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Fractional Order Thermoelastic Deflection in a Thin Circular Plate

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Abstract

In this work, a quasi-static uncoupled theory of thermoelasticity based on time fractional heat conduction equation is used to model a thin circular plate, whose lower surface is maintained at zero temperature whereas the upper surface is insulated. The edge of the circular plate is fixed and clamped. Integral transform technique is used to derive the analytical solutions in the physical domain. The numerical results for temperature distributions and thermal deflection are computed and represented graphically for Copper material.

Keywords: Quasi-static; thermoelasticity; fractional; integral transform; thermal deflection; parabolic; heat conduction

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1. Introduction

During the second half of the twentieth century, considerable amount of research in fractional calculus was published in engineering literature. Indeed, recent advances of fractional calculus are dominated by modern examples of applications in differential and integral equations, physics, signal processing, fluid mechanics, viscoelasticity, mathematical biology, and electrochemistry. There is no doubt that fractional calculus has become an exciting new mathematical method of solution of diverse problems in mathematics, science, and engineering. It is generally known that integer-order derivatives and integrals have clear physical and geometric interpretations. However, in case of fractional-order integration and differentiation, it is not so. Since the appearance of the idea of differentiation and integration of arbitrary (not necessary integer) order, there was not any acceptable geometric and physical interpretation of these operations for more than 300 year. In Podlubny (2002), it is shown that geometric interpretation of fractional integration is 'Shadows on the walls' and its Physical interpretation is 'Shadows of the past'.

The classical theory of thermoelasticity has aroused much interest in recent times due to its numerous applications in engineering discipline such as nuclear reactor design, high energy particle accelerators, geothermal engineering, advanced aircraft structure design, etc. The heat conduction of classical coupled theory of thermoelasticity is parabolic in nature and hence predicts infinite speed of propagation of heat waves. Clearly, this contradicts the physical observations. Hence, several non-classical theories such as, Lord-Shulman (1967) theory, Green Lindsay theory (1972) have been proposed, in which the Fourier law and the parabolic heat conduction equation are replaced by more complicated equations, which are hyperbolic in nature predicting finite wave propagation. Green and Naghdi (1993) developed the theory of thermoelasticity without energy dissipation. Chandrasekaraiah (1986) gave reviews of thermoelasticity with second sound. Tripathi et al. (2015a, 2015b, 2016) studied various problems in cylindrical domain in the context of generalized thermoelastic theories. Recently, Tripathi et al. (2016) studied a dynamic problem in fractional order thermoelasticity with finite wave speeds. In the last decade, study on Quasi-static thermoelasticity incorporating the time fractional derivative has gained momentum. Povstenko (2005, 2009a, 2009b, 2010, 2011, 2012) studied various problems on quasi static fractional order thermoelasticity**.** Boley and Weiner (1960) studied the problems of thermal deflection of an axisymmetric heated circular plate in the case of fixed and simply supported edges. Roy choudhury (1973) discussed the normal deflection of a thin clamped circular plate due to ramp type heating of a concentric circular region of the upper face. This satisfies the time-dependent heat conduction equation. Deshmukh and Khobragade (2005) determined a quasi-static thermal deflection in a thin circular plate due to partially distributed and axisymmetric heat supply on the outer curved surface with the upper and lower faces at zero temperature. Deshmukh et al. (2009) studied a quasi-static thermal deflection problem of a thin clamped circular plate due to heat generation. Deshmukh et al. (2014) discussed the thermal stresses in a simply supported plate with thermal bending moments with heat sources.

It is seen that the literature dealing with problems of quasi-static uncoupled fractional order thermoelasticity is limited to infinite domains and so far no one has studied problems on thermal deflection in the context of fractional order thermoelasticity. Hence, in the present study an effort has been made to develop a mathematical model to study thermal deflection in the context of fractional order thermoelasticity for a finite thin circular plate under constant temperature distribution by quasi static approach. Copper material is chosen for numerical purposes and the results for temperature and thermal deflection are discussed and illustrated graphically.

2. Formulation of the problem

Consider a thin circular plate of thickness h occupying space D defined by $0 \le r \le b, 0 \le z \le h$, whose lower surface is maintained at zero temperature whereas the upper surface is insulated. The constant heat flux Q_0 is applied on the fixed circular boundary $(r = b)$ and a mathematical model is prepared considering non-local Caputo type time fractional heat conduction equation of order α for a thin circular plate.

