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Study on the Q-Conjugacy Relations for the Janko Groups

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ABSTRACT

In this paper, we consider all the Janko sporadic groups J_1, J_2, J_3 and J_4 (with orders 175560, 604800, 50232960 and 86775571046077562880, respectively) with a new concept called the markaracter- and Q-conjugacy character tables, which enables us to discuss marks and characters for a finite group on a common basis of Q-conjugacy relationships between their cyclic subgroups. Then by using GAP (Groups, Algorithms and Programming) package we calculate all their dominant classes enabling us to find all possible Q-conjugacy characters for these sporadic groups. Finally, we prove in a main theorem that all twenty six simple sporadic groups are unmatured.

Keywords: Finite group; Sporadic, Janko; Conjugacy class; Character, Q-conjugacy; matured

MSC 2010: 20D99, 20C15

1. Introduction

In recent years, group theory has drawn wide attention of researchers in mathematics, physics and chemistry, see Fujita (1998). Many problems of the computational group theory have been researched, such as the classification, the symmetry, the topological cycle index, etc. They include not only the diverse properties of finite groups, but also their wide-ranging connections with many applied sciences, such as Nanoscience, Chemical Physics and Quantum Chemistry. For instance, see Darafsheh et al. (2008), Moghani and Najarian (2016) for more details.

Shinsaku Fujita suggested a new concept called the markaracter table, which enables us to discuss marks and characters for a finite group on a common basis. He has introduced tables of integer-valued characters and dominant classes which are acquired for such groups, Fujita

(1998). A dominant class is defined as a disjoint union of conjugacy classes corresponding to the same cyclic subgroups, which is selected as a representative of conjugate cyclic subgroups. Moreover, the cyclic (dominant) subgroup selected from a non-redundant set of cyclic subgroups of G is used to compute the Q-conjugacy characters of G as demonstrated by Fujita (2007).

The Janko groups J_1, J_2, J_3 and J_4 of orders 175560, 604800, 50232960 and 86775571046077562880, respectively are unmatured groups. The motivation for this study is outlined in Moghani (2010), Moghani et al. (2010) and Moghani (2016). The reader is encouraged to consult the papers by Aschbacher (1997) and Kerber (1999) for background material as well as basic computational techniques.

This paper is organized as follows: In Section 2, we introduce some necessary concepts, such as the maturity and Q-conjugacy character of a finite group. In Section 3, we provide all the dominant classes and Q- conjugacy characters for the Janko groups J_1, J_2, J_3 and J_4. Finally, we prove all 26 simple sporadic groups are unmatured.

2. Q-Conjugacy Relation

Throughout this paper we adopt the same notations as in ATLAS of finite groups. For instance, we will use for an arbitrary conjugacy class G of elements of order *n*, the notation *nX*, where $X = a, b, c\ldots$, see Conway et al (1985).

2.1. Dominant Class

Let G be an arbitrary finite group and $h_1, h_2 \in G$. We say h_1 and h_2 are Q-conjugate if there exists $t \in G$ such that $t^{-1} < h_1 > t = < h_2 >$. This is an equivalence relation on group G and generates equivalence classes that are called dominant classes. Therefore, G is partitioned into dominant classes, see Fujita (2007).

2.2. Maturity Property

Suppose H is a cyclic subgroup of order n of a finite group G. Then, the maturity discriminant of H denoted by $m(H)$, is an integer delineated by $|N_G(H): C_G(H)|$.

In addition, the dominant class of $K \cap H$ in the normalizer $N_G(H)$ is the union of $t = \frac{\varphi(n)}{m(H)}$ conjugacy classes of G where φ is the Euler function, i.e., the maturity of G is clearly defined by examining how a dominant class corresponding to H contains conjugacy classes. The group G should be matured group if $t = 1$, but if $t \geq 2$, the group G is unmatured concerning subgroup H, see Fujita (2007). For some properties of the maturity see the following theorem.

Theorem 2.2.1.

The wreath product of the matured groups again is a matured group, but the wreath product is an unmatured if at least one of the groups is unmatured, see Moghani (2009).

