Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 13, Issue 1 (June 2018), pp. 150–159

A New Approach to Solve Multi-objective Transportation Problem

[1]Lakhveer Kaur, [1]Madhuchanda Rakshit, & [2]Sandeep Singh

[1]Department of Applied Science
Guru Kashi University
Talwandi Sabo (Pb), India
[2]Department of Mathematics
Akal University
Talwandi Sabo (Pb), India
[1]sangha.1987.lk@gmail.com; [2]sandeepinsan86@gmail.com

Received: January 17, 2018; Accepted: April 19, 2018

Abstract

In this paper, a simple approach is proposed to obtain the best compromise solution of linear multi-objective transportation problem (MOTP). Using this approach, we get unique efficient solution. Because unique efficient extreme point obtained by proposed approach directly leads to compromise solution, which is preferred by decision maker. Also this approach is simple to use and less time consuming. For the application of proposed approach, numerical examples are considered from existing literature and are solved with proposed method.

Keywords: Transportation Problem; Multi-objective linear Programming; Multi-objective Transportation Problem; Efficient Solution; Compromise solution.

MSC 2010 No.: 42C15, 42C30, 42C05, 42B15, 90B08, 90C08

1. Introduction

Generally problem associated with supplying commodities from different sources to various destination is known as transportation problem (TP), which is basically developed by Hitchcock (1941) and then by Koopmans (1947). These transportation problems are associated with single objective. But in real life situations, all organizations wants to achieve multiple objectives while making transportation of goods. So for making decision about achieving multiple goals simultaneously, Lee et al. (1973) used goal programming approach and Zeleny (1974) generated non dominated basic feasible solution to solve multi-objective linear programming (MOLP). Diaz (1978) presented an alternative algorithm to obtain all non dominated solutions for MTOP, which dependent on satisfaction level of the closeness of any compromise solution to the ideal solution. Diaz (1979) developed a procedure to obtain all non dominated solution for MOTP. Isermann (1979) developed different procedure to obtain set of efficient solutions.

Aneja et al. (1979) presented an algorithm to obtain non dominated extreme points in the objective space instead of decision space for bi-criteria TP. In their paper, they also included bottleneck criteria as third objective. Multi-criteria simplex method to solve MOTP is applied by Gupta et al. (1983). Ringuest et al. (1987) presented two interactive algorithms to solve MOTP. Kasana et al. (2000) and Bai et al. (2011) developed different procedures to solve MOTP. Dripping method to solve bi-objective TP is established by Pandian et al. (2011) and Quddoos et al. (2013a, 2013b) used lexicographic programming and developed MMK method to solve bi-objective TP. Gupta et al. (2013) obtained compromise solution for multi-objective chance constraint capicitated TP. Yu et al. (2014) developed an approach for obtaining the solution of MOTP with interval parameters. Maity et al. (2014, 2016a) solved multi-choice MOTP and MOTP with interval goal by using utility function approach. Also Maity et al. (2015, 2016b) developed approaches for solving MOTP with non linear cost, multi-choice demand and cost reliability under uncertain environment. Nomani et al. (2017) developed weighed approach based on goal programming to obtain compromise solution of MOTP, in which according to the prioreties of the decision maker, a new weighted model is presented to obtain varying solution.

In this paper, we have developed an algorithm which is based on making allocations in the cell with minimum cost of r^{th} objectives corresponding to the row/column of cell having maximum cost of r^{th} objective. The main advantage of the proposed algorithm is that the obtained efficient solution has minimum distance from the ideal solution, which leads to a compromise solution preferred by the decision maker. Also there is no need to obtain more efficient solutions, so the compromise solution is obtained in an easy way and with less time by applying the proposed approach.

Paper organization is as follows: Section 2 contains basic definitions from existing literature. Model representation of MOTP is given in Section 3. Section 4 provides algorithms for proposed methods; numerical examples and analysis is given in Section 5. The last section contains conclusions.

2. Preliminaries

In this section, some important basic definitions are reviewed Ringuest et al. (1987).

Definition 2.1.

An ideal solution to the multi-objective transportation problem would result in each objective simultaneously realizing its minimum. That is, if $z_r^* = \min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^r x_{ij}$, then the vector $z^* = (z_1^*, \ldots, z_k^*)$ is an ideal solution. When there is a feasible extreme point x^* such that $z_1^* = z_1(x^*), \ldots, z_l^* = z_k(x^*)$.

Definition 2.2.

The best compromise solution may be defined as that solution obtained at a feasible x which is deemed closest to the ideal solution by the decision maker.

Definition 2.3.

