(k+1),\end{aligned} \tag{15}$$

$$\begin{aligned}(k+1)(k+2)\Theta(k+2) = -\left(\frac{Pr}{1+R}\right)\left(\sum_{r=0}^{k}(k-r+1)\left(\Theta(r)F(k-r+1)+F(r)\Theta(k-r+1)\right)\right)\\ -\left(\frac{1}{1+R}\right)\left(a^* e^{-\eta} + b^*\Theta(k)\right),\end{aligned} \tag{16}$$

where $F(k)$ and $\Theta(k)$ are the differential transforms of $f(\eta)$ and $\theta(\eta)$, respectively.

We choose suitable initial conditions

$$F(0) = f_w, \qquad F(1) = \frac{1}{2}, \qquad F(2) = \frac{1}{6}\ell_1, \qquad \Theta(0) = \ell_2, \qquad \Theta(1) = -\frac{1}{2}. \tag{17}$$

From Equations (15)-(16), for $k = 0, 1, ...$, and using initial values (17) we get

$$\begin{aligned}
F(3) &= \frac{1}{6}\left[2(F(1))^2 - 2F(0)F(2) + MF(1)\right] = \frac{1}{6}(0.5 - \frac{1}{3}f_w\ell_1 + 0.5M),\\
\Theta(2) &= \frac{1}{2}\left[\left(\frac{Pr}{1+R}\right)(\Theta(0)F(1) + F(0)\Theta(1)) - \left(\frac{1}{1+R}\right)(b^*\Theta(0))\right]\\
&= \frac{1}{2}\left[\left(\frac{Pr}{1+R}\right)(0.5\ell_2 - 0.5f_w) - \left(\frac{1}{1+R}\right)(b^*\ell_2)\right],\\
F(4) &= \frac{1}{24}\left[8F(1)F(2) - (6F(0)F(3) + 2F(1)F(2)) + MF(2)\right]\\
&= \frac{1}{24}\left(0.5\ell_1 - 6f_wF(3) + \frac{1}{6}M\ell_1\right),\\
\Theta(3) &= \frac{1}{6}\left[\left(\frac{Pr}{1+R}\right)(2\Theta(0)F(2) + 2F(0)\Theta(2) + 2\Theta(1)F(1)) - \left(\frac{1}{1+R}\right)(b^*\Theta(1))\right]\\
&= \frac{1}{6}\left[\left(\frac{Pr}{1+R}\right)\left(\frac{1}{3}\ell_1\ell_2 + 2f_w\Theta(2) - 0.5\right) - \left(\frac{1}{1+R}\right)(-0.5b^*)\right], \ldots .
\end{aligned} \tag{18}$$

In the same manner, the rest of components can be obtained using the Mathematica Package. Substituted the quantities listed on (18) when $\eta_0 = 0$, the approximate solution in a series form of the proposed problem (11)-(12) is given by

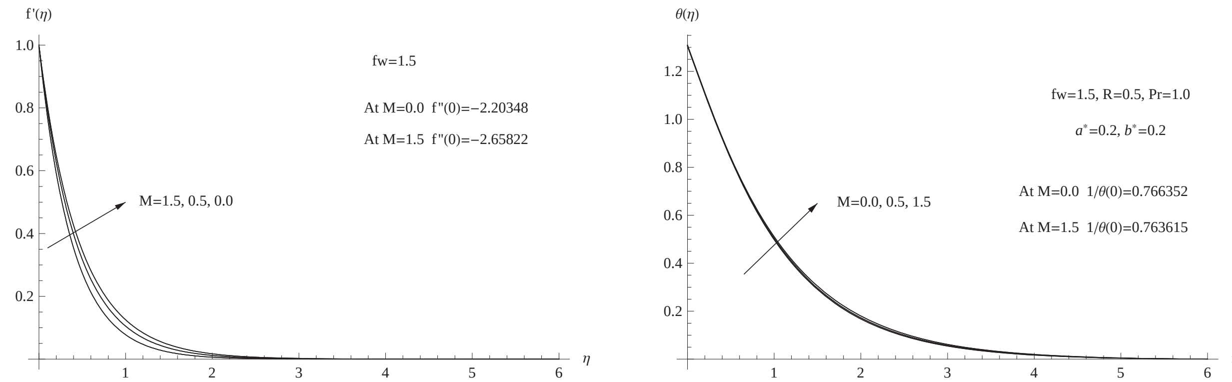

Figure 2. (a) Velocity distribution for M. (b) Temperature distribution for M

$$\begin{aligned} f(\eta) &\cong \sum_{k=0}^{n} F(k)\eta^k = F(0) + F(1)\eta + F(2)\eta^2 + F(3)\eta^3 + F(4)\eta^4 + ... + F(n)\eta^n, \\ \theta(\eta) &\cong \sum_{k=0}^{n} \Theta(k)\eta^k = \Theta(0) + \Theta(1)\eta + \Theta(2)\eta^2 + \Theta(3)\eta^3 + \Theta(4)\eta^4 + ... + \Theta(n)\eta^n. \end{aligned} \tag{19}$$