2.3. Q-conjugacy Character

Let $C_{u\times u}$ be a matrix of the character table for an arbitrary finite group G. Then, C is transformed into a more concise form called the Q-Conjugacy character table denoted by $\mathrm{C}_{\mathrm{G}}^{\mathrm{Q}}$ containing integer-valued characters. By Theorem 4 in Fujita (1998), the dimension of a Q-conjugacy character table is equal to its corresponding markaracter table, i.e., C_G^Q is a $m \times m$ –matrix where m is the number of dominant classes or equivalently the number of non-conjugate cyclic subgroups denoted by *SCSG*, see Fujita (2007) and Moghani (2010).

According to Aschbacher (1997), there are twenty six sporadic groups which are simple (i.e., it is nontrivial group but has no proper nontrivial normal subgroups). Furthermore, the number of their irreducible characters (corresponding to the number of their conjugacy classes) in their character tables are stored in Table 1; see ATLAS of finite groups in Conway et al (1985) for further properties of the sporadic groups.

Table 1: Simple Sporadic Group

Name of group *G*	**Order of Group *G***	**# irreducible Characters of *G***
Mathieu Group M_{11}	7, 920	10
Mathieu Group M_{12}	95, 040	15
Mathieu Group M_{22}	443, 520	12
Mathieu Group M_{23}	10, 200, 960	17
Mathieu Group M_{24}	244, 823, 040	26
Janko group J_1	175, 560	15
Janko group J_2	604, 800	21
Janko group J_3	50, 232, 960	21
Janko group J_4	86, 775, 571, 046, 077, 562, 880	62
Conway Group Co_1	4, 157, 776, 806, 543, 360, 000	101
Conway Group Co_2	42, 305, 421, 312, 000	60
Conway Group Co_3	495, 766, 656, 000	42
Fischer group Fi_{22}	64, 561, 751, 654, 400	65
Fischer group Fi_{23}	4, 089, 470, 473, 293, 004, 800	98
Fischer group Fi_{24}	1, 255, 205, 709, 190, 661, 721, 292, 800	183
Held Group *He*	4, 030, 387, 200	33
HigmanSims group *HS*	44, 352, 000	24
McLaughlin group M^cL	898, 128, 000	24
Rudvalis group *Ru*	145, 926, 144, 000	36
Suzuki group *Suz*	448, 345, 497, 600	43
O’Nan group *O’N*	460, 815, 505, 920	30
HaradaNorton group *HN*	273, 030, 912, 000, 000	54
Lyons group *Ly*	51, 765, 179, 004, 000, 000	53

Thompson group *Th*	90, 745, 943, 887, 872, 000	48
Baby Monster group *B*	4, 154, 781, 481, 226, 426, 191, 177, 580, 544, 000, 000	184
Monster group *M*	808, 017, 424, 794, 512, 875, 886, 459, 904, 961, 710, 757, 005, 754, 368, 000, 000, 000	194

3. Q-Conjugacy Characters of the sporadic Janko groups J_1, J_2, J_3 and J_4

Now we are equipped to compute all the dominant classes and Q-conjugacy characters for the sporadic Janko groups J_1, J_2, J_3 and J_4 with 15, 21, 21 and 62 respectively, irreducible characters in their character tables see Table 1. By using maturity concept in Fujita (2007) and GAP program in GAP (1995), it will be shown that they are unmatured finite groups.

Theorem 3.1.

The Janko group J_1 has four unmatured dominant classes of order 5, 10, 15, 19 with the reduction of 2 and 3.

Proof:

To find the numbers of dominant classes, we calculate the table of marks for J_1 via the following GAP program:

```
LogTo(″J4.txt″);
    Char:= CharacterTable("J4");
        M:= Display(TableOfMarksJankoGroup(4));
            Print("M");
        V:=List(ConjugacyClassesSubgroups(J4),x->Elements(x));
            Len:=Length(V);y:=[];
        for i in [1,2..Len]do
            if IsCyclic(V[i][1])then Add(y,i);
        fi;od;
        Display(Char);Display(y);
            LogTo();
```

The dimension of a Q-conjugacy character table (i.e., $C^{Q}_{J_1}$) is equal to its corresponding markaracter table for J_1 (i.e., $M^{C}_{J_1}$ see Table 2), see Fujita (1998) and Kerber (1999) for further details.

The markaracter table for J_1 corresponding to ten non-conjugate cyclic subgroups(i.e. $G_i \in SCS_{J1}$) of orders 1, 2, 3, 5, 6, 7, 10, 11, 15 and 19 respectively which is presented in Table 2.