A feasible vector $\bar{x} = \{\bar{x}_{ij}\}$ yields a nondominated solution to the multi-objective transportation problem if, and only if, there is no other feasible vector $x = \{x_{ij}\}$ such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r} x_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r} \bar{x}_{ij}, \forall r \text{ and } \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r} x_{ij} \neq \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r} \bar{x}_{ij}, \text{ for some } r \text{ .}$$

When this relationship holds $\bar{x}$ is said to be efficient.

3. Model Representation

In real life situations, usually every organizer wants to achieve multiple goals simultaneously while making transportation of goods. So MOTP developed by researchers to achive multiple goals. Like classical transportation problem , in MOTP, quantity (x_{ij}) is to be transported from sources $i(i = 1, 2, ..., m)$ to destinations $j(j = 1, 2, ..., n)$ with cost c_{ij}, where c_{ij} can be transportation cost, total delivery time, energy consumption or minimizing transportation risk etc. The k objectives z_1, z_2,...,z_k are to be associated with transportation cost, total delivery time, cost of damage and/or cost of security etc. So MOTP (η) can be represented mathematically as follows:

$$\text{Min } Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} {c_{ij}}^{1} x_{ij}$$

$$\vdots$$

$$\text{Min } Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} {c_{ij}}^{k} x_{ij},$$

subject to

$$\sum_{i=1}^{m} x_{ij} = a_i, a_i \geq 0, \quad (i = 1, 2, ..., m)$$

$$\sum_{j=1}^{n} x_{ij} = b_j, b_j \geq 0, \quad (j = 1, 2, ..., n)$$

$$x_{ij} \geq 0, \text{ for all i, j and } \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j,$$

where

$c_{ij}{}^{r}$ Co-efficient of the r-th objective,
a_i supply amount of the product at source i (S_i),
b_j demand of the product at destination j (D_j),
and $a_i \geq 0$, $b_j \geq 0$.

Because of the special structure of the multi-objective transportation model, the problem (η) can also be represented as Table 1.

Table 1. Tabular representation of model (η).

Destination $\rightarrow$ source $\downarrow$	D_1	D_2	$\ldots$	D_n	supply(a_i)
S_1	c_{11}^1 c_{11}^2 $\vdots$ c_{11}^k	c_{12}^1 c_{12}^2 $\vdots$ c_{12}^k	$\ldots$ $\ldots$ $\vdots$ $\ldots$	c_{1n}^1 c_{1n}^2 $\vdots$ c_{1n}^k	a_1
S_2	c_{21}^1 c_{21}^2 $\vdots$ c_{21}^k	c_{22}^1 c_{22}^2 $\vdots$ c_{22}^k	$\ldots$ $\ldots$ $\vdots$ $\ldots$	c_{2n}^1 c_{2n}^2 $\vdots$ c_{2n}^k	a_2
$\vdots$	$\vdots$	$\vdots$	$\ldots$	$\vdots$	$\vdots$
S_m	c_{m1}^1 c_{m1}^2 $\vdots$ c_{m1}^k	c_{m2}^1 c_{m2}^2 $\vdots$ c_{m2}^k	$\ldots$ $\ldots$ $\vdots$ $\ldots$	c_{mn}^1 c_{mn}^2 $\vdots$ c_{mn}^k	a_m
Demand (b_j)	b_1	b_2	$\ldots$	b_n	

Remark.

The problem (η) is always considered to be balanced, i.e., $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. Because if it is not balanced then we can make it balanced easily.

4. Algorithm for Proposed Method

As to solve multi-objective problems, it is necessary to find the efficient solution which should be very close to ideal solution. Here we have proposed an easy technique to find unique efficient solution, which leads to compromise solution. Given steps are followed to proceed with proposed algorithm:

Step 1:
Formulate the given TP into the form of problem (η) and represent as Table 1.

Step 2:
Identify

Maximum row cost (α) as α_i^r=max (c_{ij}^r), for fixed i, $1 \le j \le n$ and $1 \le r \le k$,
Maximum column cost (β) as β_j^r=max (c_{ij}^r), for fixed j, $1 \le i \le m$ and $1 \le r \le k$,
where α = $\{\alpha_1^1,\ldots,\alpha_1^r;\ldots;\alpha_m^1,\ldots,\alpha_m^r\}$ and β = $\{\beta_1^1,\ldots,\beta_1^k;\ldots;\beta_n^1,\ldots,\beta_n^k\}$.

Represent these sets α and β in multi-objective transportation table as shown in table 2.

Table 2. Tabular representation of Step 2.