Now, we will find the constants ℓ_1 and ℓ_2 using imposing the boundary conditions (13)-(14) where we take the values $M = 0.2,\ R = 1.0,\ Pr = 0.7,\ f_w = 1.0,\ a^* = 0,\ b^* = -0.3,\ \eta_\infty = 6$. These values are $\ell_1 = -0.321568,\ \ell_2 = 4.987546$. Having $F(k),\ \Theta(k)\ k = 0, 1, ..., n$, the solution are as in (19).

4. Results and discussion

This section provides the numerical evaluation for the solutions of the proposed problem and the results are illustrated graphically in the Figures 2-5. Effects of the magnetic parameter M on velocity and temperature profiles are shown in Figure 2. It is observed that the velocity decreases sharply near the wall as the magnetic parameter increases. Likewise, from Figure 2 (b), it is noticed that an increase in the magnetic parameter results in an increase in the temperature distribution. Also, the numerical results in these figures indicate that the momentum boundary layer thickness decreases in terms of η at increasing distances from the leading edge, but the reverse is true for the thermal boundary layer as the magnetic parameter increases. Likewise, the Nusselt number is reduced but the skin friction coefficient is increased with increasing for values of M. The effects of suction parameter on the fluid flow and the temperature distribution has been analyzed and the results are presented in Figures 3. From Figure 3 (a) it is clear that the velocity at any point near to the plate decreases as the suction parameter increases. From Figure 3 (b), it is evident that the fluid suction decreased the thickness of the thermal boundary layer and enhanced the rate of heat transfer. Also, it is noticed that increases in the suction parameter leads to an increase in both the local skin-friction coefficient and the local number.

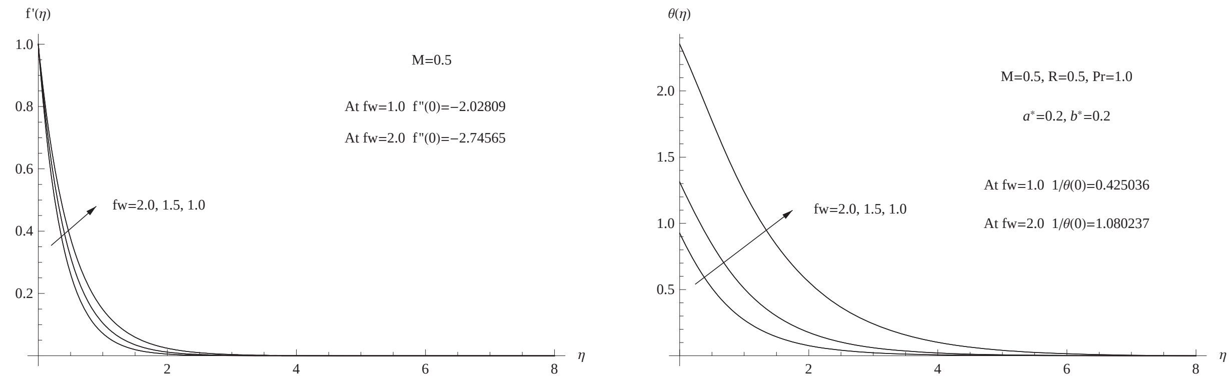

Figure 3. (a) Velocity distribution for f_w. (b) Temperature distribution for f_w

