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4-Prime cordiality of some cycle related graphs

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Abstract

Recently three prime cordial labeling behavior of path, cycle, complete graph, wheel, comb, subdivison of a star, bistar, double comb, corona of tree with a vertex, crown, olive tree and other standard graphs were studied. Also four prime cordial labeling behavior of complete graph, book, flower were studied. In this paper, we investigate the four prime cordial labeling behavior of corona of wheel, gear, double cone, helm, closed helm, butterfly graph, and friendship graph.

Keywords: Wheel; Gear; Double Cone; Helm; Butterfly Graph

MSC 2010 No.: Primary 05C78, Secondary 05C38

1. Introduction

Throughout this paper we have considered only simple and undirected graph. Terms and definitions not defined here are used in the sense of Harary (2001) and Gallian (2015).

Let $G = (V,E)$ be a (p,q) graph. The cardinality of V is called the order of G and the cardinality of E is called the size of G. Rosa (1967) introduced graceful labelling of graphs which was the foundation of the graph labelling. Consequently Graham (1980) introduced harmonious labelling. Cahit (1987) initiated the concept of cordial labelling of graphs. Ebrahim Salehi (2010) defined the notion of product cordial set. Salehi and Mukhin (2012) said a graph G of size q is fully product-cordial if its product cordial set is $\{q-2k : 0 \leq k \leq \frac{q}{2}\}$. Because the product-cordial set is the multiplicative version of the friendly index set, Kwong, Lee, and Ng (2010) called it the product-cordial index set of G and determined the exact values of the product-cordial index set of Cm and $C_m \times P_n$. Kwong et al. (2012), determined the friendly index sets and product-cordial index sets of 2-regular graphs and the graphs obtained by identifying the centers of any number of wheels. Shiu (2013) determined the product-cordial index sets of grids $C_m \times P_n$. Sundaram et al. (2005) introduced the concept of prime cordial labelling of graphs. Seoud and Salim (2010) gave an upper bound for the number of edges of a graph with a prime cordial labeling as a function of the number of vertices. For bipartite graphs they gave a stronger bound. On analogous of this, the notion of k-prime cordial labelling has been introduced by Ponraj et al. (2016). Ponraj et al. (2016) studied the k-prime cordial labelling behaviour of paths, cycles, bistars of even order. Also they studied about the 3-prime cordiality of paths, cycles, corona of tree with a vertex, comb, crown, olive tree and some more graphs. In this paper, we investigate the 4-prime cordial labelling behaviour of wheel, gear, double cone, helm, butterfly graph and closed helm.

2. Preliminaries

Let $f : V(G) \to \{1,2,\ldots,k\}$ be a function. For each edge uv, assign the label $gcd(f(u), f(v))$. f is called k-prime cordial labelling of G if $\left|v_f(i) - v_f(j)\right| \leq 1$, $i, j \in \{1,2,\ldots,k\}$ and $\left|e_f(0) - e_f(1)\right| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively the number of edges labeled with 1 and not labelled with 1. A graph with a k-prime cordial labelling is called a k-prime cordial graph.

Definition.

Let G_1 and G_2 be two graphs with vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then the join $G_1 + G_2$ is the graph whose vertex set is $V_1 \cup V_2$ and edge set is given by $E_1 \cup E_2 \cup \{uv : u \in V_1 \text{ and } v \in V_2\}$.

Definition.

The graph $C_n + K_1$ is called the wheel. In a wheel, the vertex of degree n is called the central vertex and the vertices on the cycle C_n are called rim vertices.

Definition.

The graph $C_n + \overline{K_2}$ is called a double cone.

Definition.

A gear graph is obtained from the wheel W_n by adding a vertex between every pair of adjacent vertices of the n-cycle.

Definition.

The helm H_n is the graph obtained from a wheel by attaching a pendent edge at each vertex of the n-cycle. A closed helm CH_n as the graph obtained from a helm by joining each pendent vertex to form a cycle.

Definition.

Two even cycles of the same order, say C_n, sharing a common vertex with m pendent edges attached at the common vertex is called a butterfly graph $By_{m,n}$.

Definition.

The one-point union of t copies of the cycle C_3 is called a friendship graph $C_3^{(t)}$.

Remark.

