Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Vol. 12, Issue 1 (June 2017), pp. 319 - 336

Applications and Applied Mathematics:
An International Journal
(AAM)

Mathematical modelling of Stoneley wave in a transversely isotropic thermoelastic media

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Received August 31, 2016; Accepted January 06, 2017

Abstract

This paper is concerned with the study of propagation of Stoneley waves at the interface of two dissimilar transversely isotropic thermoelastic solids without energy dissipation and with two temperatures. The secular equation of Stoneley waves is derived in the form of the determinant by using appropriate boundary conditions i.e. the stresses components, the displacement components, and temperature at the boundary surface between the two media are considered to be continuous at all times and positions . The dispersion curves giving the Stoneley wave velocity and Attenuation coefficients with wave number are computed numerically. Numerical simulated results are depicted graphically to show the effect of two temperature and anisotropy on resulting quantities. Copper material has been chosen for the medium M_1 and magnesium for the medium M_2. Some special cases are also deduced from the present investigation.

Keywords: Transversely isotropic; Stoneley wave; Two temperatures; Secular equation; Stoneley wave velocity; Attenuation Coefficient; isotropic

MSC 2010 No.: 74A10, 74 A15, 74B05, 74D05, 74 E10, 74 F05

1. Introduction

The exact number of the layers beneath the earth's surface is not known. One has, therefore, to consider various appropriate models for the purpose of theoretical investigations. These models not only provide better information about the internal composition of the earth but are also helpful in exploration of valuable materials beneath the earth surface.

Mathematical modeling of surface wave propagation along with the free boundary of an elastic half-space or along the interface between two dissimilar elastic half-spaces has been subject of continued interest for many years. These waves are well known in the study of geophysics, ocean acoustics, SAW devices and non-destructive evaluation. Stoneley (1924) studied the existence of waves, which are similar to surface waves and propagating along the plane interface between two distinct elastic solid half-spaces in perfect contact. Stoneley waves can also propagate on interfaces either two elastic media or a solid medium and a liquid medium. Stoneley (1924) derived the dispersion equation for the propagation of Stoneley waves. Tajuddin (1995) investigated the existence of Stoneley waves at an interface between two micropolar elastic half spaces.

Chen and Gurtin (1968) , Chen et al. (1968) and Chen et al. (1969) have formulated a theory of heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature φ and the thermodynamical temperature T. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in absence of heat supply, the two temperatures are identical. For time dependent problems, the two temperatures are different, regardless of the presence of heat supply. The two temperatures T, φ and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body (Boley and Tolins (1962)).The wave propagation in two temperature theory of thermoelasticity was investigated by Warren and Chen (1973).

A comprehensive work has been done in thermoelasticity theory with and without energy dissipation and thermoelasticity with two temperature. Youssef (2006), constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without energy dissipation. Youssef et al. (2007) investigated State space approach of two temperature generalized thermoelasticity of infinite body with a spherical cavity subjected to different types of thermal loading. Quintanilla (2002) investigated thermoelasticity without energy dissipation of materials with microstructure. Several researchers studied various problems involving two temperature e.g.

(Youssef and AI-Lehaibi (2007); Youssef (2011); Youssef (2013); Kumar, Sharma and Garg (2014); Kaushal et al.(2013); Kaushal et al. (2010); Ezzat and Awad (2010); Sharma and Marin (2013); Sharma and Bhargav (2014); Sharma et al. (2013); Sharma and Kumar (2013)).

Green and Naghdi (1991) postulated a new concept in thermoelasticity theories and proposed three models which are subsequently referred to as GN-I, II, and III models. The linearized version of model-I corresponds to classical thermoelastic model (based on Fourier's law). The linearised version of model-II and III permit propagation of thermal waves at finite speed. Green-Naghdi's second model (GN-II), in particular exhibits a feature that is not present in other established thermoelastic models as it does not sustain dissipation of thermal energy (1993). In this model the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables.

Green-Naghdi's third model (1992) admits dissipation of energy. In this model the constitutive equations are derived by starting with the reduced energy equation, where the thermal displacement gradient in addition to the temperature gradient, are among the constitutive variables.

Kumar and Chawla (2009) discussed the wave propagation at the imperfect boundary between transversely isotropic thermoelastic diffusive half-spaces and an isotropic elastic layer. Kumar et al. (2013) studied the reflection and transmission of plane waves at the interface between a microstretch thermoelastic diffusion solid half-space and elastic solid half-space. Recently influence of new parameters on surface waves has been investigated by many researchers (Ahmed and Abo-Dahab (2012); Abo-Dahab (2015); Abd-Alla et al. (2015); Marin (1995, 1998, 2010))

Keeping in view of these applications, dispersion equation for Stoneley waves at the interface of two dissimilar transversely isotropic thermoelastic mediums with two temperature and without energy dissipation have been derived. Numerical computations are performed for a particular model to study the variation of phase velocity and attenuation coefficient with respect to wave number. The results in this paper should prove useful in the field of material science, designers of new materials as well as for those working on the development of theory of elasticity.

2. Basic equations

Following Youssef (2006), the constitutive relations and field equations in the absence of body forces and heat sources are:

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T \,, \tag{1}$$
$$C_{ijkl}e_{kl,j} - \beta_{ij}T_{,j} = \rho\ddot{u}_i \,, \tag{2}$$
$$k^*{}_{ij}\varphi_{,ij} = \beta_{ij}T_0\ddot{e}_{ij} + \rho C_E\ddot{T} \,, \tag{3}$$

where

$$T = \varphi - a_{ij}\varphi_{,ij} \,, \tag{4}$$
$$\beta_{ij} = C_{ijkl}\alpha_{ij} \,, \tag{5}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)\, i,j = 1,2,3\ . \tag{6}$$

Here, $C_{ijkl}(C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk})$ are elastic parameters, β_{ij} is the thermal tensor, T is the temperature, T_0 is the reference temperature, t_{ij} are the components of stress tensor, e_{kl} are the components of strain tensor, u_i are the displacement components, ρ is the density, C_E is the specific heat, $k^*{}_{ij}$ is the materialistic constant, a_{ij} are the two temperature parameters, α_{ij} is the coefficient of linear thermal expansion.