Therefore, by using Table 2, the character table of J_1 and section 2.2, since $|SCS_{J1}| = 10$, the dominant classes of J_1 are 1a, 2a, 3a, 5a ∪ 5b, 6a, 7a, 10a ∪ 10b, 11a, 15a ∪ 15b and 19a ∪ 19b ∪ 19c with maturity $t = \frac{\varphi(n)}{m(H)}$ 1, 1, 1, 2, 1, 1, 2, 1, 2 and 3, respectively so the dimension of Q-conjugacy character table of J_1 which means the numbers of Q-conjugacy characters should be 10.

Furthermore J_1 has four unmatured Q-conjugacy characters χ_2, χ_6, χ_7 and χ_9 which are the sums of 2, 2, 3 and 2 irreducible characters, respectively, see Safarisabet et al (2013).
Therefore, there are four column-reductions (similarly four row-reductions) which means reduce of the numbers of columns in the character table of J_1, see Moghani et al (2011) for more details. We list all Q-conjugacy characters of J_1 in Table 3. ☐

Theorem 3.2.

The Janko group J_2 has five unmatured dominant classes of order 5, 10, 15 with the reduction of 2.

Proof:

The proof is similar to the previous one and therefore we omit it and report just the results. Since $|SCS_{J2}| = 16$, the dominant classes of J_2 are 1a, 2a, 2b, 3a, 3b, 4a, K_7 = 5a ∪ 5b, K_8 = 5c ∪ 5d, 6a, 6b, 7a, 8a, K_{13}= 10a ∪ 10b, K_{14}= 10c ∪ 10d, 12a, and K_{16}= 15a ∪ 15b with maturity 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 2, 2, 1 and 2, respectively.

Besides, J_2 contains five reducible Q-conjugacy characters μ_2, μ_3, μ_6, μ_{11} and μ_{12} which are the sums of two irreducible characters, all Q- conjugacy characters of J_2 are stored in Table 4. ☐

Theorem 3.3.

The Janko group J_3 has six unmatured dominant classes of order 5, 9, 10, 15, 17, 19 with the reductions of 2 and 3.

Proof:

Since $|SCS_{J3}| = 14$, the dominant classes of J_3 are 1a, 2a, 3a, 3b, 4a, L_6 = 5a ∪ 5b, 6a, 8a, L_9= 9a ∪ 9b ∪ 9c, L_{10}= 10a ∪ 10b, 12a, L_{12}= 15a ∪ 15b, L_{13}= 17a ∪ 17b and L_{14}= 19a ∪ 19b with maturity 1, 1, 1, 1, 1, 2, 1, 1, 3, 2, 1, 2, 2 and 2, respectively.
Besides, J_3 contains six reducible Q-conjugacy characters λ_2, λ_3, λ_5, λ_8, λ_{10} and λ_{11} which are the sums of 2, 2, 2, 2, 3 and 2 irreducible characters, respectively, see $C_{J_3}^{Q}$ in Table 5. ☐

Theorem 3.4.

The biggest Janko group J_4 has fifteen unmatured dominant classes of order 7, 14, 20, 21, 24, 28, 31, 33, 35, 37, 40, 42, 43, 66 with the reductions of 2 and 3.

Proof:

Since $\left|SCS_{J_4}\right| = 44$ the dominant classes of J_4 are 1a, 2a, 2b, 3a, 4a, 4b, 4c, 5a, 6a, 6b, 6c, M_{12}= 7a ∪ 7b, 8a, 8b, 8c, 10a, 10b, 11a, 11b, 12a, 12b, 12c, M_{23}= 14a ∪ 14b, M_{24}= 14c ∪ 14d, 15a, 16a, M_{27}= 20a ∪ 20b, M_{28}= 21a ∪ 21b, 22a, 22b, 23a, M_{32}= 24a ∪ 24b, M_{33}= 28a ∪ 28b, 29a, 30a, M_{36}= 31a ∪ 31b ∪ 31c, M_{37}= 33a ∪ 33b, M_{38}= 35a ∪ 35b, M_{39}= 37a ∪ 37b ∪ 37c, M_{40}= 40a ∪ 40b, M_{41}= 42a ∪ 42b, M_{42}= 43a ∪ 43b ∪ 43c, 44a and M_{44}= 66a ∪ 66b.