The definition of Caputo type fractional derivative is given by Podlubny (1999)

$$\frac{\partial^\alpha f(t)}{\partial t^\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_0^t t-\tau)^{n-\alpha-1} \frac{d^n f(\tau)}{d\tau^n} d\tau, \quad n-1 < \alpha < n. \tag{1}$$

For finding the Laplace transform, the Caputo derivative requires knowledge of the initial values of the function $f(t)$ and its integer derivatives of the order $k = 1, 2, ..., n-1$,

$$L\left\{\frac{\partial^\alpha f(t)}{\partial t^\alpha}\right\} = s^\alpha f^*(s) - \sum_{k=0}^{n-1} f^{(k)}(0^+) s^{\alpha-1-k}, \quad n-1 < \alpha < n, \tag{2}$$

where, the asterisk denotes the Laplace transform with respect to time, s is the Laplace transform parameter.

The temperature of the plate $T(r, z, t)$ is satisfying time fractional order differential equation,

$$a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) = \frac{\partial^\alpha T}{\partial t^\alpha}, \qquad 0 \le r \le b, 0 \le z \le h, \tag{3}$$

with boundary conditions,

$$\frac{\partial T}{\partial r} = Q_0 \quad \text{at} \quad r = b, \tag{4}$$

$$T = 0 \quad \text{at} \quad z = 0, \tag{5}$$

$$\frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = h, \tag{6}$$

and under zero initial condition

$$T = 0 \qquad \text{at} \quad t = 0, \;\; 0 < \alpha < 1, \tag{7}$$

$$\frac{\partial T}{\partial t} = 0 \qquad \text{at} \quad t = 0, \;\; 1 < \alpha < 2. \tag{8}$$

2.1. Thermal deflection $\omega(r,t)$

The differential equation satisfying the deflection function $\omega(r,t)$ is given as Deshmukh et al. (2009),

$$\nabla^4 \omega = -\frac{\nabla^2 M_T}{D(1-\nu)}, \tag{9}$$

where M_T is the thermal moment of the plate defined as,

$$M_T = a_t\, E \int_0^h T(r,z,t) z\, dz\ . \tag{10}$$

D is the flexural rigidity of the plate denoted as,

$$D = \frac{Eh^3}{12(1-\nu^2)}\ . \tag{11}$$

a_t, E and ν are the coefficients of the linear thermal expansion, the Young's modulus and Poisson's ratio of the plate material respectively and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\ . \tag{12}$$

Since, the edge of the circular plate is fixed and clamped, i.e., built-in edge,

$$\omega = \frac{\partial \omega}{\partial r} = 0 \text{ at } r = b\,. \tag{13}$$

Equations (3) to (13) constitute the mathematical formulation of the problem.

3. Solution

To obtain the expression for temperature function $T(r,z,t)$; we first define the finite Fourier transform and its inverse transform over the variable z in the range $0 \le z \le h$ defined in Boley and Weiner (1960) as

$$\overline{T}(r,\eta_p,t)=\int_{z'=0}^{h}K(\eta_p,z').T(r,z',t)dz', \tag{14}$$

$$T(r,z,t)=\sum_{p=1}^{\infty}K(\eta_p,z).\overline{T}(r,\eta_p,t), \tag{15}$$

where

$$K(\eta_p,z)=\sqrt{\frac{2}{h}}\sin(\eta_p z),$$

and $\eta_1,\eta_2,\ldots$, are the positive roots of the transcendental equation

$$\cos(\eta_p h)=0,\ p=1,2,\ldots\ .$$

Taking the integral transform of Equations (3) – (8) and with the aid of transform Equation (14), one obtains,

$$\frac{\partial^2\overline{T}}{\partial r^2}+\frac{1}{r}\frac{\partial\overline{T}}{\partial r}-\eta_p^2\overline{T}=\frac{1}{a}\frac{\partial^\alpha\overline{T}}{\partial t^\alpha}, \qquad 0\le r\le b, 0\le z\le h, \tag{16}$$

$$\frac{\partial\overline{T}}{\partial r}=\overline{Q}_0\ , \qquad \text{at } r=b, \tag{17}$$

$$\overline{T}=0, \qquad \text{at } t=0,\ 0<\alpha<2, \tag{18}$$

$$\frac{\partial\overline{T}}{\partial t}=0, \qquad \text{at } t=0,\ 1<\alpha<2. \tag{19}$$

