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Transient Solution of an *M*/*M*/1 Retrial Queue with Reneging from Orbit

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Abstract

In this paper, the transient behavior of an *M*/*M*/1 retrial queueing model is analyzed where the customers in the orbit possess the reneging behavior. There is no waiting room in the system for the arrivals. If the server is not free when the occurrence of an arrival, the arriving customer moves to the waiting group, known as orbit and retries for his service. If the server is idle when an arrival occurs (either coming from outside the queueing system or from the waiting group), the arrival immediately gets the service and leaves the system. Each individual customer in the orbit, retrying for his service, becomes impatient and starts reneging from the orbit. Here the reneging of customers is due to the long wait in the orbit. Using continued fractions, the transient probabilities of orbit size for this model are derived explicitly. Average and variance of orbit size at time t are also obtained. Further, numerical illustrations of performance measures are done to analyze the effect of parameters.

Keywords: *M/M*/1 queue; Retrial; Reneging from orbit; Continued fractions; Transient probabilities

MSC 2010 No.: 60K25, 90B22, 68M20

1. Introduction

Retrial queue is the queue in which the arrivals finding the server busy will retry again for their service after some random time period (see Artalejo (1991, 1999), Falin (1991), Yang and Templeton (1987)). Retrial queues are widely used in modeling many problems in the field of telephone networks, computer and communication systems, call centers, etc. If the server is free during the occurrence of an arrival that comes from outside the system, the arriving customer is served immediately. If the server is busy when an arrival occurs, the arrival will join the orbit. If the server is not busy while a customer reattempts for service from the waiting group, that customer is instantly getting the service and moves out of the system after getting completed his service. Otherwise, the customer joins the orbit once again and repeats the same until the completion of his service.

Customers waiting in the orbit (i.e., reattempts for their service) lose their patience due to the long waiting time for service. Every customer in the orbit independently activates an impatience timer of fixed duration. If a customer finds the idle server when he/she reattempts before the expiry of timer, he gets the chance to be served and moves out from the system after the service completion. If the timer expires before he is getting a chance to be served, he quits before the commencement of his service and never return to the system.

Mohanty et al. (1993) derived the time-dependent distributions of the number present in the system and the length of a busy period for a finite birth-death process. Choi et al. (2001) analyzed an *M/M/*1 queueing model with impatient behavior of unaware customers. They have taken two classes of customers, high priority class 1 customers and low priority class two customers. They also considered two impatience rules for class 1 customers. That is, the first one is that the waiting class 1 customers were moving out before getting served and the second one is that class 1 customers quit before finishing their service as their timer expires.

Choudhury (2008) studied the single server queueing model where the customers were reneging the system if their waiting time was more than the threshold they fixed. Kim (2010) derived the solution of an *M/M/1* queueing model for the stationary case where he had taken the situation that the customer coming from outside the service area or from the waiting group collided with the one in service during the server was busy and both of them were shifted to the orbit. Using generating function, Pavai Madheswari et al. (2016) computed the stationary probabilities of a single server retrial queueing model in which they considered the influence of retaining the impatient customers from the orbit.

The *M/M/1*, *M/M/c* and $M/M/\infty$ queueing models were investigated by Perel and Yechiali (2010) in which they assumed both fast and slow service modes. Also the customers in the queue possessed impatience behavior during the slow service mode. They derived the probabilities of such models for the steady-state case. Parthasarathy and Sudhesh (2007) computed the system size probabilities for the *M/M/1* retrial queueing model with state-dependent rates using continued fraction for the transient case. Sudhesh and Azhagappan (2016) investigated the impatience nature of an *M/M/1* vacation queueing model where the server waits dormant in the system for certain period after returning from vacation. They also computed the average and variance of size of the system for the time-dependent case and presented some numerical illustrations.

Sudhesh et al. (2017) described the *M/M/1* queueing model in which the server starts working vacation (single and multiple) whenever the system becomes empty and the customers possess impatience behavior during working vacation. They have derived the system size probabilities explicitly using continued fraction for the transient case. They also obtained the performance measures like mean and variance and carried out numerical simulations. In this paper, we have extended the work of Parthasarathy and Sudhesh (2007) by introducing the impatient behavior of customers in the orbit.