3. Formulation of the problem

We consider a homogeneous, transversely isotropic thermoelastic half-space M_1 overlying another homogeneous, transversely isotropic thermoelastic half-space M_2 connecting at the interface $x_3 = 0$. We take origin of co-ordinate system (x_1, x_2, x_3) on $x_3 = 0$ .We choose $x_1 -$ axis in the direction of wave propagation in such a way that all the particles on a line parallel to x_2 axis are equally displaced, so that the field component $u_2 = 0$ and u_1, u_3 and φ are independent of x_2. Medium M_2 occupies the region $-\infty < x \leq 0$ and the medium M_1 occupies the region $0 \leq x < \infty$. The plane $x_3 = 0$ represents the interface between the two media M_1 and M_2 . We define all the quantities without bar for the medium M_1 and with bar for medium M_2. We have used appropriate transformations following Slaughter (2002) on the set of Equations (1) - (3) to derive the equations for transversely isotropic thermoelastic solid with two temperature and without energy dissipation and we restrict our analysis to the two dimensional problem with

$$\vec{u} = (u_1, 0, u_3), \tag{7}$$

Equations (1) - (3) with the aid of (7) take the form

$$c_{11}\frac{\partial^2 u_1}{\partial x_1^2} + c_{44}\frac{\partial^2 u_1}{\partial x_3^2} + (c_{13} + c_{44})\frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \beta_1 \frac{\partial}{\partial x_1}\left\{\varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\} = \rho \frac{\partial^2 u_1}{\partial t^2}\ , \tag{8}$$

$$(c_{13} + c_{44})\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + c_{44}\frac{\partial^2 u_3}{\partial x_1^2} + c_{33}\frac{\partial^2 u_3}{\partial x_3^2} - \beta_3 \frac{\partial}{\partial x_3}\left\{\varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\} = \rho \frac{\partial^2 u_3}{\partial t^2}, \tag{9}$$

$$k_1{}^*\frac{\partial^2 \varphi}{\partial x_1^2} + k_3{}^*\frac{\partial^2 \varphi}{\partial x_3^2} = T_0 \frac{\partial^2}{\partial t^2}\left(\beta_1 \frac{\partial u_1}{\partial x_1} + \beta_3 \frac{\partial u_3}{\partial x_3}\right) + \rho C_E \frac{\partial^2}{\partial t^2}\left\{\varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\}, \tag{10}$$

$$t_{11} = c_{11}e_{11} + c_{13}e_{33} - \beta_1 T,$$

$$t_{33} = c_{13}e_{11} + c_{33}e_{33} - \beta_3 T,$$

$$t_{13} = 2c_{44}e_{13},$$

where

$$e_{11} = \frac{\partial u_1}{\partial x_1}, \qquad e_{33} = \frac{\partial u_3}{\partial x_3}, \quad e_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right), T = \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right),$$

$$\beta_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3, \qquad \beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3.$$

In the above equations we use the contracting subscript notations ($1 \to 11, 2 \to 22, 3 \to 33, 4 \to 23, 5 \to 31, 6 \to 12$) to relate c_{ijkl} to c_{mn}

The initial and regularity conditions are given by

$$u_1(x_1, x_3, 0) = 0 = \dot{u_1}(x_1, x_3, 0),$$

$$u_3(x_1, x_3, 0) = 0 = \dot{u_3}(x_1, x_3, 0),$$

$$\varphi(x_1, x_3, 0) = 0 = \dot{\varphi}(x_1, x_3, 0), \text{ for } x_3 \geq 0, \quad -\infty < x_1 < \infty. \tag{11}$$

$$u_1(x_1, x_3, t) = u_3(x_1, x_3, t) = \varphi(x_1, x_3, t) = 0 \text{ , for } t > 0 \text{ when } x_3 \to \infty \tag{12}$$

To facilitate the solution, following dimensionless quantities are introduced:

$$x_1' = \frac{x_1}{L},\ x_3' = \frac{x_3}{L},\ u_1' = \frac{\rho c_1^2}{L\beta_1 T_0}u_1,\ u_3' = \frac{\rho c_1^2}{L\beta_1 T_0}u_3,$$
$$T' = \frac{T}{T_0}, t' = \frac{c_1}{L}t, t_{11}' = \frac{t_{11}}{\beta_1 T_0}, t_{33}' = \frac{t_{33}}{\beta_1 T_0},$$
$$t_{31}' = \frac{t_{31}}{\beta_1 T_0}, \varphi' = \frac{\varphi}{T_0}, a_1' = \frac{a_1}{L}, a_3' = \frac{a_3}{L}, \tag{13}$$

where $c_1^2 = \frac{c_{11}}{\rho}$ and L is a constant of dimension of length.

Using the dimensionless quantities defined by (13) into (8) - (10) and after that suppressing the primes we obtain

$$\frac{\partial^2 u_1}{\partial x_1^2} + \delta_1 \frac{\partial^2 u_1}{\partial x_3^2} + \delta_2 \frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + a_3 \frac{\partial^2}{\partial x_3^2}\right)\right] \frac{\partial \varphi}{\partial x_1} = \frac{\partial^2 u_1}{\partial t^2}, \tag{14}$$

$$\delta_4 \frac{\partial^2 u_3}{\partial x_3^2} + \delta_1 \frac{\partial^2 u_3}{\partial x_1^2} + \delta_2 \frac{\partial^2 u_1}{\partial x_1 \partial x_3} - p_5 \left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + a_3 \frac{\partial^2}{\partial x_3^2}\right)\right] \frac{\partial \varphi}{\partial x_3} = \frac{\partial^2 u_3}{\partial t^2}, \tag{15}$$

$$\frac{\partial^2 \varphi}{\partial x_1^2} + p_3 \frac{\partial^2 \varphi}{\partial x_3^2} - \zeta_1 \frac{\partial^2}{\partial t^2} \frac{\partial u_1}{\partial x_1} - \zeta_2 \frac{\partial^2}{\partial t^2} \frac{\partial u_3}{\partial x_3} = \zeta_3 \left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + c^2 a_3 \frac{\partial^2}{\partial x_3^2}\right)\right] \frac{\partial^2 \varphi}{\partial t^2}, \tag{16}$$

where

$$\delta_1 = \frac{c_{44}}{c_{11}}, \quad \delta_2 = \frac{c_{13} + c_{44}}{c_{11}}, \delta_4 = \frac{c_{33}}{c_{11}}, p_5 = \frac{\beta_3}{\beta_1}, p_3 = \frac{k_3^*}{k_1^*},$$
$$\zeta_1 = \frac{T_0 \beta_1^2}{k_1 \rho}, \zeta_2 = \frac{T_0 \beta_3 \beta_1}{k_1 \rho}, \zeta_3 = \frac{C_E c_{11}}{k_1}, c_{11} = \rho c_1^2.$$