Secondly, we define the finite Hankel transform and its inverse transform over the variable r in the range $0\le r\le b$ as,

$$\overline{\overline{T}}(\beta_m,\eta_p,t)=\int_{r'=0}^{b}r'.K_0(\beta_m,r').\overline{T}(r',\eta_p,t)dr'\ , \tag{20}$$

$$\overline{T}(r,\eta_p,t)=\sum_{m=1}^{\infty}K_0(\beta_m,r)\,\overline{\overline{T}}(\beta_m,\eta_p,t), \tag{21}$$

where

$$K_0(\beta_m,r)=\frac{\sqrt{2}}{b}\frac{J_0(\beta_m r)}{J_0(\beta_m b)},$$

and $\beta_1,\beta_2,\ldots$, are the positive roots of the transcendental equation

$$J_1(\beta_m b) = 0\,.$$

Now, we apply the integral transform to Equations (16) - (19) and with the aid of transform Equation (20), one obtains,

$$\frac{\partial^\alpha \overline{\overline{T}}(\beta_m, \eta_p, t)}{\partial^\alpha t} + a\left(\beta_m^2 + \eta_p^2\right)\overline{\overline{T}}(\beta_m, \eta_p, t) = abK_0(\beta_m, b).\overline{Q}_0\ , \tag{22}$$

$$\overline{\overline{T}} = 0\,, \quad \text{at } t = 0,\ \ 0 < \alpha \le 2\,, \tag{23}$$

$$\frac{\partial \overline{\overline{T}}}{\partial t} = 0\,, \quad \text{at } t = 0,\ \ 1 < \alpha \le 2\,. \tag{24}$$

Taking the Laplace transform of Equation (22) and applying initial conditions (23) - (24), we get the solution as follows,

$$s^\alpha \overline{\overline{T}}^*(\beta_m, \eta_p, s) + a\left(\beta_m^2 + \eta_p^2\right)\overline{\overline{T}}^*(\beta_m, \eta_p, s) = abK_0(\beta_m, b)\frac{\overline{Q}_0}{s}\ . \tag{25}$$

On rearranging the terms in Equation (25), we get,

$$\overline{\overline{T}}^*(\beta_m, \eta_p, s) = abK_0(\beta_m, b)\overline{Q_0}\ \frac{1}{s[s^\alpha + a(\beta_m^2 + \eta_p^2)]}\,. \tag{26}$$

On applying inverse Laplace transform to Equation (26), we get,

$$\overline{\overline{T}}(\beta_m, \eta_p, s) = abK_0(\beta_m, b)\,\overline{Q}_0\ \left[1 - E_\alpha\left(-a\ (\beta_m^2 + \eta_p^2)\ t^\alpha\right)\right], \tag{27}$$

where

$$L^{-1}\left[\frac{1}{s[s^\alpha + a(\beta_m^2 + \eta_p^2)]}\right] = \left[1 - E_\alpha\left(-a\ (\beta_m^2 + \eta_p^2)\ t^\alpha\right)\right]. \tag{28}$$

Here $E_\alpha(.)$ represents the Mittag-Leffler function.

The resulting double transform of temperature is inverted successively by means of the inversion formulae (15) and (21). We obtain the expression of temperature $T(r, z, t)$ as,

$$T(r,z,t)=\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}K(\eta_p,z)K_0(\beta_m,r)abK_0(\beta_m,b)$$
$$\times\; Q_0\,\frac{\sqrt{2}}{\eta_p h}\left[1-\cos\eta_p h\right]\left[1-E_\alpha\left(-a\;(\beta_m^2+\eta_p^2)\;t^\alpha\right)\right]. \tag{29}$$