Kumar and Sharma (2018) derived the transient system size probabilities for a single server queueing model with retention of reneging customers using generating function method where the arriving customers become impatient due to the long waiting time and the system implement certain retention strategies to retain those impatient customers. Sudhesh and Azhagappan (2018) obtained the time-dependent system size probabilities of an *M/M/1* queueing model with working vacations and variant impatient behavior of customers.

In our queueing model, we have derived the transient solution of an *M/M/*1 queue with retrial customers and reneging of customers from the orbit. Our model differs from Kumar and Sharma (2018) as follows: *Retrial customers* and *reneging of customers from the orbit* are the newly added parameters in our model.

The remaining sections of this paper are organized as follows. The *M/M/1* retrial queueing model with customers in the orbit becoming impatient is analyzed in section 2. Using continued fraction, the orbit size probabilities of such model are derived explicitly for the transient case. In section 3, performance measures such as mean and variance are computed in time t. In section 4, the effect of performance measures for different ξ values are analyzed graphically. Section 5 concluded our research work and directed the future extension.

2. Model Description

Consider an *M/M/*1 retrial queueing model with impatient behavior of customers in the orbit. Arrival pattern follows Poisson process with parameter λ and the service time distribution is exponential with parameter μ. It is assumed that there is no waiting place for the customers. If the server is not free during an arrival, the arriving customer moves to the waiting group, called an orbit, and retries for his/her service which follows an exponential distribution with the rate θ. If the server is idle when an arrival occurs (either from the orbit or from outside the queueing system), the arrival immediately gets the service and leaves the system.

Each individual customer in the orbit, retrying for his/her service, becomes impatient due to the long wait and activates an independent impatience timer which follows an exponential distribution with rate ξ. If the customer in the orbit gets a chance to be served before the expiry of his timer, he gets his service and moves out from the system. If the timer expires before his/her successful reattempts for service, he/she leaves the system without waiting for service and never returns. The inter-arrival durations, service time durations, inter-retrial times and impatience time durations are all assumed as independent. First Come First Served (FCFS) is the service discipline for the arrivals. Figure 1 shows the state transition diagram of this model.

Let $\{\mathbb{N}(t), t \geq 0\}$ be the size of the orbit at time t and $\Omega(t)$ be the server state at time t, which is given as follows:

$\Omega(t)$ = 0, if the server is idle at time t; = 1, if the server is busy at time t.

Then, $\{\mathbb{N}(t), \Omega(t), t \geq 0\}$ is a continuous-time Markov chain on the state space

$$S = \{k, n : k = 0 \; or \; 1; n = 0,1,2, \; \ldots\}.$$

Let

$$P_{0,n}(t) = P\{\Omega(t) = 0, \mathbb{N}(t) = n\}, \; n = 0, 1, \cdots .$$
$$P_{1,n}(t) = P\{\Omega(t) = 1, \mathbb{N}(t) = n\}, \; n = 0, 1, \cdots .$$

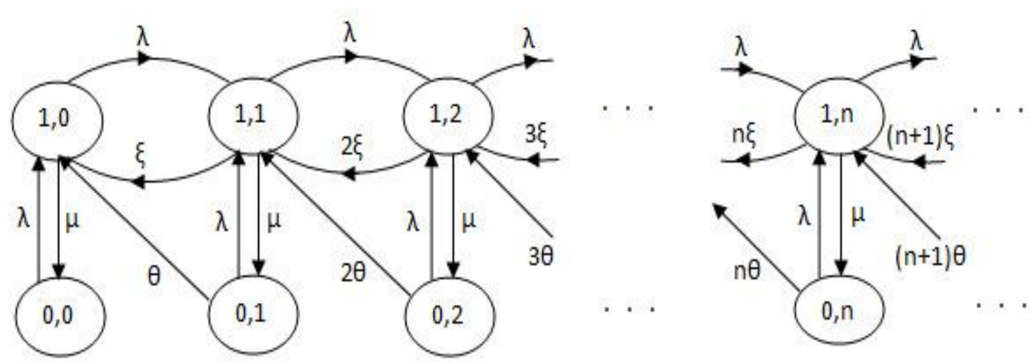

Figure 1. State Transition Diagram

The state transition rates are given as

$$q_{(1,\,n)(1,\,n+1)} = q_{(0,\,n)(1,\,n)} = \lambda,\; n = 0,\, 1,\, 2,\, \ldots,$$

$$q_{(1,\,n)(0,\,n)} = \mu,\; n = 0,\, 1,\, 2,\, \ldots,$$

$$q_{(1,\,n)(1,\,n-1)} = n\xi,\; n = 1,\, 2,\, 3,\, \ldots,$$

$$q_{(0,\,n)(1,\,n-1)} = n\theta,\; n = 1,\, 2,\, 3,\, \ldots.$$

From the state transition rates and the state transition diagram (given in Figure 1), the differential difference equations given in (1), (2) and (3) are developed.