4. Solution of the problem

We assume the solution of the form

$$(u_1, u_3, \varphi) = (u_1^*, u_3^*, \varphi^*) e^{i\xi(x_1 - ct)}, \tag{17}$$

where ξ is the wave number, $\omega = \xi c$ is the angular frequency and c is the phase velocity of the wave Using (17) in Equations (14) - (16) and satisfying the radiation condition $u_1, u_3, \varphi \to 0\ as\ x_3 \to \infty$, we obtain the values of u_1, u_3, φ for the medium M_1.
Substitute in (14) - (16)

$$\xi^2(c^2-1)u_1^* + \delta_1 \frac{d^2u_1^*}{dx_3^2} + i\xi\delta_2 \frac{du_3^*}{dx_3} - \left[1 + \left(a_1\xi^2 - a_3 \frac{d^2}{dx_3^2}\right)\right] i\xi\varphi^* = 0, \tag{18}$$

$$(\xi^2c^2 - \delta_1\xi^2)u_3^* + i\xi\delta_2 \frac{du_1^*}{dx_3} + \delta_4 \frac{d^2u_1^*}{dx_1^2} - p_5\left[1 + \left(a_1\xi^2 - a_3 \frac{d^2}{dx_3^2}\right)\right]\frac{d\varphi^*}{dx_3} = 0, \tag{19}$$

$$\zeta_1\xi^2c^2 i\xi u_1^* + \zeta_2\xi^2c^2 \frac{du_3^*}{dx_3} + \left(\zeta_3\xi^2c^2 - \xi^2 + \zeta_3\xi^4a_1c^2\right)\varphi^* - \zeta_3\xi^2c^2a_3 \frac{d^2}{dx_3^2}\varphi^* = 0\ . \tag{20}$$

These equations have nontrivial solutions if the determinant of the coefficient $(u_1^*, u_3^*, \varphi^*)$ vanishes, which yield the following characteristic equation

$$\left(P \frac{d^6}{dx_3^6} + Q \frac{d^4}{dx_3^4} + R \frac{d^2}{dx_3^2} + S\right)(u_1^*, u_3^*, \varphi^*) = 0\ , \tag{21}$$

where

$$P = \delta_1\left(-\delta_4\zeta_3a_3\xi^2 - \delta_4p_3 + \zeta_2p_5a_3\xi^2c^2\right),$$

$$Q = \left(-\zeta_3a_3\xi^2c^2 - p_3\right)\left\{\left(-\xi^2 - \xi^2c^2\right)\delta_4 - \delta_1\left(\delta_1\xi^2 - \xi^2c^2\right) + \delta_2^2\xi^2\right\} + \delta_1\delta_4\left\{\xi^2 + \zeta_3\xi^2c^2 + \xi^4\zeta_3c^2a_1\right\} - \xi^2c^2\zeta_2\left\{a_3p_5\left(\xi^2 - \xi^2c^2\right) + \delta_1p_5\left(a_1\xi^2 + 1\right)\right\} + \xi^4c^2\left\{\delta_4a_3\left(p_5\zeta_1 + \zeta_2 - \zeta_1\right)\right\},$$

$$R = \left(1 + a_1\xi^2\right)\left\{\left(\xi^2 - \xi^2c^2\right)\zeta_2p_5\xi^2c^2 - \xi^4c^2\left(p_5\zeta_1\delta_2 + \zeta_2\delta_2 - \zeta_1\delta_4\right)\right\} + \left(\delta_1\xi^2 - \xi^2c^2\right)\left\{(\xi^2 - \xi^2c^2)\left(-\xi^2c^2\zeta_3a_3 - p_3\right) - \delta_1\left(\xi^2 + \zeta_3\xi^2c^2 + \zeta_3c^2a_1\xi^4\right) + \xi^4a_3\zeta_1c^2\right\} + \left(\xi^2 + \zeta_3\xi^2c^2 + \zeta_3c^2\xi^4a_1\right)\left\{-\left(\xi^2 - \xi^2c^2\right)\delta_4 + \delta_2^2\xi^2\right\},$$

$$S = \left(\delta_1\xi^2 - \xi^2c^2\right)\left\{(\xi^2 - \xi^2c^2)\left(\xi^2 + \zeta_3\xi^2c^2 + \zeta_3c^2a_1\xi^4\right) - \xi^6(1 + a_1\xi^2)\zeta_1c^2\right\}.$$

For the medium $\mathbf{M_1}$

$$(u_1, u_3, \varphi) = \sum_{j=1}^{3}(1, d_j, l_j)\, e^{-m_jx_3}A_je^{i\xi(x_1-ct)}\ , \tag{22}$$

$$d_j = \frac{-m_j{}^3P^* - m_jQ^*}{m_j{}^4R^* + m_j{}^2S^* + T^*}\ ,$$

$$l_j = \frac{m_j{}^2P^{**} + Q^{**}}{m_j{}^4R^* + m_j{}^2S^* + T^*}\ , \tag{23}$$

where

$$P^* = i\xi\left\{\zeta_1p_5a_3\xi^2c^2 - \delta_2\left(\zeta_3a_3\xi^2c^2 + p_3\right)\right\},$$

$$Q^* = \delta_2\left(\xi^2 + \zeta_3\xi^2c^2 + \zeta_3a_1\xi^4c^2\right) - p_5\zeta_1(1 + a_1\xi^2)\xi^2c^2,$$
$$R^* = \zeta_2p_5a_3\xi^2c^2 - \delta_4\left(\zeta_3a_3\xi^2c^2 + p_3\right),$$

$$S^* = (\xi^2 + \zeta_3\xi^2c^2 + \zeta_3\xi^4c^2a_1)\delta_4 + (\delta_1\xi^2 - \xi^2c^2)\left(\zeta_3a_3\xi^2c^2 + p_3\right) - \zeta_2p_5\xi^2c^2(1 + a_1\xi^2)\ ,$$

$$T^* = -\xi^4(\delta_1 - c^2)\left(1 + \zeta_3c^2 + \zeta_3\xi^2a_1c^2\right),$$

$$P^{**} = -\left(\zeta_2\delta_2 - \zeta_1\delta_4\right)ic^2\xi^3,$$

$$Q^{**} = -\xi^4c^2\zeta_1(\delta_1 - c^2) + i\xi\{\zeta_1p_5a_3\xi^2c^2 - \delta_2\left(\zeta_3a_3\xi^2c^2 + p_3\right)\}\,. \tag{24}$$