3.1. Determination of thermal deflection $\omega(r,t)$

Using Equation (29) in Equation (10), one obtains

$$M_T=-a_t\,E\,\sqrt{2h}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\eta_p^2}\sin(\eta_p h)K_0(\beta_m,r)a.bK_0(\beta_m,b)$$
$$\times\; Q_0\,\frac{\sqrt{2}}{\eta_p h}\left[1-\cos\eta_p h\right]\left[1-E_\alpha\left(-a\;(\beta_m^2+\eta_p^2)\;t^\alpha\right)\right]\quad . \tag{30}$$

Assume that the solution of Equation (9), satisfies the condition (13) as

$$\omega(r,t)=\sum_{m=1}^{\infty}C_m(t)\left[J_0(\beta_m r)-J_0(\beta_m b)\right], \tag{31}$$

where β_m' are the positive roots of the transcendental equation $J_1(\beta_m b)=0$.

It can be easily shown that

$$\omega(r,t)=0 \text{ at } r=b. \tag{32}$$

Now,

$$\frac{\partial\omega}{\partial r}=\sum_{m=1}^{\infty}C_m(t).\beta_m\;J_1(\beta_m r), \tag{33}$$
$$\frac{\partial\omega}{\partial r}=0 \text{ at } r=b.$$

Hence, the solution (31) satisfies the condition (13). Now,

$$\nabla^4\omega=\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)^2\sum_{m=1}^{\infty}C_m(t)\left[J_0(\beta_m r)-J_0(\beta_m b)\right]. \tag{34}$$

Using the following well known result

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)J_0(\beta_m r)=-\beta_m^2\, J_0(\beta_m r), \tag{35}$$

in Equation (34), one obtains,

$$\nabla^4\omega=\sum_{m=1}^{\infty}C_m(t)\beta_m^4\, J_0(\beta_m r)\,. \tag{36}$$

Also,

$$\begin{aligned}\nabla^2 M_T &= -\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)a_t\, E\sqrt{2h}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\eta_p^2}\sin\left(\eta_p h\right)K_0\left(\beta_m, r\right)a.b\,K_0\left(\beta_m, b\right)\\ &\quad\times\; Q_0\,\frac{\sqrt{2}}{\eta_p h}\left[1-\cos\eta_p h\right]\left[1-E_\alpha\left(-a\;(\beta_m^2+\eta_p^2)\;t^{\alpha}\right)\right].\end{aligned} \tag{37}$$

On simplifying above equation, we get,

$$\begin{aligned}\nabla^2 M_T &= a_t\, E\sqrt{2h}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\eta_p^2}\sin(\eta_p h)\beta_m^2\, K_0(\beta_m, r)a.b.K_0(\beta_m, b)\\ &\quad\times\; Q_0\,\frac{\sqrt{2}}{\eta_p h}\left[1-\cos\eta_p h\right]\left[1-E_\alpha\left(-a\;(\beta_m^2+\eta_p^2)\;t^{\alpha}\right)\right].\end{aligned} \tag{38}$$

Substituting Equations (36) and (38) into Equation (9), one obtains,

$$\begin{aligned}\sum_{m=1}^{\infty}C_m(t)\beta_m^4\, J_0(\beta_m r) &= -\frac{a_t\, E\sqrt{2h}}{D(1-\nu)}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\eta_p^2}\sin(\eta_p h)\;\beta_m^2\;K_0(\beta_m r)a.b.K_0(\beta_m b)\\ &\quad\times\; Q_0\,\frac{\sqrt{2}}{\eta_p h}\left[1-\cos\eta_p h\right]\left[1-E_\alpha\left(-a\;(\beta_m^2+\eta_p^2)\;t^{\alpha}\right)\right],\end{aligned} \tag{39}$$

$$\begin{aligned}C_m(t) &= -\frac{a_t\, E\sqrt{2h}}{D(1-\nu)}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\eta_p^2}\sin(\eta_p h)\;\frac{1}{\beta_m^2}\;\frac{\sqrt{2}}{b}\,\frac{1}{J_0(\beta_m b)}\,a.b.K_0(\beta_m b)\\ &\quad\times\; Q_0\,\frac{\sqrt{2}}{\eta_p h}\left[1-\cos\eta_p h\right]\left[1-E_\alpha\left(-a\;(\beta_m^2+\eta_p^2)\;t^{\alpha}\right)\right].\end{aligned} \tag{40}$$