Besides, J_4 has fifteen reducible Q-conjugacy characters π_2, π_3, π_4, π_6, π_8, π_{10}, π_{11}, π_{12}, π_{15}, π_{24}, π_{26}, π_{27}, π_{34}, π_{39} and π_{40} which are sums of 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3 and 3 irreducible characters, respectively, see $C_{J_4}^{Q}$ in Table 6. ☐

Table 2. The markaracter Table of J_1

$M_{J_1}^{C}$	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}
$J_1(/G_1)$	175560	0	0	0	0	0	0	0	0	0
$J_1(/G_2)$	87780	60	0	0	0	0	0	0	0	0
$J_1(/G_3)$	58520	0	20	0	0	0	0	0	0	0
$J_1(/G_4)$	35112	0	0	12	0	0	0	0	0	0
$J_1(/G_5)$	29260	60	10	0	2	0	0	0	0	0
$J_1(/G_6)$	25080	0	0	0	0	6	0	0	0	0
$J_1(/G_7)$	17556	60	0	6	0	0	6	0	0	0
$J_1(/G_8)$	15960	0	0	0	0	0	0	10	0	0
$J_1(/G_9)$	11704	0	4	4	0	0	0	0	4	0
$J_1(/G_{10})$	9240	0	0	0	0	0	0	0	0	6

According to our previous studies on other sporadic groups, we are ready to present the following theorem for each sporadic groups.

Table 3: The Q-Conjugacy Character Table of Janko group J_1,

wherein D_4= 5a ∪ 5b D_7= 10a ∪ 10b, D_9= 15a ∪ 15b and D_{10}= 19a ∪ 19b ∪ 19c.

$C_{J_1}^{Q}$	**1a**	**2a**	**3a**	D_4	**6a**	**7a**	D_7	**11a**	D_9	D_{10}
χ_1	1	1	1	1	1	1	1	1	1	1
χ_2	112	0	4	2	0	0	0	2	-1	-2
χ_3	76	4	1	1	1	-1	-1	-1	1	0
χ_4	76	-4	1	1	-1	-1	1	-1	1	0
χ_5	77	5	-1	2	-1	0	0	0	-1	1
χ_6	154	-6	4	-1	0	0	-1	0	-1	2
χ_7	360	0	0	0	0	3	0	-3	0	-1
χ_8	133	5	1	-2	-1	0	0	1	1	0
χ_9	266	-6	-4	1	0	0	-1	2	1	0
χ_{10}	209	1	-1	-1	1	-1	1	0	-1	0

Table 4: The Q-Conjugacy Character Table of Janko group J_2, wherein $K_7 = 5a \cup 5b$, $K_8 = 5c \cup 5d$, $K_{13} = 10a \cup 10b$, $K_{14} = 10c \cup 10d$ and $K_{16} = 15a \cup 15b$.

$C^Q_{J_2}$	**1a**	**2a**	**2b**	**3a**	**3b**	**4a**	**K_7**	**K_8**	**6a**	**6b**	**7a**	**8a**	**K_{13}**	**K_{14}**	**12a**	**K_{16}**
μ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
μ_2	28	-4	4	10	-2	4	3	3	2	-2	0	0	-1	1	-2	0
μ_3	42	10	-6	6	0	2	7	2	-2	0	0	-2	-1	0	2	1
μ_4	36	4	0	9	0	4	-4	1	1	0	1	0	0	-1	1	-1
μ_5	63	15	-1	0	3	3	3	-2	0	-1	0	1	-1	0	0	0
μ_6	140	-20	-4	14	2	4	5	0	-2	2	0	0	5	0	-2	-1
μ_7	90	10	6	9	0	-2	5	0	1	0	-1	0	1	0	1	-1
μ_8	126	14	6	-9	0	2	1	1	-1	0	0	0	1	-1	-1	1
μ_9	160	0	4	16	1	0	-5	0	0	1	-1	0	-1	0	0	1
μ_{10}	175	15	-5	-5	1	-1	0	0	3	1	0	-1	0	0	-1	0
μ_{11}	378	-6	-6	0	0	-6	3	3	0	0	0	2	-1	-1	0	0
μ_{12}	448	0	-8	16	-2	0	-2	-2	0	-2	0	0	2	0	0	1
μ_{13}	225	-15	5	0	3	-3	0	0	0	-1	1	-1	0	0	0	0
μ_{14}	228	0	4	0	-3	0	3	-2	0	1	1	0	-1	0	0	0
μ_{15}	300	-20	0	-15	0	4	0	0	1	0	-1	0	0	0	1	0
μ_{16}	366	16	0	-6	0	0	-4	1	-2	0	0	0	0	1	0	-1

Table 5: The Q-Conjugacy Character Table of Janko group J_3, wherein $L_6 = 5a \cup 5b$, $L_9 = 9a \cup 9b \cup 9c$, $L_{10} = 10a \cup 10b$, $L_{12} = 15a \cup 15b$, $L_{13} = 17a \cup 17b$ and $L_{14} = 19a \cup 19b$.