For the medium $\mathrm{M_2}$

We attach bars for the medium $\mathrm{M_2}$ and write the appropriate values $\bar{u}_1, \bar{u}_3, \bar{\varphi}$ for the medium $\mathrm{M_2}(\mathrm{x_3} < 0)$satisfying the radiation conditions as

$$(\bar{u}_1, \bar{u}_3, \bar{\varphi}) = \sum_{j=1}^{3}[\overline{A}_j(1, \bar{d}_j, \bar{l}_j)e^{\overline{m}_j x_3}]e^{i\xi(x_1 - ct)}\,, \tag{25}$$

where $\bar{d}_j, \bar{l}_j, \overline{P^*}, \overline{Q^*}, \overline{R^*}, \overline{P^{**}}, \overline{Q^{**}}$ are obtained from equations (23) and (24) by attaching bar to all the quantities.

5. Boundary conditions

We assume that the half spaces are in perfect contact. Thus, there is continuity of components of displacement vector, normal stress vector, tangential stress vector, temperatures and temperature change at the interface

i) $t_{33} = \overline{t_{33}}\,,$ at $x_3 = 0$ (26)
ii) $t_{31} = \overline{t_{31}}\,,$ at $x_3 = 0$ (27)
iii) $\varphi = \bar{\varphi}\,,$ at $x_3 = 0$ (28)
iv) $u_1 = \bar{u}_1,$ at $x_3 = 0$ (29)
v) $u_3 = \bar{u}_3,$ at $x_3 = 0$ (30)
vi) $k_3^*\frac{\partial\varphi}{\partial x_3} = \overline{k_3^*}\frac{\partial\bar{\varphi}}{\partial x_3}\,,$ at $x_3 = 0$ (31)

In case the half spaces are not in perfect contact i.e., the interface between the half -spaces is frictionless, then tangential stress is absent and the tangential displacement is discontinuous.

6. Derivation of secular equations

Making use of Equations (14) and (15) for medium M_1 and corresponding equations with bar for medium M_2 in Equations (26)-(31) along with (22) and (25) we obtain a system of six simultaneous homogeneous secular equations

$$\sum_{j=1}^{3}\eta_{qj}A_j+\eta_{qj+3}\overline{A_j}=0\,,(q=1,2,3,4,5,6)$$

$$\eta_{1j}=\frac{c_{31}}{\rho c_1^2}i\xi-\frac{c_{33}}{\rho c_1^2}d_j\mathrm{m}_j-\frac{\beta_3}{\beta_1}l_j+\frac{\beta_3}{\beta_1 T_0}a_3 l_j\mathrm{m}_j{}^2-\frac{\beta_3}{\beta_1}l_j a_1\xi^2\,,\ j=1,2,3.$$

$$\eta_{1j+3}=-\frac{\overline{c_{31}}}{\overline{\rho c_1}^2}i\xi-\frac{\overline{c_{33}}}{\overline{\rho c_1}^2}\overline{d_j}\overline{\mathrm{m}_j}+\frac{\overline{\beta_3}}{\overline{\beta_1}}\overline{l_j}-\frac{\overline{\beta_3}}{\overline{\beta_1 T_0}}a_3\overline{l_j}\overline{\mathrm{m}_j}^2+\frac{\overline{\beta_3}}{\overline{\beta_1}}\overline{l_j}a_1\xi^2,\ j=1,2,3.$$

$$\eta_{2j}=-\frac{c_{44}}{\rho c_1^2}\mathrm{m}_j+\frac{c_{44}}{\rho c_1^2}i\xi d_j,\ j=1,2,3.$$

$$\eta_{2j+3}=-\frac{c_{44}}{\overline{\rho c_1}^2}\overline{\mathrm{m}_j}-\frac{c_{44}}{\overline{\rho c_1}^2}i\xi\overline{d_j}j=1,2,3.$$

$$\eta_{3j}=l_j j=1,2,3.$$

$$\eta_{3j+3}=-\overline{l_j},j=1,2,3.$$

$$\eta_{4j}=\delta_1\mathrm{m}_j{}^2-i\xi d_j\mathrm{m}_j-i\xi(1+a_1\xi^2)l_j+i\xi a_3 l_j\mathrm{m}_j{}^2,j=1,2,3.$$

$$\eta_{4j+3}=-\delta_1\overline{\mathrm{m}_j}^2-i\xi\overline{d_j}\overline{\mathrm{m}_j}+i\xi(1+a_1\xi^2)\overline{l_j}-i\xi a_3\overline{l_j}\overline{\mathrm{m}_j}^2,j=1,2,3.$$

$$\eta_{5j}=\zeta_1 i\xi-\mathrm{m}_j d_j-\zeta_3 a_3 l_j\mathrm{m}_j{}^2,\ \ j=1,2,3.$$

$$\eta_{5j+3}=-\overline{\zeta_1}i\xi-\overline{\mathrm{m}_j}\overline{d_j}+\overline{\zeta_3}a_3\overline{l_j}\overline{\mathrm{m}_j}^2,j=1,2,3.$$

$$\eta_{6j}=-k_3^*l_j\mathrm{m}_j,j=1,2,3,\eta_{6j+3}=\overline{-k_3^*}\overline{l_j}\,\overline{\mathrm{m}_j},j=1,2,3. \tag{32}$$

$\overline{\zeta_1},\overline{\zeta_3}$ can be obtained from ζ_1 and ζ_3 by attaching bar to all the quantities. The system of Equations (32) has a non-trivial solution if the determinant of unknowns A_j, $\overline{A_j}$, j=123 vanishes i.e.,

$$\left|\eta_{ij}\right|_{6\times 6}=0.$$

7. Particular cases

(i) If $a_1=a_3=0$, from Equations (26) - (31), we obtain the corresponding expressions for displacements, and stresses and conductive temperature for transversely isotropic thermoelastic solid without two temperature and without energy dissipation.

(ii) If we take $c_{11}=c_{33}=\lambda+2\mu$, $c_{12}=c_{13}=\lambda$, $c_{44}=\mu$, $\beta_1=\beta_3=\beta$, $\alpha_1=\alpha_3=\alpha$, $K_1=K_3=K$ in Equations (26) -(31), we obtain the corresponding expressions for displacements, and stresses and conductive temperature in isotropic thermoelastic solid with two temperature and without energy dissipation.