Substituting Equation (40) into Equation (31), one obtains,

$$\omega(r,t) = -a_t\, E\sqrt{2h}\,\frac{1}{D(1-\nu)}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\eta_p^2}\sin(\eta_p h)\,\frac{1}{\beta_m^2}\,\frac{\sqrt{2}}{b}\,\frac{[J_0(\beta_m r) - J_0(\beta_m b)]}{J_0(\beta_m b)}$$
$$\times\ a.b.K_0(\beta_m b)\, Q_0\,\frac{\sqrt{2}}{\eta_p h}\left[1-\cos\eta_p h\right]\left[1 - E_\alpha\left(-a\ (\beta_m^2 + \eta_p^2)\ t^\alpha\right)\right]. \tag{41}$$

4. Numerical calculations

4.1. Dimensions

For the sake of convenience, we choose:

radius of a thin circular plate $b = 1\mathrm{m}$, and
thickness of a thin circular plate $h = 0.1\ \mathrm{m}$.

4.2. Material properties

The numerical calculation has been carried out for a Copper (Pure) thin circular plate with the material properties as:

thermal diffusivity $a = 112.34\times 10^{-6}\,(m^2 s^{-1})$,
thermal conductivity $k = 386(W/mk)$,
density $\rho = 8954 kg/m^3$,
specific heat $c_p = 383 J/kgK$,
Poisson ratio $\nu = 0.35$,
coefficient of linear thermal expansion $a_t = 16.5\times 10^{-6}\,\frac{1}{K}$, and
Lamé constant $\mu = 26.67$.

4.3. Roots of transcendental equation

$\beta_1 = 3.8317,\ \beta_2 = 7.0156,\ \beta_3 = 10.1735,\ \beta_4 = 13.3237,\ \beta_5 = 16.470$, and
$\beta_6 = 19.6159,\ \beta_7 = 22.7601,\ \beta_8 = 25.9037,\ \beta_9 = 29.0468,\ \beta_{10} = 32.18,$

are the roots of transcendental equation $J_1(\beta b) = 0$.

We set for convenience, $X = -a_t\, E\, h / D(1-\nu)$.

The graphs are plotted for fractional order parameter $\alpha = 0.5, 1, 1.5$, depicting weak, normal and strong conductivity and fixed time $t = 0.5$. Figures 1 and 2 depict the distributions of temperature, thermal deflection along the radial direction for various values of fractional order parameter

α. The numerical calculation has been carried out in Matlab 2013a programming environment. The Mittag-Leffler functions used in the paper were evaluated following Podlubny (1999).