$C^Q_{J_3}$	**1a**	**2a**	**3a**	**3b**	**4a**	**L_6**	**6a**	**8a**	**L_9**	**L_{10}**	**12a**	**L_{12}**	**L_{13}**	**L_{14}**
λ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
λ_2	170	10	-10	8	2	0	-2	-2	2	0	2	0	0	-1
λ_3	646	6	16	-2	6	1	0	-2	-2	1	0	1	0	0
λ_4	324	4	9	0	4	-1	1	0	0	-1	1	-1	1	1
λ_5	1292	-20	14	-4	4	2	-2	0	2	0	-2	-1	0	0
λ_6	816	-16	6	6	0	1	2	0	0	-1	0	1	0	-1
λ_7	1140	20	15	6	-4	0	-1	0	0	0	-1	0	1	0
λ_8	2430	30	0	0	6	0	0	2	0	0	0	0	-1	-2
λ_9	1615	15	-5	-5	-1	0	3	-1	1	0	-1	0	0	0
λ_{10}	5760	0	0	9	0	0	0	0	0	0	0	0	-3	3
λ_{11}	3876	4	6	-12	-4	1	-2	0	0	-1	2	1	0	0
λ_{12}	2432	0	-16	2	0	2	0	0	-1	0	0	-1	1	0
λ_{13}	2754	-14	9	0	-2	-1	1	0	0	1	1	-1	0	-1
λ_{14}	3078	-10	-9	0	2	-2	-1	0	0	0	-1	1	1	0

Table 6: The Q-Conjugacy Character Table of Janko group J_4

$C_{J_4}^{Q}$	1a	2a	2b	3a	4a	4b	4c	5a	6a	6b	6c
π_1	1	1	1	1	1	1	1	1	1	1	1
π_2	2666	106	-22	20	-22	10	-6	6	-20	4	-4
π_3	598734	-306	462	90	-178	14	14	-6	90	-6	-6
π_4	1775556	1476	-1212	90	68	-28	20	6	90	-6	-6
π_5	889111	2071	727	55	87	39	-1	6	55	7	7
π_6	2374290	1170	-750	-90	-110	-14	34	0	-90	6	6
π_7	1776888	2808	120	99	120	24	8	8	99	3	3
π_8	6806298	-1254	-1254	-132	154	26	-6	28	132	12	-12
π_9	4290927	1647	175	141	175	31	7	-8	-99	-3	13
π_{10}	64614726	-12474	198	0	198	102	-42	6	0	0	0
π_{11}	65794214	-10586	-1498	20	422	70	-42	-6	-20	4	-4
π_{12}	70822290	20370	6930	210	530	50	98	0	110	18	18
π_{13}	95288172	25452	364	231	44	108	28	7	-189	15	-5
π_{14}	230279749	11333	6853	-308	197	37	21	14	308	-4	4
π_{15}	519550080	13440	13440	420	640	138	0	0	-420	-12	12
π_{16}	300364890	34650	-7910	420	-550	42	-14	0	0	12	-8
π_{17}	366159104	-2816	-1792	440	768	0	0	-6	-440	-8	8
π_{18}	393877506	-10494	7106	561	66	-110	-22	1	-99	9	5
π_{19}	394765284	-9756	-2716	309	804	4	-28	-1	-351	-3	-7
π_{20}	460559498	24458	6986	329	-54	90	-14	-7	329	-7	-7
π_{21}	493456605	16605	7645	-120	285	29	13	0	540	0	4
π_{22}	690839247	23247	-10801	21	79	15	-49	7	441	-3	17
π_{23}	786127419	16443	3003	252	187	-37	-21	14	-252	12	-12
π_{24}	1572254838	32886	6006	-252	374	-74	-42	28	252	-12	12
π_{25}	789530568	49608	-440	-111	-440	40	-8	8	-351	-15	1
π_{26}	1770515712	48384	-16128	0	768	0	0	42	0	0	0
π_{27}	2032814336	55552	19712	-112	768	0	0	-14	112	16	-16
π_{28}	1085604531	-17229	-9933	330	307	19	35	6	330	-6	-6
π_{29}	1089007680	14400	-3520	-330	320	-64	-64	0	-90	6	-10
π_{30}	1182518964	-32076	10164	99	-369	-12	-28	-1	99	3	3
π_{31}	1183406741	-31339	341	-154	341	101	-35	1	-154	-10	-10
π_{32}	1183406741	39061	6677	440	-363	-75	-27	6	-440	-8	-8
π_{33}	1184295852	-29268	10284	-99	-276	12	-20	7	-99	-3	-3
π_{34}	4337827830	-39690	20790	0	630	-90	126	0	0	0	0
π_{35}	1509863773	-5027	-7139	-176	-99	45	-27	-7	-176	16	16
π_{36}	1579061136	31632	4752	-384	528	-48	-48	-14	384	0	0
π_{37}	1842237992	1064	3752	-385	-472	8	56	42	-385	-1	-1
π_{38}	1903741279	26719	-737	385	-737	-17	7	-6	385	1	1
π_{39}	5945425920	0	0	0	0	0	0	0	0	0	0
π_{40}	6003455535	-31185	-31185	0	495	-81	63	0	0	0	0
π_{41}	2267824128	-49152	0	384	0	0	0	8	384	0	0
π_{42}	2692972480	-34880	3520	-230	-320	64	-64	0	-230	10	10
π_{43}	2727495848	25256	-4312	-385	-88	8	56	-42	-385	-1	-1
π_{44}	3054840657	1617	-495	231	495	33	-7	-8	231	-9	-9