8. Numerical results and discussion

Following Youssef (2006), Copper material is chosen for the purpose of numerical calculation for the medium M_1which is transversely isotropic

$c_{11} = 18.78 \times 10^{10} Kgm^{-1}s^{-2}$, $a_1 = 0.03, a_3 = 0.06$, $c_{11} = \rho c_1^2$, $c_{12} = 8.76 \times 10^{10}\ Kgm^{-1}s^{-2}$, $c_{13} = 8.0 \times\ 10^{10} Kgm^{-1}s^{-2}$, $c_{33} = 17.2 \times 10^{10} Kgm^{-1}s^{-2}$, $c_{44} = 5.06 \times 10^{10} Kgm^{-1}s^{-2}$, $C_E = 0.6331 \times 10^3 JKg^{-1}K^{-1}$, $\alpha_1 = 2.98 \times 10^{-5} K^{-1}$, $\alpha_3 = 2.4 \times 10^{-5} K^{-1}$, $\rho = 8.954 \times 10^3 Kgm^{-3}$, $T_0 = 293°$, $k_1^* = 0.04 \times 10^2 Nsec^{-2}deg^{-1}$, $k_3^* = 0.02 \times 10^2 Nsec^{-2}deg^{-1}$, $\beta_1 = 7.543 \times 10^6 Nm^{-2}deg^{-1}, \beta_3 = 9.0208 \times 10^6 Nm^{-2}deg^{-1}$,with non-dimensional parameter L=1.

Following Dhaliwal and Singh (1980), magnesium material has been taken for the medium M_2, as

$\overline{c_{11}} = 5.974 \times 10^{10} Nm^{-2}, \overline{c_{33}} = 6.17 \times 10^{10} Nm^{-2}, \overline{c_{13}} = 2.17 \times 10^{10} Nm^{-2}, \overline{c_{12}} = 2.624 \times 10^{10} Kgm^{-1}s^{-2}, \overline{c_{44}} = 3.278 \times 10^{10} Nm^{-2}, \bar{\rho} = 1.74 \times 10^3 Kgm^{-3}, \overline{T_0} = 298°K$,
$\overline{C_E} = 1.04 \times 10^3 JKg^{-1}deg^{-1}, \overline{\beta_1} = 2.68 \times 10^6 Nm^{-2}deg^{-1}$,
$\overline{\beta_3} = 2.68 \times 10^6 Nm^{-2}deg^{-1}, \overline{k_1^*} = 0.02 \times 10^2 Nsec^{-2}deg^{-1}$,
$\overline{k_3^*} = 0.02 \times 10^2 Nsec^{-2}deg^{-1}, \overline{c_{11}} = \bar{\rho}\bar{c_1}^2, \overline{a_1} = 0.104, \overline{a_3} = 0.0204$, with non-dimensional parameter L=1

For particular case (ii): For the medium M_1, we take numerical data from Youssef (2006) and take Copper material with values of constants as

$\lambda = 7.76 \times 10^{10} Nm^{-2}$, $c_1^2 = \frac{\lambda+2\mu}{\rho}, a_1 = 0.03, a_3 = 0.06, \mu = 3.86 \times 10^{10} Nm^{-2}$, ${k_1}^* = {k_3}^* = k^* = 0.02 \times 10^2 Nsec^{-2}deg^{-1}$, $\beta_1 = \beta_3 = \beta = 5.518 \times 10^6 Nm^{-2}deg^{-1}, \rho = 8954\ Kgm^{-3}$, $T_0 = 293\text{K}$, $C_E = 383.1\ Jkg^{-1}K^{-1}, k_3^* = 0.04 \times 10^2 Nsec^{-2}deg^{-1}, \alpha_1 = \alpha_3 = \alpha = 1.78 \times 10^{-5} K^{-1}$, with non-dimensional parameter L=1.

For the medium M_2, we consider Magnesium material with numerical data from Dhaliwal and Singh (1980)

$\bar{\lambda} = 2.17 \times 10^{10} Nm^2, \bar{\mu} = 3.278 \times 10^{10} Nm^{-2}, \overline{k_1^*} = \overline{k_3^*} = \bar{k} = 0.04 \times 10^2 Nsec^{-2}deg^{-1}$, $\bar{\alpha}_1 = \bar{\alpha}_3 = \bar{\alpha} = .2488 \times 10^{-5} K^{-1}$, $\overline{\beta_1} = \overline{\beta_3} = \bar{\beta} = 2.68 \times 10^6\ Nm^{-2}deg^{-1} \bar{\rho} = 1.74 \times 10^3 Kgm^{-3}, \overline{T_0} =$ 298K, $\overline{C_E} = 1.04 \times 10^3 Jkg^{-1}deg^{-1}$, $c_1^2 = \frac{\lambda+2\mu}{\rho}$, $\overline{a_1} = 0.02, \overline{a_3} = 0.104, 0.02 \times 10^2 Nsec^{-2}deg^{-1}$,with non-dimensional parameter L=1.

Mathcad software has been used for numerical computation of the resulting quantities. The values of determinant of secular equations, Stoneley wave velocity and attenuation coefficients with respect to frequency ξ have been computed and are depicted graphically in Figures 1 - 9.

i) Figures 1- 3 correspond to the variations in determinant of secular equations with respect to the wave number ξ subject to two temperature, without two temperatures (i.e. particular case (i)), and isotropic material (particular case (ii)) respectively.

ii) Figures 4, 5 and 6 correspond to the variations in velocity of Stoneley wave with respect to the wave number ξ subject to two temperatures, without two temperatures (i.e. particular case (i)), and isotropic material (particular case (ii)), respectively.

iii) Figures 7, 8 and 9 correspond to the variations in Attenuation coefficient with respect to the wave number ξ subject to two temperatures, without two temperature (i.e. particular case(i)), and isotropic material (particular case (ii)), respectively.

Determinant of secular equations

Important phenomena are noticed in all the physical quantities. It is evident from Figure 1, that initially, variation in the determinant of secular equations is steady state with respect to wave number, but near ξ =0.7, it results in sudden variations similar to Dirac delta function and when it is away from ξ =0.8, again variations in $|\eta_{ij}|$ are negligible. Without two temperature, the resulting variations are very close to the two temperature case as can be examined from Figure 2. The difference noticed is of magnitude and of small variations for the range $0.7 \leq \xi \leq 0.8$ and $0.9 \leq \xi \leq 1$,as without two temperature, these variations are not present. In Figure 3, for the range $0 \leq \xi \leq .6$, variations are zero but away from this range, variations result in small jolt waves followed by large jolt waves and these variations are similar to seismic waves used for earthquake measures.