Then, $P_{i,j}(t)$, $i = 0,1$; $j = 0,1,2, \; \ldots$, satisfy the differential equations which are given as follows:

$$P'_{0,n}(t) = -(\lambda + n\theta)P_{0,n}(t) + \mu P_{1,n}(t),\ n \ge 0, \tag{1}$$

$$P'_{1,0}(t) = -(\lambda + \mu)P_{1,0}(t) + \lambda P_{0,0}(t) + \theta P_{0,1}(t) + \xi P_{1,1}(t), \tag{2}$$

$$\begin{aligned} P'_{1,n}(t) &= -(\lambda + \mu + n\xi)P_{1,n}(t) + \lambda P_{1,n-1}(t) + (n+1)\xi P_{1,n+1}(t) + \lambda P_{0,n}(t) \\ &\quad + (n+1)\theta P_{0,n+1}(t),\ n \ge 1, \end{aligned} \tag{3}$$

with $P_{00}(0) = 1$.

2.1. Transient solution

The system size probabilities for the model under consideration are derived in this section for the transient case. Let $\hat{P}_{i,n}(s)$ be the Laplace transform of $P_{i,n}(t)$.
Taking Laplace Transform of (1), for $n = 0$ and rewriting, we get

$$\hat{P}_{0,0}(s) = \frac{1}{(s+\lambda) - \mu \dfrac{\hat{P}_{1,0}(s)}{\hat{P}_{0,0}(s)}} \tag{4}$$

and for $n = 1, 2, 3, \ldots,$

$$\frac{\hat{P}_{0,n}(s)}{\hat{P}_{1,n}(s)} = \frac{\mu}{s + \lambda + n\theta}. \tag{5}$$

From (2), for $n = 0$, we get

$$\frac{\hat{P}_{1,0}(s)}{\hat{P}_{0,0}(s)} = \frac{\lambda}{(s+\lambda+\mu) - \xi \dfrac{\hat{P}_{1,1}(s)}{\hat{P}_{1,0}(s)} - \theta \dfrac{\hat{P}_{0,1}(s)}{\hat{P}_{1,0}(s)}}. \tag{6}$$

From (3), for $n = 1, 2, 3, \ldots,$ we have

$$\frac{\hat{P}_{1,n}(s)}{\hat{P}_{1,n-1}(s)} = \frac{\lambda}{(s+\lambda+\mu+n\xi) - \lambda \dfrac{\hat{P}_{0,n}(s)}{\hat{P}_{1,n}(s)} - (n+1)\xi \dfrac{\hat{P}_{1,n+1}(s)}{\hat{P}_{1,n}(s)} - (n+1)\theta \dfrac{\hat{P}_{0,n+1}(s)}{\hat{P}_{1,n}(s)}}. \tag{7}$$

From (5) and (7), for $n = 1, 2, 3, \ldots,$ we get

$$\hat{F}_n(s)=\frac{\hat{P}_{0,n}(s)}{\hat{P}_{1,n}(s)}\times\frac{\hat{P}_{1,n}(s)}{\hat{P}_{1,n-1}(s)},$$

$$\hat{F}_n(s)=\frac{\mu}{(s+\lambda+n\theta)}\times\frac{\lambda}{(s+\lambda+\mu+n\xi)-\lambda\frac{\hat{P}_{0,n}(s)}{\hat{P}_{1,n}(s)}-(n+1)\xi\frac{\hat{P}_{1,n+1}(s)}{\hat{P}_{1,n}(s)}-(n+1)\theta\frac{\hat{P}_{0,n+1}(s)}{\hat{P}_{1,n}(s)}},$$