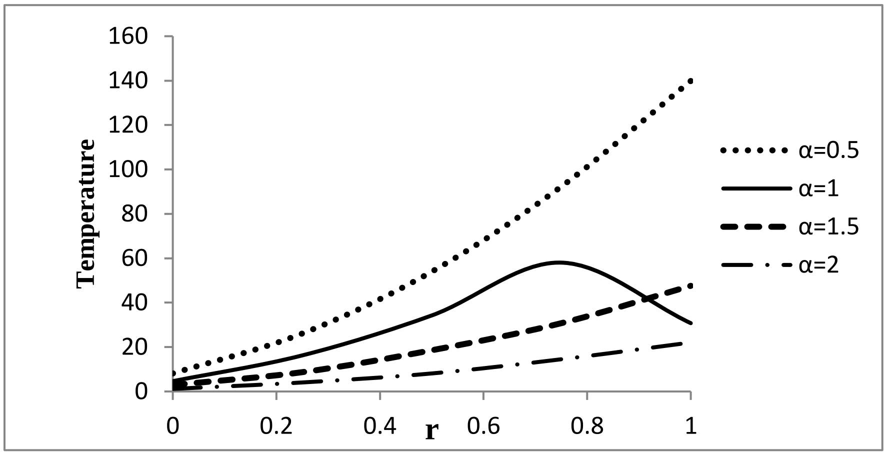

Figure 1. Temperature Distribution Function

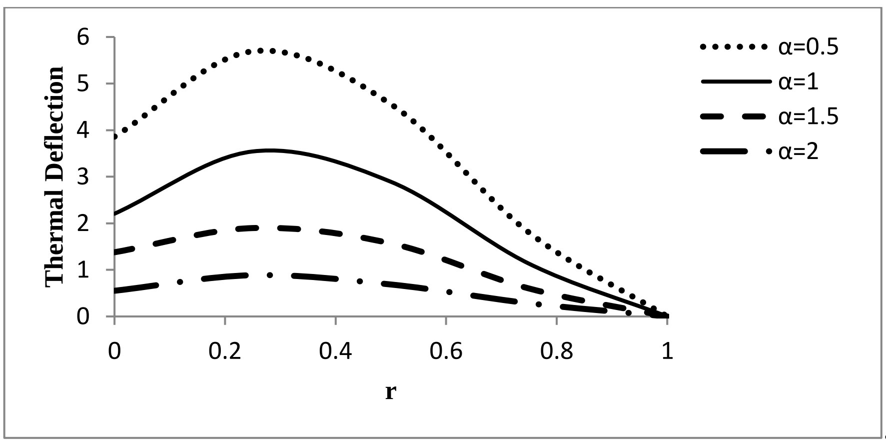

Figure 2. Thermal Deflection Function $\frac{\sigma_{rr}}{X}$

Figure 1, represents the temperature distributions along the radial direction. For the case $\alpha = 0.5, 1.5, 2$, the values of the temperature show an increase with respect to radius. For the case $\alpha = 1$, the values of the temperature initially increase up to $r = 0.72$ and then, decrease in the range $0.72 \le r \le 1$. For the case $\alpha = 1$, depicting classical thermoelasticity the pattern of graphs is completely different as compared to fractional thermoelasticity $\alpha = 0.5, 1.5, 2$. It should be noted that for range $0 < \alpha < 1$, the graphs show weak conductivity. For $\alpha = 1$, the graphs describe normal conductivity and for $1 < \alpha < 2$, the graphs depict strong conductivity. The case $\alpha = 2$ coincides with Green and Naghdi theory.

Figure 2, represents thermal deflection along the radial direction. It can be observed that for the cases $\alpha = 0.5, 1, 1.5, 2$, the deflection increases with increase in radius and attains a maximum at $r=0.3$ and then, gradually decreases to zero at $r=1$. It is zero at the outer circular edge which coincides with the boundary condition imposed on the thin circular cylinder. When the fractional order parameter $\alpha=0.5$ (describing weak conductivity), its deflection is high whereas for cases $\alpha=1$ (normal conductivity) and $\alpha=1.5,\ 2$ (strong conductivity), its deflection is less. Hence, one can say that, as thermal conductivity of metal decreases its deflection increases.

It is noted from the graphs that changing values of fractional order parameter α, the speed of wave propagation is affected. Hence, it can be an important factor for designing new materials applicable to real life situations.

5. Conclusions

The theory of Thermoelasticity based on time fractional heat wave equation proposed by Povstenko [8] is used to model a finite cylinder. The cases $0<\alpha<1$ and $1<\alpha<2$ correspond to weak and strong conductivity, respectively, while $\alpha=1$ corresponds to normal conductivity.
We restrict ourselves to the quasi-static uncoupled theory neglecting the inertia term in the equation of motion and the coupling term. The quasi-static statement of a thermoelastic problem is possible if the relaxation time of mechanical oscillations is considerably less than the relaxation time of the heat conduction process. The motivation behind the consideration of the fractional theory is that it predicts retarded response to physical stimuli, as seen in nature.
In real life situations, the problems dealing with finite domains are important but unfortunately, due to the complexity involved in modeling a finite domain, the literature is limited to problems on infinite domains. Hence, this problem was developed for a finite cylinder.

We can summarize that, in a thin circular plate, the temperature and deflection are inversely proportional to the thermal conductivity of metal.

In the case $1<\alpha<2$, the time fractional heat conduction equation interpolates the standard parabolic heat conduction equation and the hyperbolic wave equations. Likewise, the thermoelasticity interpolates the classical theory of thermal stresses without energy dissipation introduced by Green and Naghdi (1993) and admitting the propagation of second sound (see, Chandrasekaraiah (1986)).

REFERENCES

Boley B. A. and Weiner J. H. (1960). Theory of Thermal Stresses, Wiley; New York, USA.

Chandrasekharaiah, D.S. (1986). Thermoelasticity with Second sound: A review, Appl. Mech. Rev., Vol. 39, pp. 355-376.