A 2-prime cordial labeling is a product cordial labeling. [Sundaram et al. (2004)]

In the next section we investigate the 4-prime cordiality of some graphs.

3. Main results

First we investigate the 4-prime cordiality of the wheel W_n and a double cone $C_n + \overline{K_2}$.

Theorem 1.

The wheel W_n is a 4-prime cordial if and only if $n \neq 3,4,7$.

Proof:

Let $C_n : u_1u_2 \dots u_nu_1$ be the cycle and let $V(K_1) = \{u\}$. Obviously W_n has $n+1$ vertices and $2n$ edges. First we consider the case when $n = 3$. Clearly it is isomorphic to K_4. Since there are four vertices, each should be labelled with different labels from the set $\{1,2,3,4\}$. If it is so, then $e_f(0) = 1$ and $e_f(1) = 5$. This gives $e_f(1) - e_f(0) > 1$, so there does not exists a 4-prime cordial labelling f of W_3. For W_4 let g be a 4-prime cordial labelling. Then $v_g(1) = 2$, $v_g(2) = v_g(3) = v_g(4) = 1$ (or) $v_g(2) = 2$, $v_g(1) = v_g(3) = v_g(4) = 1$ (or) $v_g(3) = 2$, $v_g(1) = v_g(2) = v_g(4) = 1$ (or) $v_g(4) = 2$, $v_g(1) = v_g(2) = v_g(3) = 1$. Clearly it is not too hard to show that $\left|e_g(1) - e_g(0)\right| \leq 3$. So W_4 does not admit any 4-prime cordial labelling. For completing the proof of nonexistence part, it remains to show that W_7 does not permit any 4-prime cordial labeling. Here each label must be used exactly twice. Then the number of edges not labelled 1 is at most 6. Thus there does not exist a 4-prime cordial labelling for W_7. This completes the nonexistence part.

Now we consider the existence part. We divide the proof into four cases.

Case 1. $n \equiv 0 \pmod 4$ and $n \neq 4$.

In this case we can take the value of n be of the form $4t$, $t > 1$. We construct a vertex labelling

h for this case as follows: Assign the label 2 to the vertices u_1, u_2, $\ldots$, u_t and u. The vertices u_{t+1}, u_{t+2}, $\ldots$, u_{2t} should be labelled with 4. Then the next two consecutive vertices u_{2t+1}, u_{2t+2} are labelled with 3. Use the label 1 to the vertices u_{2t+3} and u_{2t+4}. Now, from u_{2t+5} to u_{4t}, we put the labels 3, 1 alternately as $h(u_{2t+5}) = 3$, $h(u_{2t+6}) = 1$, and so on. It is easy to check that $h(u_{4t}) = 1$. From the above labelling h, we have the vertex and edge conditions respectively are $v_h(2) = t+1$, $v_h(1) = v_h(3) = v_h(4) = t$ and $e_h(0) = e_h(1) = 4t$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4t+1$. Assign the labels to the vertices of u_1, $u_2, \ldots, u_{2t}$ and u as in case 1. Put the label 4 to the vertex u_{2t+1}. Then use the labels 3 and 1 in this order alternately to label the vertices u_{2t+2}, $u_{2t+3}, \ldots$, u_{4t+1}. The vertex and edge conditions of the above labelling μ are given by $v_\mu(1) = v_\mu(3) = t$, $v_\mu(2) = v_\mu(4) = t+1$ and $e_\mu(0) = e_\mu(1) = 4t+1$ respectively.

Case 3. $n \equiv 2 \pmod 4$.

Take $n = 4t+2$. A 4-prime cordial ϕ for this case is given below. Assign the labels to the vertices u_1, $u_2, \ldots$, u_{2t+1} and u as in case 2. Then put the label 3 to the vertices u_{2t+2} and u_{2t+3}. Then the remaining rim vertices u_{2t+4}, $u_{2t+5}, \ldots, u_{4t+2}$ are labelled with 1 and 3 in this order alternately. Note that the vertex u_{4t+2} received the label 1. For this labelling, we have the following vertex and edge conditions: $v_\phi(1) = t$, $v_\phi(2) = v_\phi(3) = v_\phi(4) = t+1$ and $e_\phi(0) = e_\phi(1) = 4t+2$.