$$\hat{F}_n(s)=\frac{\hat{P}_{0,n}(s)}{\hat{P}_{1,n-1}(s)}=\frac{\frac{\lambda\mu}{(s+\lambda+n\theta)}}{(s+\lambda+\mu+n\xi)-\lambda\frac{\hat{P}_{0,n}(s)}{\hat{P}_{1,n}(s)}-(n+1)\xi\frac{\hat{P}_{1,n+1}(s)}{\hat{P}_{1,n}(s)}-(n+1)\theta\frac{\hat{P}_{0,n+1}(s)}{\hat{P}_{1,n}(s)}},$$

$$=\frac{\lambda\mu}{\chi_n(s)-}\,\frac{\phi_n(s)}{\chi_{n+1}(s)-}\,\frac{\phi_{n+1}(s)}{\chi_{n+2}(s)-}. \tag{8}$$

Substitute (6) in (4), we have

$$\hat{P}_{0,0}(s)=\frac{1}{(s+\lambda)-\cfrac{\lambda\mu}{(s+\lambda+\mu)-\xi\frac{\hat{P}_{1,1}(s)}{\hat{P}_{1,0}(s)}-\theta\frac{\hat{P}_{0,1}(s)}{\hat{P}_{1,0}(s)}}}.$$

Using (7) and (8) in the above equation, we get

$$\hat{P}_{0,0}(s)=\cfrac{1}{(s+\lambda)-\cfrac{\lambda\mu}{(s+\lambda+\mu)-\cfrac{(s+\lambda+\theta)\lambda\xi+\lambda\mu\theta}{\chi_1(s)-\cfrac{\phi_1(s)}{\chi_2(s)-\cfrac{\phi_2(s)}{\chi_3(s)-...}}}}},$$

which can be rewritten as

$$\hat{P}_{0,0}(s)=\frac{1}{(s+\lambda)-}\,\frac{\lambda\mu}{(s+\lambda+\mu)-}\,\frac{(s+\lambda+\theta)\lambda\xi+\lambda\mu\theta}{\chi_1(s)-}\,\frac{\phi_1(s)}{\chi_2(s)-}\,\frac{\phi_2(s)}{\chi_3(s)-}..., \tag{9}$$

where

$$\phi_n(s)=(s+\lambda+n\theta)\left[(s+\lambda+(n+1)\theta)(n+1)\lambda\xi+(n+1)\lambda\mu\theta\right],$$

$$\chi_n(s) = (s+\lambda+\mu+n\xi)(s+\lambda+n\theta) - \lambda\mu.$$

Using (5) and (8) in (7), we obtain

$$\begin{aligned}
\hat{G}_n(s) &= \frac{\hat{P}_{1,n}(s)}{\hat{P}_{1,n-1}(s)}, \\
&= \cfrac{\lambda}{(s+\lambda+\mu+n\xi) - \cfrac{\lambda\mu}{s+\lambda+n\theta} - (n+1)\xi\cfrac{\hat{P}_{1,n+1}(s)}{\hat{P}_{1,n}(s)} - (n+1)\theta\cfrac{\lambda\mu}{\chi_{n+1}(s)-}\,\cfrac{\phi_{n+1}(s)}{\chi_{n+2}(s)-}\,\cfrac{\phi_{n+2}(s)}{\chi_{n+3}(s)-}}, \\
&= \cfrac{\lambda}{s+\lambda+\mu+n\xi - \cfrac{\lambda\mu}{s+\lambda+n\theta} - \cfrac{(n+1)\lambda\xi + \cfrac{(n+1)\lambda\mu\theta}{s+\lambda+(n+1)\theta}}{s+\lambda+\mu+(n+1)\xi - \cfrac{\lambda\mu}{s+\lambda+(n+1)\theta} - \ldots}}.
\end{aligned} \tag{10}$$

For the details of continued fractions and their properties, one can refer Jones and Thron (1980). Using (7) and (8) in (6), for $n = 1$, we get

$$\begin{aligned}
\hat{J}(s) = \frac{\hat{P}_{1,0}(s)}{\hat{P}_{0,0}(s)} &= \frac{\lambda}{(s+\lambda+\mu) - \xi\hat{G}_1(s) - \theta\hat{F}_1(s)} \\
&= \lambda\sum_{k=0}^{\infty}\sum_{r=0}^{k}\binom{k}{r}\frac{\xi^{k-r}\theta^r}{(s+\lambda+\mu)^{k+1}}\hat{G}_1^{k-r}(s)\hat{F}_1^{r}(s).
\end{aligned} \tag{11}$$

$$\hat{P}_{1,0}(s) = \hat{J}(s)\hat{P}_{0,0}(s). \tag{12}$$

From (10) and (11), for n=1, 2, 3, …, we get

$$\hat{P}_{1,n}(s) = \hat{G}_n(s)\hat{P}_{1,n-1}(s) = \hat{G}_n(s)\times G_{n-1}(s)\times G_{n-2}(s)\times \ldots\times G_1(s)\times\hat{J}(s)\times\hat{P}_{0,0}(s). \tag{13}$$