Stoneley wave velocity

Figure 4, exhibits variation of Stoneley wave velocity with respect to ξ when a_1=0.03,a_3=0.06. Here for the range $0 \leq \xi \leq 0.6$, small variations are noticed whereas away from this range variations increase and reach to maximum for the range $.7 \leq \xi \leq .8$. For $\xi \geq .7$, first we notice a small increase in Stoneley wave velocity above boundary surface; afterwards there is a decrease below boundary surface and then variations remain stationary in the small interval followed by a sharp decrease with high amplitude below boundary surface and are similar to negation of Heaviside function Figure 5 represents variations of Stoneley waves velocity with respect to wave number ξ when a_1=0,a_3=0 (particular case(i)). Here variations are similar to Figure 4 with change in magnitude of amplitudes. Figure 6 exhibits variation of Stoneley wave velocity with respect to ξ corresponding to particular case (ii). For the range $0 \leq \xi \leq .7$, small variations are observed but afterwards, variations occur with high frequency. Variations in Stoneley wave velocity for $\xi \geq .7$ are similar to seismic waves with amplitude increasing monotonically.

Attenuation coefficient

Figure 7 depicts the variations in Attenuation coefficient with respect to wave number ξ. It is noticed that values of Attenuation coefficient are zero for the range $0 \leq \xi \leq .5$. and for $.5 \leq \xi \leq .8$, values lie below boundary surface in the form of small jolt wave followed by high jolt wave in downwards direction. For the range $.8 \leq \xi \leq .95$, small variations are noticed whereas for $.95 \leq \xi \leq 1$, we notice high momentary increase. Figure 8 represents variations in Attenuation coefficient with respect to ξ corresponding to particular case (i). Here small variations above boundary surface are noticed for the range $0 \leq \xi \leq .8$ whereas variations lie below boundary surface in the rest. Figure 9 represents variations in Attenuation coefficient with respect to ξ corresponding to particular case (ii). We notice that values are zero for the range $0 \leq \xi \leq .5$ whereas instant increases above boundary surface are noticed in the rest except for the range $.9 \leq \xi \leq .95$ as here, the variations are similar to negation of Dirac delta function.

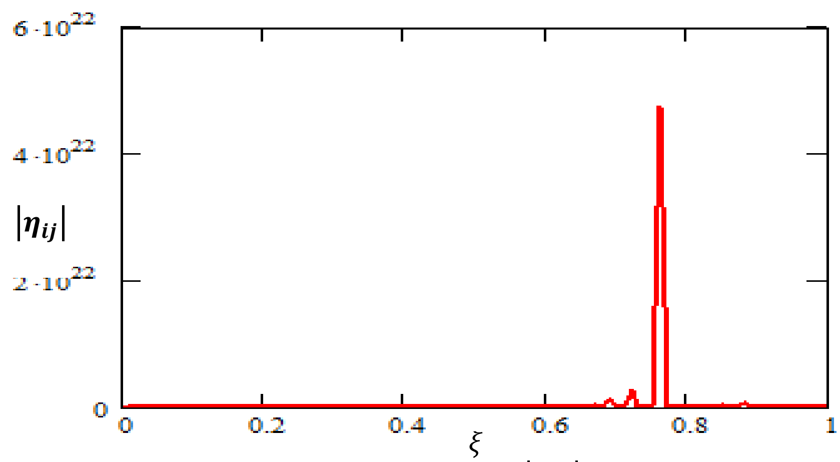

Figure 1. Variation of determinant of secular equation $|\eta_{ij}|$ respect to ξ when a_1=0.03,a_3=0.06

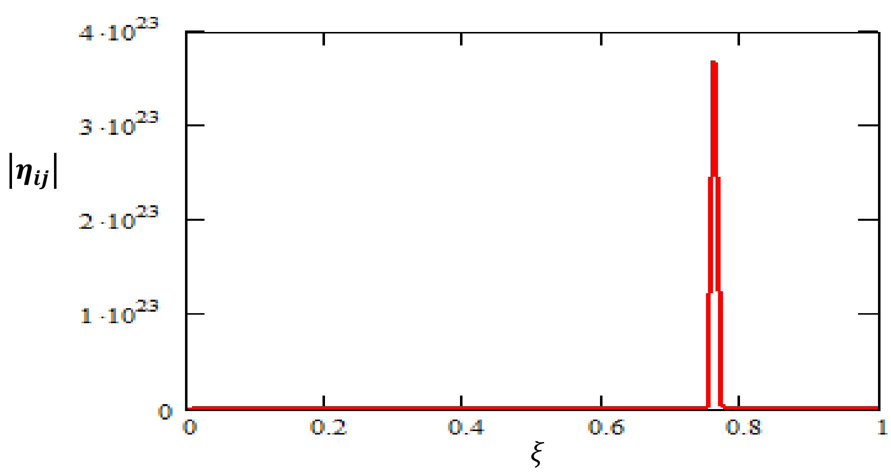

Figure 2. Variation of determinant of secular equation $|\eta_{ij}|$ with respect to ξ for particular case (i)

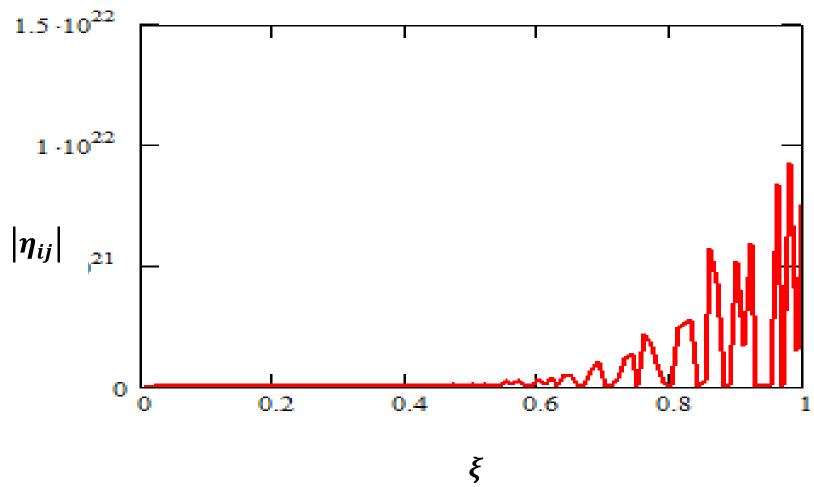

Figure 3. Variation of determinant of secular equation $|\eta_{ij}|$ with respect to ξ for isotropic, particular case (ii)