Case 4. $n \equiv 3 \pmod 4$ and $n \neq 3, 7$.

Let $n = 4t+3$. We describe a 4-prime cordial labelling ψ as follows: as in case 3, we assign the labels to the vertices u_1, $u_2, \ldots$, u_{2t+3} and u. Then the vertices u_{2t+4}, $u_{2t+5}, \ldots$, u_{4t} are labelled with 3 and 1 in this order alternately. Clearly the label of u_{4t} is 3. Finally the last three vertices, namely, u_{4t+1}, u_{4t+2} and u_{4t+3} are labelled by 1. With reference to the vertex condition $v_\psi(1) = v_\psi(2) = v_\psi(3) = v_\psi(4) = t+1$ and the edge condition $e_\psi(0) = e_\psi(1) = 4t+3$, ψ is a 4-prime cordial labelling for this case. From the above four cases, we can conclude that W_n is 4-prime cordial for $n \neq 3, 4, 7$ and this completes the existence part. □

Example 1.

A 4-prime cordial labelling of W_{11} is given in Figure 1.

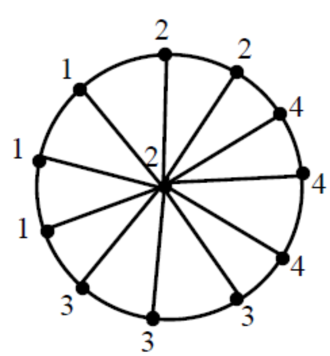

Figure 1: A 4-prime cordial labelling of W_{11}

Corollary 1.

If $n \equiv 0,\ 3 \pmod 4$, $n \neq 3,\ 4,\ 7$ then the double cone $C_n + \overline{K_2}$ is 4-prime cordial.

Proof:

Take the vertex and edge sets as in Theorem 1Let v be the new vertex joined to the rim vertices of W_n. Here $p = n+2$ and $q = 3n$. We divide the proof into two cases.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4t$, $t > 1$. Assign the label 4 to the vertex v. Then assign the remaining vertices as in Theorem 1.If f denote this labelling then $v_f(1) = v_f(3) = t$, $v_f(2) = v_f(4) = t+1$ and $e_f(0) = e_f(1) = 6t$.

Case 2. $n \equiv 3 \pmod 4$.

Let $n = 4t+3$, $t > 1$. Assign the label 2 to the vertex v and then assign the remaining vertices as in Theorem 1. If g denote this labelling then $v_g(2) = t+2$, $v_g(1) = v_g(3) = v_g(4) = t+1$ and $e_g(0) = 6t+4$, $e_g(1) = 6t+5$. Thus for $n \neq 3,\ 4,\ 7$ and $n \equiv 0,3 \pmod 4$, $C_n + \overline{K_2}$ is 4-prime cordial. □

The next investigation is about the 4-prime cordiality of the gear graph G_n.

Theorem 2.

The gear graph G_n is 4-prime cordial for all n.

Proof:

Consider a wheel W_n with vertex and edge sets are as in Theorem 1. Now subdivide each edge u_iu_{i+1}, u_nu_1 with the vertices v_i where i runs from 1 to $n-1$ and v_n respectively. We observe that the order and size of G_n are $2n+1$ and $3n$ respectively. We divide the proof into four cases.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4t$. We label the central vertex u by 2. Now consider the rim vertices u_i where $1 \leq i \leq n$. Assign the label 2 to the vertices u_1, $u_2, \ldots$, u_{2t}. Then put the labels 3 and 1 respectively to the vertices u_{2t+1} and u_{2t+2}. Now put the label 3 to all the remaining vertices u_{2t+3}, $u_{2t+4}, \ldots$, u_{4t}. Next we consider the vertices v_i where $1 \leq i \leq n$. The first $2t$ vertices v_1, $v_2, \ldots$, v_{2t} are labelled with 4 then for v_{2t+1} and v_{2t+2}, we use the labels 3 and 1 respectively. The remaining vertices from the set $\{v_{2t+3}, \ldots, v_{4t}\}$ are labelled by 1. If f denotes the above mentioned labeling then we have the following vertex and edge conditions: $v_f(1) = v_f(3) = v_f(4) = 2t$, $v_f(2) = 2t+1$ and $e_f(0) = e_f(1) = 6t$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4t+1$. Here, we construct a 4-prime cordial labelling g as follows: assign the labels to the vertices u, u_1, $u_2, \ldots$, u_{2t} and v_1, $v_2, \ldots$, v_{2t} as in case 1. Then put the label 4 to u_{2t+1}. Now consider the remaining rim vertices u_{2t+2}, $u_{2t+3}, \ldots$, u_{4t+1}. All these vertices are labelled with 3. Similarly, we assign the label 1 to all the vertices from the set $\{v_{2t+1}, \ldots, v_{4t+1}\}$. Note that,