Table 6 (Cont.), wherein M_{12}= 7a ∪ 7b, M_{23}= 14a ∪ 14b, M_{24}= 14c ∪ 14d and M_{27}= 20a ∪ 20b.

$C_{J_4}^{Q}$	M_{12}	8a	8b	8c	10a	10b	11a	11b	12a	12b	12c	M_{23}	M_{24}	15a	16a	M_{27}
π_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
π_2	-1	2	-6	2	6	-2	4	4	-4	4	0	1	-1	0	-2	-2
π_3	-4	-2	-2	-2	-6	2	26	4	2	2	2	2	0	0	-2	2
π_4	-1	12	4	-4	6	-2	2	2	2	2	2	-1	-1	0	0	-2
π_5	-1	-1	-5	7	3	6	2	3	3	3	3	-1	-1	0	1	2
π_6	-5	10	2	-6	0	0	28	6	-2	-2	-2	1	-1	0	-2	0
π_7	1	0	8	0	8	0	3	3	3	3	-1	1	1	-1	0	0
π_8	-5	10	-6	-6	-4	-4	48	4	4	-4	0	-1	-1	-2	2	4
π_9	4	-5	-1	3	-8	0	25	3	1	1	1	2	0	1	1	0
π_{10}	-6	-18	6	-2	6	-2	0	0	0	0	0	0	2	0	2	-2
π_{11}	-4	-2	6	-2	-6	2	-20	2	-4	4	0	-2	0	0	2	2
π_{12}	0	10	2	10	0	0	22	0	2	2	2	0	0	0	-2	0
π_{13}	0	4	-4	-4	7	-1	-54	1	-1	3	1	0	0	1	0	-1
π_{14}	0	5	-3	-3	-2	-2	51	-4	-4	4	0	0	0	2	1	2
π_{15}	0	0	0	0	0	0	38	-6	4	-4	0	0	0	0	0	0
π_{16}	0	-10	-6	6	0	0	56	1	-4	0	-2	0	0	0	0	0
π_{17}	-4	0	0	0	-6	-2	36	3	0	0	0	-2	0	0	0	-2
π_{18}	1	6	2	-2	1	1	0	0	-3	1	-1	-1	1	1	0	1
π_{19}	4	4	4	-4	-1	-1	1	1	-3	1	-1	2	0	-1	0	-1
π_{20}	0	6	2	6	-7	1	-19	3	-3	1	1	0	0	-1	0	1
π_{21}	5	5	-3	-3	0	0	-29	4	0	-3	2	1	1	0	0	0
π_{22}	0	-1	-1	-1	7	-1	32	-1	1	-4	-1	0	0	1	-1	1
π_{23}	0	-5	3	3	-2	-2	-22	0	4	-4	-3	0	0	2	-1	1
π_{24}	0	-10	6	6	-4	-4	-44	0	-4	4	-4	0	0	-2	-2	2
π_{25}	1	0	-8	0	8	0	2	8	1	1	4	-1	1	-1	0	4
π_{26}	0	0	0	0	-6	2	-90	-2	0	1	1	0	0	0	0	0
π_{27}	0	0	0	0	2	2	58	-8	0	0	0	0	0	-2	0	0
π_{28}	4	-5	3	-5	6	2	33	0	-2	-2	-2	-2	0	0	-1	0
π_{29}	5	0	0	0	0	0	57	2	2	2	2	1	1	0	0	2
π_{30}	4	4	-4	4	-1	-1	0	0	3	3	3	-2	0	-1	0	0
π_{31}	6	1	-3	1	1	1	0	0	2	2	2	0	-2	1	-1	-1
π_{32}	-1	5	-3	-3	6	2	0	0	0	0	0	1	-1	0	1	2
π_{33}	5	4	4	4	7	-1	3	3	-3	-3	-3	-1	1	1	0	-1
π_{34}	0	-30	-18	-6	0	0	-27	6	0	0	0	0	0	0	0	0
π_{35}	5	1	-3	-7	-7	1	0	0	0	0	0	-1	1	-1	-1	1
π_{36}	-5	0	0	0	2	2	4	4	0	0	0	-1	-1	1	0	-2
π_{37}	0	0	8	0	-6	2	45	1	-1	-1	-1	0	0	0	0	-2
π_{38}	6	-5	7	-5	-6	-2	0	0	1	1	1	0	-2	0	1	-2
π_{39}	0	0	0	0	0	0	-72	-6	0	0	0	0	0	0	0	0
π_{40}	0	15	-9	15	0	0	0	0	0	0	0	0	0	0	3	0
π_{41}	-4	0	0	0	8	0	20	-2	0	0	0	2	0	-1	0	0
π_{42}	-5	0	0	0	0	0	-11	0	-2	-2	-2	1	-1	0	0	0
π_{43}	0	0	8	0	6	-2	0	0	-1	-1	-1	0	0	0	0	2
π_{44}	-6	5	1	-3	-8	0	0	0	3	3	3	0	2	1	-1	0