Deshmukh, K. C. and Khobragade, N. L. (2005). An inverse quasi-static thermal deflection problem for a thin clamped circular plate, J. Therm. Stresses, Vol. 28, pp. 353-361.

Deshmukh, K. C., Warbhe, S.D. and Kulkarni, V.S. (2009). Quasi-static thermal deflection of a thin clamped circular plate due to heat generation, J. Therm. Stresses, Vol. 32, pp. 877-886.

Deshmukh, K.C., Khandait, M.V. and Kumar, R. (2014). Thermal stresses in a simply supported plate with thermal bending moments with heat sources, Materials Physics and Mechanics, Vol. 21, pp. 135-146.

Green, A.E, and Lindsay, K.A. (1972). Thermoelasticity, J. Elasticity, Vol. 2, pp. 1-7.

Green, A.E, and Naghdi, P.M. (1993). Thermoelasticity without energy dissipation, J. Elasticity, Vol. 31, pp.181-208.

Lord, H, and Shulman, Y. A. (1967). Generalized Dynamical theory of thermoelasticity, Journal of the Mechanics and Physics of solids, Vol. 15, pp. 299-307.

Podlubny, I. (2002). Geometric and physical interpretation of fractional integration and fractional differentiation, Fractional Calculus and Applied Analysis, pp. 5:4.

Podlubny, I. (1999). Fractional Differential Equations. An Introduction to Fractional Derivatives, Fractional Differential Equations, Some Methods of Their Solution and Some of Their Applications. Academic Press, San Diego-Boston-New York-London-Tokyo-Toronto.

Povstenko, Y. (2005). Fractional heat conduction equation and associated thermal stresses, J. Therm. Stresses, Vol. 28, pp. 83–102.

Povstenko, Y. (2009a). Thermoelasticity which uses fractional heat conduction equation, J. Math. Sci, Vol. 162, pp.296–305.

Povstenko, Y. (2009b). Theory of thermoelasticity based on the space-time-fractional heat conduction equation, Phys. Scr. T. , Vol. 136, pp. 1-6.
DOI: 10.1088/0031-8949/2009/T136/014017

Povstenko, Y. (2010) Signaling problem for time-fractional diffusion-wave equation in a half-plane in the case of angular symmetry, Nonlinear Dyn.,Vol. 59, pp. 593–605.

Povstenko, Y. (*2011).* Fractional Cattaneo-type equations and generalized thermoelasticity, J. Therm. Stresses, Vol. 34, pp. 97–114.

Povstenko, Y. (2012). Theories of thermal stresses based on space-time-fractional telegraph equations, Comp. Math. Appl., Vol. 64, pp. 3321–3328.

Roy Choudhuri, S. K. (1973). A Note On Quasi-Static Thermal Deflection of a Thin Clamped Circular Plate due to Ramp-type Heating of a Concentric Circular Region of the Upper Face, Journal of the Franklin Institute, Vol. 296, pp. 213-219.

Tripathi, J. J., Kedar, G. D., and Deshmukh, K. C. (2015a). Generalized thermoelastic diffusion problem in a thick circular plate with axisymmetric heat supply, Acta Mech., Vol. 226, pp. 2121-2134. DOI: 10.1007/s00707-015-1305-7.

Tripathi, J. J., Kedar, G. D., and Deshmukh, K. C. (2015b). Two dimensional generalized thermoelastic diffusion in a half space under axisymmetric distributions, Acta Mech., Vol. 226, No. 10, pp. 3263-3274. DOI: 10.1007/s00707-015-1383-6.

Tripathi, J. J., Kedar, G. D., and Deshmukh, K. C. (2016a). A Brief Note on Generalized Thermoelastic Response in a Half Space due to a Periodically Varying Heat Source under Axisymmetric Distribution, International Journal of Thermodynamics, Vol. 19, pp. 1-6.
DOI: 10.5541/ijot.5000145489.

Tripathi, J. J., Kedar, G. D., and Deshmukh, K. C. (2016b*).* Dynamic problem of fractional order thermoelasticity for a thick circular plate with finite wave speeds, J. Therm. Stresses, Vol. 39, No.2, pp. 220-230. DOI: 10.1080/01495739.2015.1124646