$v_g(1) = 2t$, $v_g(2) = v_g(3) = v_g(4) = 2t+1$ and $e_g(0) = 6t+1$, $e_g(1) = 6t+2$.

Case 3. $n \equiv 2 \pmod 4$.

Take $n = 4t+2$. Assign the labels 2 to the vertices u, u_1, $u_2, \ldots, u_{2t}$ and the label 4 to the vertices v_1, $v_2, \ldots, v_{2t}$ as in case 1. For the vertices u_{2t+1} and v_{2t+1}, we assign the labels 2, 4 respectively. The next two rim vertices, namely, u_{2t+2}, u_{2t+3} are labelled with 3, 1 respectively. Then the remaining rim vertices u_{2t+4}, $u_{2t+5}, \ldots, u_{4t+2}$ are labelled by 3. The vertices v_{2t+2}, v_{2t+3} are assigned with 3, 1 respectively. Finally, the remaining unlabeled vertices $v_{2t+4}, v_{2t+5}, \ldots, v_{4t+2}$ are labelled by 1. If the above mentioned labelling is taken as ϕ then $v_\phi(1) = v_\phi(3) = v_\phi(4) = 2t+1$, $v_\phi(2) = 2t+2$ and $e_\phi(0) = e_\phi(1) = 6t+3$.

Case 4. $n \equiv 3 \pmod 4$.

Put $n = 4t+3$. We describe a 4-prime cordial labelling ψ as follows: as in case 1, assign the labels to the vertices u, u_1, $u_2, \ldots, u_{2t}$ and v_1, $v_2, \ldots, v_{2t}$. Then we consider the remaining rim vertices u_{2t+1}, $u_{2t+2}, \ldots, u_{4t+3}$. First we put the labels 2, 4 to the vertices u_{2t+1}, u_{2t+2} respectively. The remaining rim vertices are labelled by 1. For the vertices v_{2t+1}, $v_{2t+2}, \ldots, v_{4t+3}$, we use the label 4 to v_{2t+1} and then the rest vertices v_{2t+2}, $v_{2t+3}, \ldots, v_{4t+3}$ are labelled with 3. It is easy to verify the vertex and edge conditions of the above labelling ψ are $v_\psi(1) = 2t+1$, $v_\psi(2) = v_\psi(3) = v_\psi(4) = 2t+2$ and $e_\psi(0) = 6t+4$, $e_\psi(1) = 6t+5$.

Hence, all gears are 4-prime cordial. □

Next we prove that, for all values of n, the helm H_n is 4-prime cordial. Also we discuss the 4-prime cordiality of closed helm CH_n.

Theorem 3.

The helm H_n is 4-prime cordial for all n.

Proof:

Consider the wheel W_n with vertex and edge sets as in Theorem 1. Let v_1, v_2, $\ldots$, v_n be the pendent vertices adjacent to u_1, u_2, $\ldots$, u_n respectively. Clearly $p = 2n+1$ and $q = 3n$. We construct four different types of labelling so that H_n is 4-prime cordial. For this, we split the proof into four cases.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4t$. We present a 4-prime cordial labelling f as follows: first we consider the rim vertices u_i where $1 \le i \le n$. The vertices u_1, $u_2, \ldots, u_{2t}$ are labelled with the integer 2. The remaining rim vertices u_{2t+1}, u_{2t+2}, $\ldots$, u_{4t} are labelled alternately with 3 and 1 in this order so that u_{4t} received the label 1. For the central vertex, we use the label 2. Now, we consider the pendent vertices v_i where $1 \le i \le n$. Assign the label 4 to the pendent vertex v_i, $1 \le i \le 2t$ in which v_i is a neighbour of the vertex u_i such that the label of u_i is 2. Assign the labels 3, 1 to the vertices v_{2t+1}, v_{2t+2} respectively. The remaining pendent vertices v_i from v_{2t+3} to v_{4t} are labelled with 1 and 3 according as i is odd or even. For this vertex labelling, we have the vertex condition as $v_f(1) = v_f(3) = v_f(4) = 2t$, $v_f(2) = 2t+1$ and $e_f(0) = e_f(1) = 6t$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4t+1$. In this case, we fix the labels to the vertices u, u_i, $1 \le i \le 2t$ and v_i, $1 \le i \le 2t$ as in case 1. Consider the rim vertices u_i, $2t+1 \le i \le 4t+1$. Put the number 4 to u_{2t+1}. The remaining vertices are labelled with 3 or 1 according as i is even or odd. Now, the only unlabeled vertices are v_i, $2t+1 \le i \le 4t+1$. Assign the labels 3, 3, 1 to the vertices v_{2t+1}, v_{2t+2}, v_{2t+3} respectively. Finally, the vertices v_i, $2t+4 \le i \le 4t+1$ are labeled with 1 or 3 according as its neighbour u_i, $2t+4 \le i \le 4t+1$ received the label 3 or 1. Let g denote the above mentioned labelling. It is easy to check that $v_g(1) = 2t$, $v_g(2) = v_g(3) = v_g(4) = 2t+1$ and $e_g(0) = 6t+2$, $e_g(1) = 6t+1$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4t+2$. Here, we assign the labels to the vertices u, u_i, v_i, $1 \le i \le 2t$ and u_i, v_i, $2t+4 \le i \le 4t+1$ as in case 2. Next we assign the labels 3, 1 to the vertices u_{4t+2}, v_{4t+2} respectively. Then assign the labels to the vertices u_{2t+1}, u_{2t+2}, u_{2t+3} respectively with 2, 3, 1 and v_{2t+1}, v_{2t+2}, v_{2t+3} with 4, 3, 1 respectively. The above labelling ϕ satisfies the vertex condition $v_\phi(1) = v_\phi(3) = v_\phi(4) = 2t+1$, $v_\phi(2) = 2t+2$ and the edge condition $e_\phi(0) = e_\phi(1) = 6t+3$.

Case 4. $n \equiv 3 \pmod 4$.

Put $n = 4t+3$. In this case, we assign the labels to the vertices u, u_i, v_i, $1 \le i \le 2t+1$ are as in case 3. Now consider the remaining rim vertices. Assign the labels 3, 1, 3 to the vertices u_{2t+2}, u_{2t+3}, u_{2t+4} respectively. Then the vertices u_i, $2t+5 \le i \le 4t+3$ are labeled with 3 or 1 according as i is odd or even. For the pendent vertices $v_{2t+2}, \dots, v_{4t+3}$, we first allocate the labels 3, 3, 1 to the vertices v_{2t+2}, v_{2t+3}, v_{2t+4} respectively. The rest of the pendent vertices are labeled with 1 or 3 according as i is odd or even. Let ψ be the above said labelling then it is easy to verify that $v_\psi(1) = 2t+1$, $v_\psi(2) = v_\psi(3) = v_\psi(4) = 2t+2$ and $e_\psi(0) = 6t+5$, $e_\psi(1) = 6t+4$.

From Figure 2, it is easy to see that H_3 is 4-prime cordial.

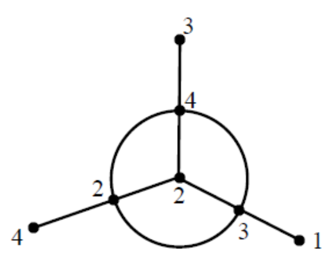

Figure 2: A 4-prime cordial labeling of H_3

Hence, the helm H_n is 4-prime cordial for all values of n. □

Corollary 2.

For every values of n, the closed helm CH_n is 4-prime cordial.

Proof:

Clearly, the order and size of CH_n are $2n+1$, $4n$ respectively. We divide the proof into four cases.

Case 1. $n \equiv 0 \pmod 4$.