Table 6 (Cont.); wherein M_{28}= 21a ∪ 21b, M_{32}= 24a ∪ 24b, M_{33}= 28a ∪ 28b, M_{36}= 31a ∪ 31b ∪ 31c, M_{37}= 33a ∪ 33b, M_{38}= 35a ∪ 35b, M_{39}= 37a ∪ 37b ∪ 37c, M_{40}= 40a ∪ 40b, M_{41}= 42a ∪ 42b, M_{42}= 43a ∪ 43b ∪ 43c and M_{44}= 66a ∪ 66b.

$C_{J_4}^{Q}$	**M_{28}**	**22a**	**22b**	**23a**	**M_{32}**	**M_{33}**	**29a**	**30a**	**M_{36}**	**M_{37}**	**M_{38}**	**M_{39}**	**M_{40}**	**M_{41}**	**M_{42}**	**44a**	**M_{44}**
π_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
π_2	-1	-4	0	-2	0	1	-2	0	0	-2	-1	2	2	1	0	0	2
π_3	-1	2	0	-2	-2	0	0	0	0	2	1	0	-2	-1	2	-2	2
π_4	-1	2	-2	2	-2	-1	2	0	0	2	-1	0	2	-1	0	2	2
π_5	-1	3	1	0	1	-1	0	0	0	0	-1	1	0	-1	0	-1	0
π_6	1	4	-2	0	2	-1	2	0	0	-2	0	0	0	1	2	0	-2
π_7	1	3	-1	0	-1	1	0	-1	-1	0	1	0	0	1	-1	-1	0
π_8	4	1	0	0	0	1	-2	2	0	0	0	0	0	-1	0	0	0
π_9	1	-3	-1	1	-1	0	0	1	0	-2	-1	0	0	-1	0	-1	0
π_{10}	0	0	0	-2	0	0	0	0	0	0	-1	-2	2	0	2	0	0
π_{11}	-1	-4	-2	0	0	0	0	0	0	-2	1	0	-2	1	0	4	2
π_{12}	0	-2	0	0	2	0	-2	0	0	1	0	-2	0	0	0	2	1
π_{13}	0	-2	1	0	-1	0	1	1	0	0	0	0	-1	0	0	0	-2
π_{14}	0	3	0	0	0	0	0	-2	0	0	0	0	0	0	0	-1	0
π_{15}	0	-2	-2	-2	0	0	0	0	0	2	0	2	0	0	0	2	-2
π_{16}	0	0	-1	1	0	0	0	0	0	2	0	0	0	0	0	0	0
π_{17}	-1	0	1	1	0	0	0	0	0	0	1	0	0	1	0	-2	0
π_{18}	1	0	0	-1	-1	-1	-1	1	0	0	1	0	1	-1	0	0	0
π_{19}	1	1	1	0	1	0	0	-1	0	1	-1	0	-1	-1	0	0	1
π_{20}	0	5	1	0	-1	0	0	-1	0	-1	0	0	1	0	0	1	-1
π_{21}	-1	-5	0	0	0	-1	0	0	0	1	0	0	0	1	0	-1	1
π_{22}	0	4	1	0	-1	0	0	1	0	-1	0	0	-1	0	0	2	1
π_{23}	0	-2	0	0	0	0	1	-2	0	-1	0	0	0	0	0	0	1
π_{24}	0	-4	0	0	0	0	2	2	0	1	0	0	0	0	0	0	-1
π_{25}	1	-2	0	0	1	-1	0	-1	0	-1	1	0	0	-1	0	0	1
π_{26}	0	6	-2	0	0	0	-2	0	2	0	0	0	0	0	0	-2	0
π_{27}	0	2	0	0	0	0	2	2	0	-2	0	0	0	0	0	-2	2