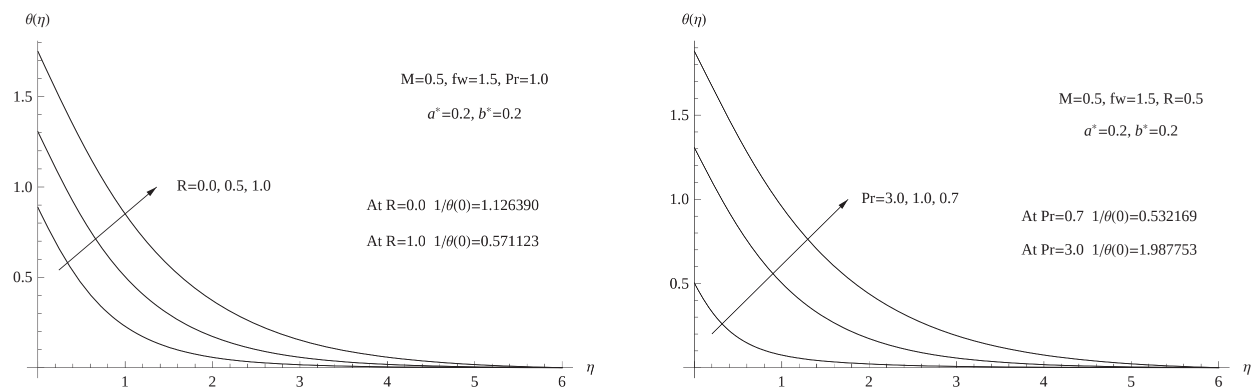

Figure 4. (a) Temperature distribution for R. (b) Temperature distribution for Pr

Figure 4 (a) displays the effect of the radiation parameter R on the temperature profile. It is observed that both the temperature distribution and the surface temperature $\theta(0)$ increases with an increase in the value of the radiation parameter R, i.e., the thermal boundary layer thickness increases as R increases. In the second part for the same figure, we observe that, figure 4 (b) shows the effect of Prandtl number on the temperature profiles above the sheet. It is noticed that, the larger Prandtl number has a relatively lower thermal diffusivity. Therefore a rapid increase in the Prandtl number Pr decreases the temperature and the thermal boundary layer thickness. On the other hand, an increase in the Prandtl number causes an increase in the Nusselt number.

Figure 5 illustrates the effect of heat generation or absorption parameters a^*, b^* on the temperature profile. It is shown that the effect of heat absorption parameter $a^* < 0$ and $b^* < 0$ causes a drop in the temperature as the heat following from the sheet is absorbed. Also, when $a^* > 0$ and $b^* > 0$ it is clear that as the heat generation or absorption parameters a^* and b^* increases the temperature of the fluid increase. Moreover, from the same figure, it is noticed that increases in the values of the

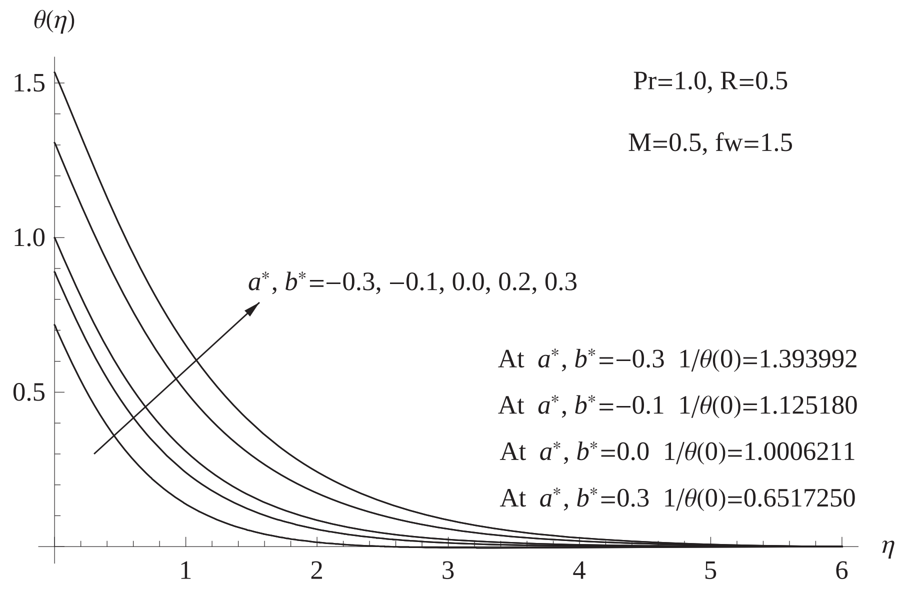

Figure 5. The behavior of the temperature distribution for various values of a^* and b^*

heat generation parameter leads to a decrease in the Nusselt number. Also, it is observed that the Nusselt number increases with the increase of the heat absorption parameter.

5. The discussion and conclusion

We applied the DTM to solve the system of ODEs which describe the MHD fluid flow which caused solely by an exponentially stretching porous sheet with non-uniform internal heat generation/absorption and thermal radiation. It was found that the effect of increasing values of the heat generation and the radiation parameter reduce the local Nusselt number. On the other hand it was observed that the local Nusselt number increases as the Prandtl number, suction parameter and heat absorption parameter increases. Moreover, it is interesting to find that as the suction parameter, viscosity parameter and radiation parameter increases in magnitude, causes the fluid to slow down past the sheet, the skin-friction coefficient increases in magnitude. Moreover, numerical results indicate that, the skin-friction coefficient as well as Nusselt number are strongly affected by the suction parameter and the magnetic parameter.

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