For $n = 4$, the Figure 3 shows that CH_4 is 4-prime cordial.

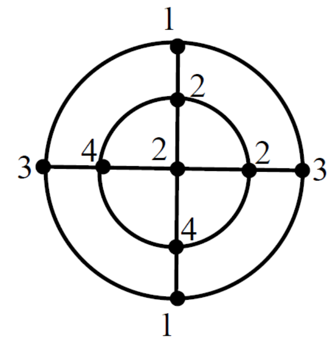

Figure 3: A picture of a gull.

When $n > 4$, let $n = 4t$ and we assign the labels to the vertices as in case 1 of Theorem 3. Then relabel the vertices u_{4t-1}, v_{4t-1} with 1, 3 respectively.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4t + 1$. Assign the labels to the vertices as in case 2 of Theorem 3.

Case 3. $n \equiv 2 \pmod 4$.

For $n = 6$, the Figure 4 shows that CH_6 is 4-prime cordial.

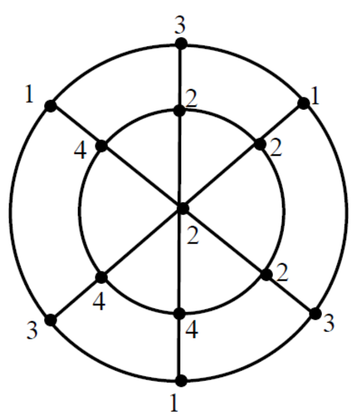

Figure 4: A 4-prime cordial labeling of CH_6

When $n > 6$, let $n = 4t + 2$, $t > 1$ and assign the labels to the vertices as in case 3 of Theorem 3. Then relabel the vertices u_{4t+2}, v_{4t+2} with 1, 3 respectively.

Case 4. $n \equiv 3 \pmod 4$.

A 4-prime cordial labelling of CH_3 is given in Figure 5.

For $n > 3$, let $n = 4t + 3$, $t > 0$ and assign the labels to the vertices as in case 4 of Theorem 3.

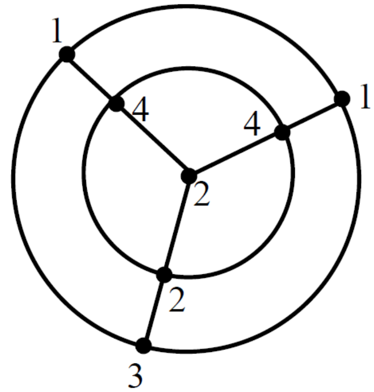

Figure 5: A 4-prime cordial labeling of CH_3

Note that in each case, the number of edges labelled with 1 and not labelled with 1 are $2n$, $2n$ respectively.

Hence, the closed helm CH_n is 4-prime cordial. □

Next we examine the butterfly graph.

Theorem 4.

The butterfly graph $By_{m,n}$ is 4-prime cordial.

Proof:

Let u_1, u_2, …, u_n be the vertices of the first copy of the cycle C_n and v_1, v_2, …, v_n be that of the second copy. By symmetry, without loss of generality, we can unify u_1 and v_1. Let w_1, w_2, …, w_m be the pendent vertices of $By_{m,n}$. Now, we depict a 4-prime cordial μ as follows: assign the first copy of the vertices of the cycle with the labels 2, 4 such that each label assigned to exactly $\frac{n}{2}$ vertices in any order. For the second copy, since $v_1 = u_1$ is already labelled by 2, we can focus the remaining vertices v_2, …, v_n. These vertices are labelled by 3 and 1 alternately in this order. If we do like this, one can easily check that the label of v_n is 3. Also, at this phase, each labels are used exactly $\frac{n}{2}$ times except the label 1 which is used one less that of the others in number. Now we shift into the pendant vertices. Here, we have four cases.

Case 1. $m \equiv 0 \pmod 4$.

Let $m = 4t$, $t \geq 1$. Here, we use each labels exactly t times to label the vertices w_1, w_2, …, w_m in any order.

Case 2. $m \equiv 1 \pmod 4$.

Let $m = 4t+1$, $t \geq 0$. In this case, assign the labels to the vertices w_1, w_2, …, w_{m-1} as in case 1. Then put the integer 1 to w_m.