From (8) and (13), for n = 1, 2, 3, …, we obtain

$$\hat{P}_{0,n}(s) = \hat{F}_n(s)\hat{P}_{1,n-1}(s) = \hat{F}_n(s)\times G_{n-1}(s)\times G_{n-2}(s)\times \ldots\times G_1(s)\times\hat{J}(s)\times\hat{P}_{0,0}(s). \tag{14}$$

From the normalization condition, we have

$$\hat{P}_{0,0}(s) + \sum_{n=0}^{\infty}\hat{P}_{1,n}(s) + \sum_{n=1}^{\infty}\hat{P}_{0,n}(s) = \frac{1}{s}. \tag{15}$$

From (12) to (14), we have

$$\hat{P}_{0,0}(s)\left\{1+\hat{J}(s)\left[1+\sum_{n=1}^{\infty}(\hat{G}_n(s)+\hat{F}_n(s))\prod_{i=1}^{n-1}\hat{G}_i(s)\right]\right\}=\frac{1}{s},$$

$$\hat{P}_{0,0}(s)=\frac{1}{s}\sum_{k=0}^{\infty}(-1)^k[\hat{J}(s)]^k\sum_{r=0}^{k}\binom{k}{r}\left\{\sum_{n=1}^{\infty}(\hat{G}_n(s)+\hat{F}_n(s))\prod_{i=1}^{n-1}\hat{G}_i(s)\right\}^r. \tag{16}$$

Let $G_n(t), F_n(t)$ and $J(t)$ be the inverse Laplace transform of $\hat{G}_n(s), \hat{F}_n(s)$ and $\hat{J}(s)$ respectively. On Laplace inversion of (13), (14), (12) and (16), we get

$$P_{1,n}(t)=G_1(t)*G_2(t)*\ \dots *G_n(t)*J(t)*P_{0,0}(t),\ \text{for}\ n=1,2,3,\ \dots, \tag{17}$$

$$P_{0,n}(t)=G_1(t)*G_2(t)*\ \dots *G_{n-1}(t)*F_n(t)*J(t)*P_{0,0}(t),\ \text{for}\ n=1,2,3,\ \dots, \tag{18}$$

$$P_{1,0}(t)=J(t)*P_{0,0}(t) \tag{19}$$

and

$$P_{0,0}(t)=\int_0^t\sum_{k=0}^{\infty}(-1)^k[J(u)]^{*k}*\sum_{r=0}^{k}\binom{k}{r}$$
$$\times\left\{\sum_{n=1}^{\infty}(G_n(u)+F_n(u))*G_1(u)*G_2(u)*\ \dots *G_{n-1}(u)\right\}du, \tag{20}$$

where "$*$" represents the convolution and $[R(t)]^{*k}$ is the k-fold convolution of $[R(t)]^k$. The equations (17), (18), (19) and (20) can be obtained easily as the convolution product of inverse Laplace transforms of corresponding functions respectively from (13), (14), (12) and (16).

2.2. Special case

When $\xi=0$, the results $P_{1,n}(t)$, $P_{0,n}(t)$, $P_{1,0}(t)$ and $P_{0,0}(t)$ coincide with the corresponding results of Parthasarathy and Sudhesh (2007).

3. Performance measures

The average and variance number of customers in the orbit at time t are obtained in this section.