Destination → source ↓	D_1	D_2	$\ldots$	D_n	supply(a_i)	α
S_1	c_{11}^1 c_{11}^2 $\vdots$ c_{11}^k	c_{12}^1 c_{12}^2 $\vdots$ c_{12}^k	$\ldots$ $\ldots$ $\vdots$ $\ldots$	c_{1n}^1 c_{1n}^2 $\vdots$ c_{1n}^k	a_1	α_1^1 α_1^2 $\vdots$ α_1^k
S_2	c_{21}^1 c_{21}^2 $\vdots$ c_{21}^k	c_{22}^1 c_{22}^2 $\vdots$ c_{22}^k	$\ldots$ $\ldots$ $\vdots$ $\ldots$	c_{2n}^1 c_{2n}^2 $\vdots$ c_{2n}^k	a_2	α_2^1 α_2^2 $\vdots$ α_2^k
$\vdots$	$\vdots$	$\vdots$	$\ldots$	$\vdots$	$\vdots$	$\vdots$
S_m	c_{m1}^1 c_{m1}^2 $\vdots$ c_{m1}^k	c_{m2}^1 c_{m2}^2 $\vdots$ c_{m2}^k	$\ldots$ $\ldots$ $\vdots$ $\ldots$	c_{mn}^1 c_{mn}^2 $\vdots$ c_{mn}^k	a_m	α_m^1 α_m^2 $\vdots$ α_m^k
Demand (b_j)	b_1	b_2	$\ldots$	b_n		
β	β_1^1 β_1^2 $\vdots$ β_1^k	β_2^1 β_2^2 $\vdots$ β_2^k	$\ldots$ $\ldots$ $\vdots$ $\ldots$	β_n^1 β_n^2 $\vdots$ β_n^k		

Step 3:
Choose

$$Q = \max_{1 \le i \le m, 1 \le j \le n} (\alpha_i^r, \beta_j^r), \forall r.$$

Step 4:
Choose the cell (C) having Q as one of its objective value. If there exist more than one cell (C) then select the one, which has maximum cost for another objectives.

Step 5:
Choose the cell containing min$\left(\sum_{i=1}^{m} c_{ij}^r, \text{ for fixed } j\right)$ in corresponding row or column of cell chosen in Step 4. If tie occurs, then select the one to which maximum allocation is possible.

Step 6:

Make maximum possible allocation to the cell selected in Step 5 and cross out the row or column for which supply/demand is satisfied.

Step 7:
Repeat the procedure of Step 3 to Step 6 for remaining sources and destinations until whole supply or demand requirements are not met.

Now we interpret the proposed method with the help of example as follows:

Step 1.
Consider MOTP in tabular form as same as table 1.

Table 3. Input data of MOTP considered for interpretation of proposed method.

Destination → source ↓	D_1	D_2	D_3	supply(a_i)
S_1	3 5	4 2	5 1	8
S_2	4 3	5 4	2 3	5
S_3	5 2	1 3	2 1	2
Demand (b_j)	7	4	4	

Step 2.
Calculate maximum row cost α as

$$\alpha_1^1 = \max\{3, 4, 5\} = 5,\ \alpha_1^2 = \max\{5, 2, 1\} = 5,$$
$$\alpha_2^1 = \max\{4, 5, 2\} = 5,\ \alpha_2^2 = \max\{3, 4, 3\} = 4,$$
$$\alpha_3^1 = \max\{5, 1, 2\} = 5,\ \alpha_3^2 = \max\{2, 3, 1\} = 3.$$

and maximum column cost β as

$$\beta_1^1 = \max\{3, 4, 5\} = 5,\ \beta_1^2 = \max\{5, 3, 2\} = 5,$$
$$\beta_2^1 = \max\{4, 5, 1\} = 5,\ \beta_2^2 = \max\{2, 4, 3\} = 4,$$
$$\beta_3^1 = \max\{5, 2, 2\} = 5,\ \beta_3^2 = \max\{1, 3, 1\} = 3.$$

Represent α and β in table 4.

Step 3.
Calculate

$$Q = \max_{1 \le i \le 3, 1 \le j \le 3} (\alpha_i^r, \beta_j^r), \forall r = \max\{5, 4, 3\} = 5.$$

Table 4. Initial procedure of proposed algorithm.

Destination $\rightarrow$ source $\downarrow$	D_1	D_2	D_3	supply(a_i)	α
S_1	3 5	4 2	5 1	8	5 5
S_2	4 3	5 4	2 3	5	5 4
S_3	5	1 2	2	2	5
	2	3	1		3
Demand (b_j)	7	4	4		
β	5 5	5 4	5 3		

Step 4.
Cells c^r_{11}, c^r_{13}, c^r_{22}, c^r_{31} have one of objective value 5. But we have to select only one cell. So according to proposed method, from selected cells, c^r_{22} has maximum cost 4 for another objective. Therefore we select c^r_{22}.