Case 3. $m \equiv 2 \pmod 4$.

Let $m = 4t+2$, $t \geq 0$. In this case, assign the labels to the vertices w_1, w_2, …, w_{m-1} as in case

2. Then put the integer 2 to w_m.

Case 4. $m \equiv 3 \pmod 4$.

Let $m = 4t+3$, $t \geq 0$. Here, assign the labels to the vertices w_1, w_2, $\ldots$, w_{m-1} as in case 2. Then put the integer 1 to w_m.

We can refer the Table 1 for the existence of a 4-prime cordial labelling μ of $By_{m,n}$.

Table 1. Vertex and edge conditions of 4-prime cordial labelling μ of $By_{m,n}$

Values of n	$v_\mu(1)$	$v_\mu(2)$	$v_\mu(3)$	$v_\mu(4)$	$e_\mu(0)$	$e_\mu(1)$
$m \equiv 0 \pmod 4$	$\frac{n}{2}-1+t$	$\frac{n}{2}+t$	$\frac{n}{2}+t$	$\frac{n}{2}+t$	$n+2t$	$n+2t$
$m \equiv 1 \pmod 4$	$\frac{n}{2}+t$	$\frac{n}{2}+t$	$\frac{n}{2}+t$	$\frac{n}{2}+t$	$n+2t$	$n+2t+1$
$m \equiv 2 \pmod 4$	$\frac{n}{2}+t$	$\frac{n}{2}+t+1$	$\frac{n}{2}+t$	$\frac{n}{2}+t$	$n+2t+1$	$n+2t+1$
$m \equiv 3 \pmod 4$	$\frac{n}{2}+t+1$	$\frac{n}{2}+t+1$	$\frac{n}{2}+t$	$\frac{n}{2}+t$	$n+2t+1$	$n+2t+2$

Hence, $By_{m,n}$ is a prime cordial graph. □

Finally we look into the friendship graph.

Theorem 5.

The friendship graph $C_3^{(t)}$ is 4-prime cordial if and only if $t \neq 1$.

Proof:

Let $V(C_3^{(t)}) = \{u, u_i, v_i : 1 \leq i \leq t\}$ and $E(C_3^{(t)}) = \{uu_i, u_iv_i, v_iu : 1 \leq i \leq t\}$. Obviously the order and size of this graph are respectively $2t+1$ and $3t$. Assign the label 2 to the vertex u. Then we consider the following cases to label the vertices u_i, v_i.

Case 1. $t \equiv 0 \pmod 2$.

Let $t = 2r$, $r > 0$. Assign the label 2 to the vertices u_1, u_2, $\ldots$, u_r. Then put the label 3 to the vertices u_{r+1}, u_{r+2}, $\ldots$, u_{2r}. Similarly, we allocate the integer 4 to the vertices v_1, v_2, $\ldots$, v_r. The remaining unlabeled vertices are labelled by 1. If ϕ denotes the above mentioned labelling then we have $v_\phi(1) = v_\phi(3) = v_\phi(4) = r$, $v_\phi(2) = r+1$ and $e_\phi(0) = e_\phi(1) = 3r$.

Case 2. $t \equiv 1 \pmod 2$.

Let $t = 2r+1$, $r > 0$. Assign the labels to the vertices u_i, v_i : $1 \leq i \leq 2r$, as in case 1. Then put the labels 3, 4 to the vertices u_{2r+1}, v_{2r+1}. If f denotes the above mentioned labelling then we have $v_f(1) = v_f(4) = r$, $v_f(2) = v_f(3) = r+1$ and $e_f(0) = 3r+1$, $e_f(1) = 3r+2$.

Thus the friendship graph $C_3^{(t)}$ is 4-prime cordial if and only if $t \neq 1$. □

4. Conclusion

Graph labeling plays an important role of various fields of science and few of them are astronomy, coding theory, x-ray crystallography, radar, circuit design, communication network addressing, database management, secret sharing schemes, and models for constraint programming over finite domains. So study of graph labeling is an essential one. In this paper, we have studied the 4-prime cordiality of wheel, gear, double cone, helm, butterfly graph and closed helm. Investigation of 4-prime cordiality of grid, caterpillars, dragon, prism are open problems for future researchers.

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