3.1. Expected orbit size

The expected size of the orbit, *m(t)*, is given by

$$m(\mathrm{t})=\sum_{n=1}^{\infty}n\left(P_{0,n}(t)+P_{1,n}(t)\right).$$

From (1) and (3), we have

$$m'(\mathrm{t}) = \lambda\sum_{n=1}^{\infty} P_{1,n}(t) - \xi\sum_{n=1}^{\infty} nP_{1,n}(t) - \theta\sum_{n=1}^{\infty} nP_{0,n}(t).$$

Integrating the above differential equation, we can get

$$m(\mathrm{t}) = \lambda\sum_{n=1}^{\infty}\int_0^t P_{1,n}(u)du - \xi\sum_{n=1}^{\infty} n\int_0^t P_{1,n}(u)du - \theta\sum_{n=1}^{\infty} n\int_0^t P_{0,n}(u)du. \tag{21}$$

3.2. Variance of orbit size

The variance, $g(t)$, of the size of the orbit at time t is defined as

$$g(\mathrm{t}) = h(\mathrm{t}) - (\mathrm{m(t)})^2,$$

where

$$h(\mathrm{t}) = \sum_{n=1}^{\infty} n^2\left(P_{0,n}(t) + P_{1,n}(t)\right).$$

From the equations (1) and (3), we get

$$h'(\mathrm{t}) = \lambda\sum_{n=1}^{\infty}(2n = 1)P_{1,n}(t) - \xi\sum_{n=1}^{\infty} n(2n-1)P_{1,n}(t) - \theta\sum_{n=1}^{\infty} n(2n-1)P_{0,n}(t).$$

On integration, the above equation yields

$$h(\mathrm{t}) = \lambda\sum_{n=1}^{\infty}(2n-1)P_{1,n}(t) - \xi\sum_{n=1}^{\infty} n(2n-1)P_{1,n}(t) - \theta\sum_{n=1}^{\infty} n(2n-1)P_{0,n}(t). \tag{22}$$

4. Numerical illustrations

In this section, we have presented the graphical representation of the average and variance of orbit size as time t varies. The expected value and variance of the orbit size for various values of ξ are plotted in Figures 2 and 3 for $\lambda = 1, \mu = 1.5, \theta = 0.6.$ From Figures 2 and 3, it is evident that the average and variance number of units present in the orbit gets reduced as the reneging rate ξ increases.

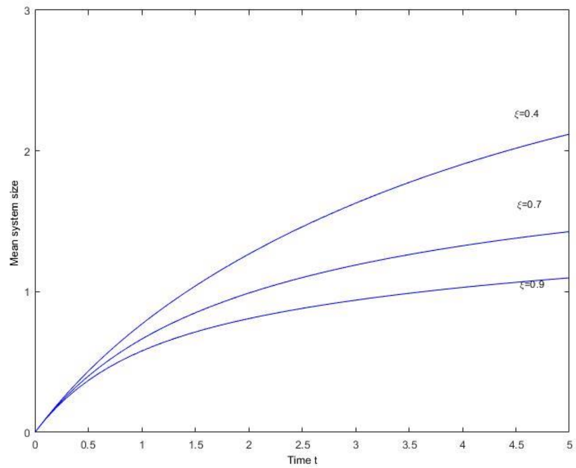

Figure 2. Mean orbit size verses time t for different values of ξ

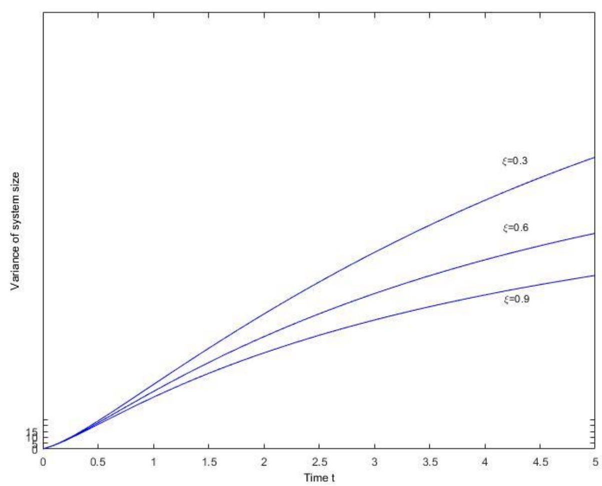

Figure 3. Variance of orbit size verses time t for different values of ξ

5. Conclusion and future work

The objective of this paper is to analyze the impatience behavior of waiting customers in the orbit for an *M/M/1* retrial queueing model. The orbit size probabilities of this model are derived explicitly for the transient case using the continued fraction. The average and variance of size of the orbit are also obtained. This work may be extended to the system with state dependent rates.

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