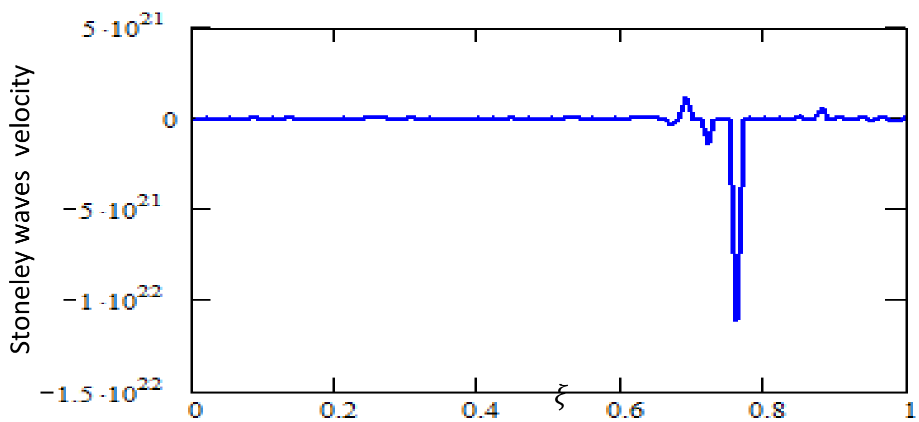

Figure 4 .Variation of stoneley waves velocity with respect to ξ when a_1=0.03,a_3=0.06

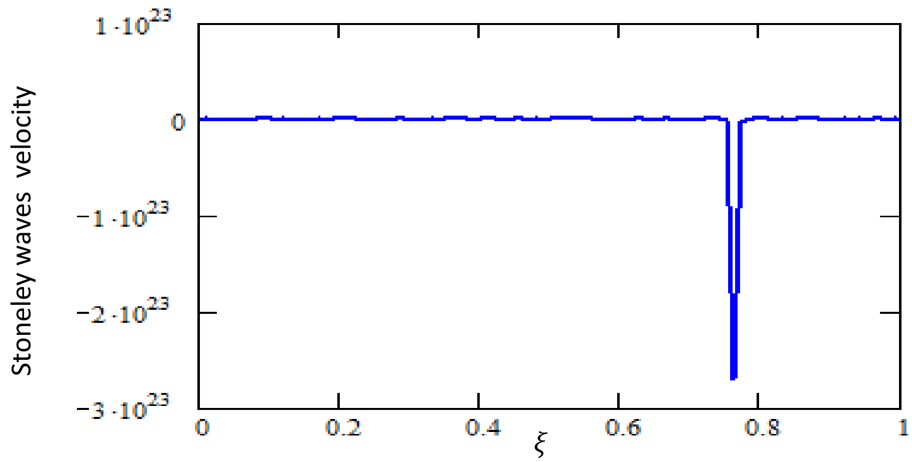

Figure 5. Variation of stoneley waves velocity with respect to ξ when $a_1=0=a_3$.

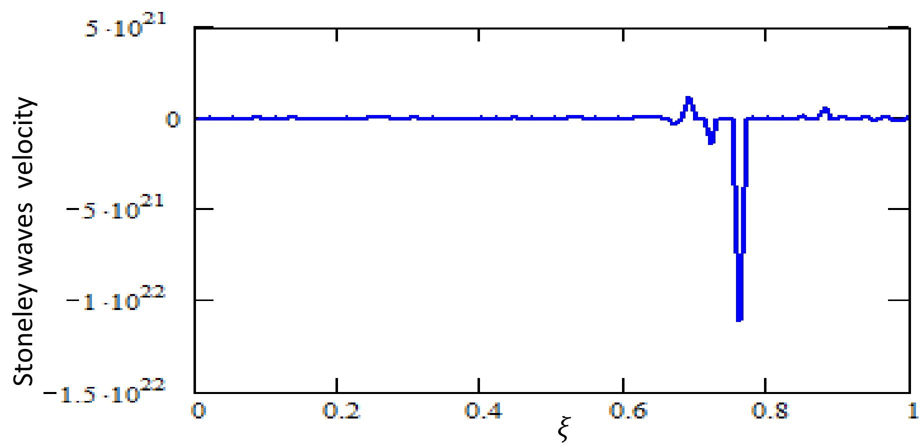

Figure 4 .Variation of Stoneley waves velocity with respect to ξ when a_1=0.03,a_3=0.06.

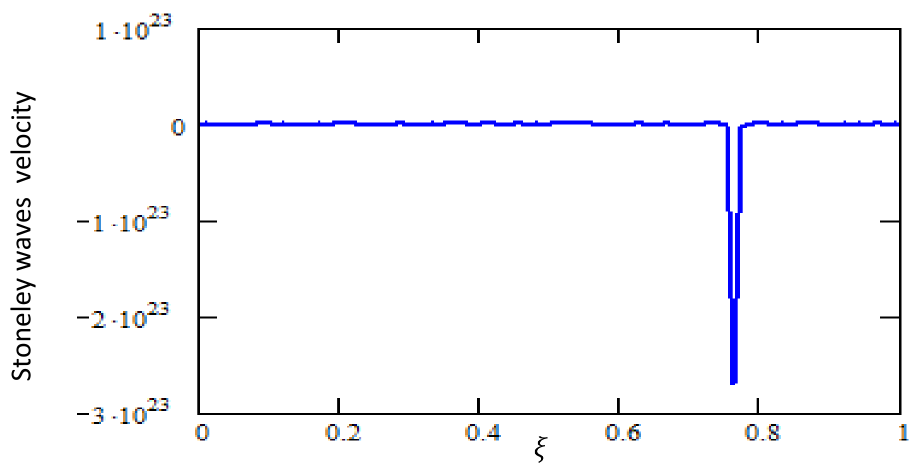

Figure 5. Variation of Stoneley waves velocity with respect to ξ when $a_1=0=a_3$