Step 5.
As c^r_{32} has minimum (1 + 3 =4) cost, so we make allocation min{2, 4}=2 in the cell c^r_{32} and cross out 3^{rd} row of table 4 for which supply S_3 is satisfied.

Again value of Q for remaining rows and columns is

$$Q = \max_{1\leq i\leq 2, 1\leq j\leq 3}(\alpha^r_i, \beta^r_j), \forall r = \max\{5, 4, 3\} = 5,$$

and on repeating Step 3 to Step 5 for uncrossed rows and columns, we get second allocation as 4 in the cell c^r_{23} and cross out the 3^{rd} column, for which demand D_3 is satisfied.

So by proceeding with the procedure of step 3 to step 5 for making allocations in remaining rows and columns, we get 3^{rd}, 4^{th} and 5^{th} allocations as 2, 1, and 6 in cells c^r_{12}, c^r_{21} and c^r_{11} respectively and are shown in table 5. Therefore the obtained efficient solution from table 5 is (40, 55).

5. Numerical Examples and Analysis

For proving the effectiveness of our proposed algorithm, we have considered 2 examples from existing literature and solved using our proposed method. Input data of examples is given in table 6.

On obtaining the solution of both examples by applying proposed method and ideal solution according to def. 2.1, the results are given in table 7.

So from table 7, it is analyzed that our proposed method gives efficient solution with less calcu-

Table 5. Final solution table.

Destination → source ↓	D_1	D_2	D_3	supply(a_i)	α
S_1	3 (6) 5	4 (2) 2	5 1	8	5 5
S_2	4 (1) 3	5 4	2 (4) 3	5	5 4
S_3	5 2	1 (2) 3	2 1	2	5 3
Demand (b_j)	7	4	4		
β	5 5	5 4	5 3		

Table 6. Input data and source of Examples taken from literature.

Ex.	Input Data	Source
1	$[c_{ij}^1]_{3\times4}$=[1 2 7 7; 1 9 3 4; 8 9 4 6]; $[c_{ij}^2]_{3\times4}$=[4 4 3 4; 5 8 9 10 ; 6 2 5 1]; $[a_i]_{3\times1}$=[8, 19, 17]; $[b_j]_{1\times4}$=[11, 3, 14, 16]	Aneja et al. (1979)
2	$[c_{ij}^1]_{4\times5}$=[9 12 9 6 9; 7 3 7 7 5; 6 5 9 11 3; 6 8 11 2 2]; $[c_{ij}^2]_{4\times5}$=[2 9 8 1 4; 1 9 9 5 2; 8 1 8 4 5; 2 8 6 9 8]; $[c_{ij}^3]_{4\times5}$=[2 4 6 3 6; 4 8 4 9 2; 5 3 5 3 6; 6 9 6 3 1]; $[a_i]_{4\times1}$=[5, 4, 2, 9]; $[b_j]_{1\times5}$=[4, 4, 6, 2, 4]	Diaz (1979)

Table 7. Result (efficient solution using proposed algorithm and ideal solution).

Ex.	Obtained Allocations	Obtained Cost	Ideal Solution
1	x_{11} = 0,x_{12} = 3, x_{13} = 5,x_{14} = 0,x_{21} = 11,x_{22} = 0 x_{23} = 8, x_{24} = 0, x_{31} = 0, x_{32}=0,x_{33} = 1,x_{34} = 16	(176, 175)	(143, 167)
2	x_{11} = 3,x_{12} = 0,x_{13} = 0,x_{14} = 2,x_{15} = 0,x_{21} = 0, x_{22} = 2,x_{23} = 2, x_{24} = 0,x_{25} = 0x_{31} = 0, x_{32} = 2, x_{33} = 0,x_{34} = 0, x_{35} = 0,x_{41} = 1,x_{42} = 0,x_{43} = 4,x_{44} = 0,x_{45} = 4	(127, 104, 76)	(102, 72, 64)

lation and in simple way. It has minimum distance from ideal solution. Because for example 1, Ringuest et al. (1987) obtained (156, 200),Aneja et al. (1979) and Quddoos et al. (2013a) computed (176, 175) as efficient solution and all algorithms applied by these researchers need long procedure to obtain efficient solutions and are difficult to apply. But it is very easy to obtain most efficient solution by our proposed algorithm. Also for example 2, Nomani et al. (2017) obtained (127, 104, 76) as the efficient solution which is same as our proposed approach. But their algorithm takes more time and efforts comparative to our proposed algorithm.

6. Conclusions

From above analysis, it is analyzed that the proposed algorithm gives unique efficient solution. There is no need to obtain efficient solution more than one. Because obtained unique solution used to give compromise solution, which is preferred by decision maker. This method can be easily used to solve large scale MOTP and better decisions can be made in less time. So, it can be more suitable to apply for each decision maker.

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