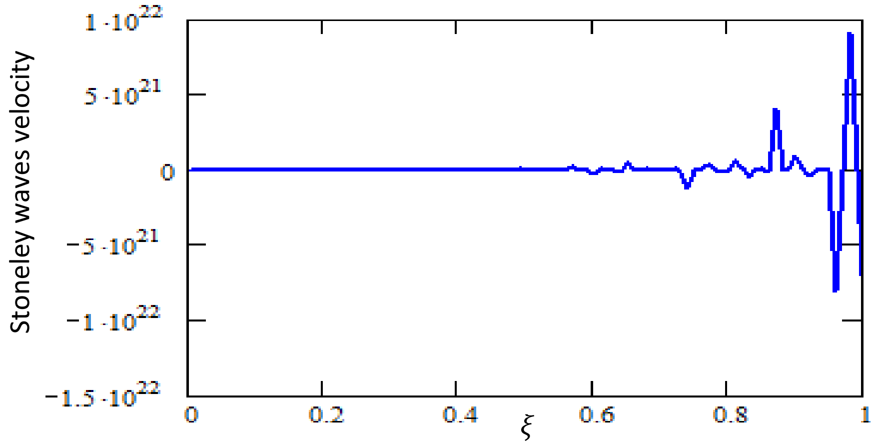

Figure 6. Variation of Stoneley waves velocity with respect to ξ for isotropic, particular case (ii)

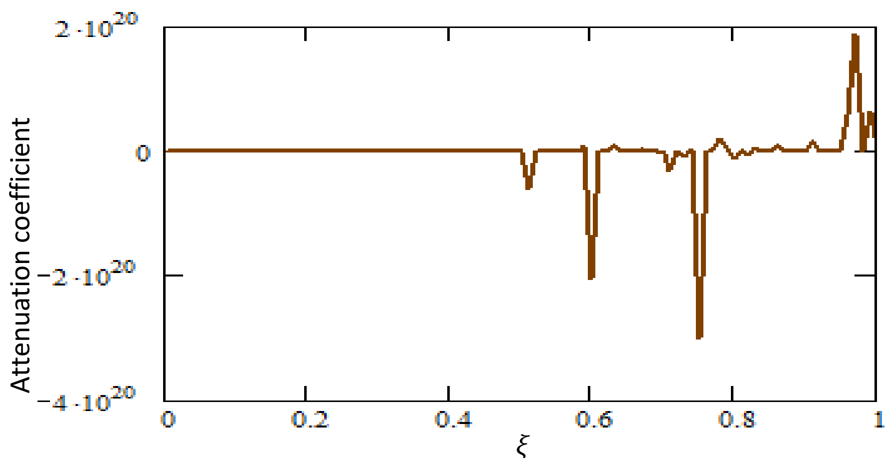

Figure 7. Variation of attenuation coefficient of Stoneley waves with respect to ξ when a_1 = 0.03, a_3 = 0.06.

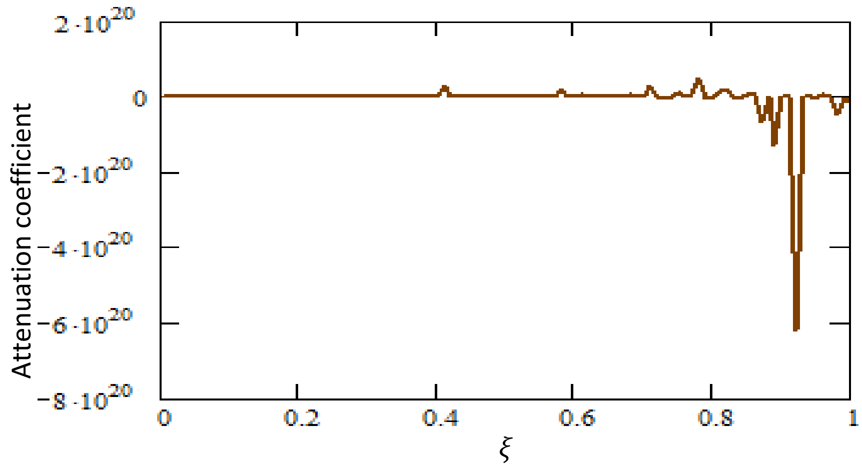

Figure 8. Variation of attenuation coefficient of Stoneley waves with respect to ξ when $a_1=0=a_3$

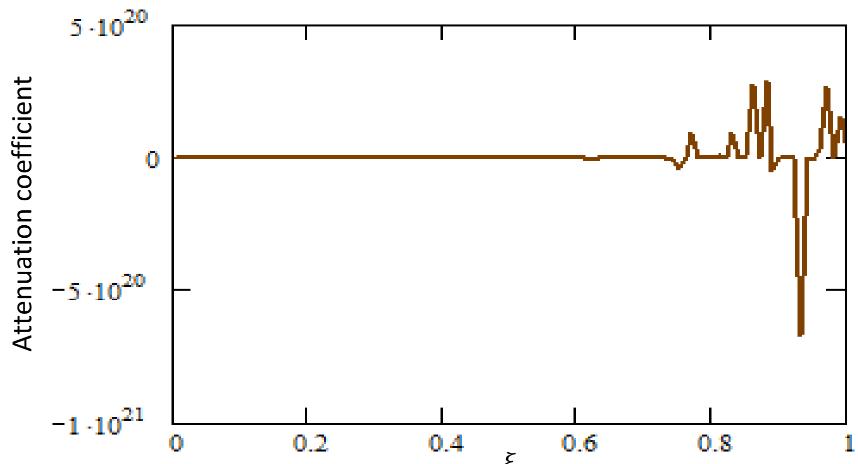

Figure 9 .Variation of attenuation coefficient of Stoneley waves with respect to ξ for isotropic, particular case (ii)

9. Conclusion

In this work, the influence of two temperature and anisotropy on determinant of secular equations of Stoneley waves, Stoneley wave velocity, attenuation coefficient is studied. We observed the following significant facts which reflect the influence of these parameters on the physical quantities.

1. Anisotropy has a great effect on the various quantities. As in isotropic case variations are moving just similar to seismic waves.

2. It is found that, the two temperature theory of thermoelasticity and theory of thermoelasticity produce close results with difference in magnitudes of variations.
3. Variations are more pronounced for higher values of wave number initially for small values of wave number; variations are very small for all the quantities.
4. The present theoretical results may provide interesting information for experimental scientists/ researchers/seismologist working on this subject.

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