π_{28}	1	-3	0	0	0	0	0	0	0	0	1	0	0	1	0	-1	0
π_{29}	-1	1	0	0	0	-1	-1	0	0	0	0	0	0	1	0	1	-2
π_{30}	1	0	0	0	-1	0	0	-1	1	0	-1	0	-1	1	1	0	0
π_{31}	0	0	0	0	0	0	0	1	0	0	1	-1	1	0	0	0	0
π_{32}	-1	0	0	0	0	1	0	0	0	0	-1	-1	0	1	0	0	0
π_{33}	-1	3	-1	0	1	1	0	1	0	0	0	0	-1	-1	0	-1	0
π_{34}	0	9	0	0	0	0	0	0	0	0	0	0	0	0	-1	3	0
π_{35}	-1	0	0	0	0	1	1	-1	0	0	0	0	1	-1	0	0	0
π_{36}	1	-4	0	0	0	1	0	-1	0	1	0	0	0	-1	0	0	-1
π_{37}	0	-3	1	0	-1	0	0	0	0	0	0	0	0	0	0	1	0
π_{38}	0	0	0	-1	1	0	0	0	0	0	1	0	0	0	0	0	0
π_{39}	0	0	0	3	0	0	-3	0	-3	0	0	1	0	0	3	0	0
π_{40}	0	0	0	-3	0	0	3	0	-1	0	0	0	0	0	0	0	0
π_{41}	-1	-4	0	1	0	0	0	-1	1	-1	1	0	0	-1	0	0	-1
π_{42}	1	1	1	0	0	-1	0	0	0	1	0	0	0	1	-1	-1	1
π_{43}	0	0	0	0	-1	0	-1	0	1	0	0	0	0	0	0	0	0
π_{44}	0	0	0	0	1	0	0	1	0	0	0	0	0	0	-1	0	0

Theorem 3.5.

Every sporadic group is an unmatured group.

Proof:

There are 26 sporadic groups which are simple, see Conway et al (1985) and Aschbacher (1997). Let G be an arbitrary sporadic group, we will calculate by using section 2.2 and GAP program, to find at least a conjugacy class nX with a reduction $t \geq 2$.

By using Theorems 3.1 to 3.4, the author's published papers (like Gilani and Moghani (2010), Moghani et al. (2011), Safarisabet et al. (2013), Moghani (2010), Moghani and Safarisabet (2011), Moghani and Najarian (2016) and Moghani (2016)), similarly we find maturities of the remain results for sporadic groups like Baby Monster Group (B) and Monster Group (M) reported in Table 7.

Table 7: The Selected Conjugacy Classes of the simple Sporadic Groups with the desired reductions.

Name	The Reduction in nX	n	t	Name	The Reduction in nX	n	t
Mathieu Groups	M_{11}	**11**	**2**	Janko Groups	J_1	**19**	**3**
	M_{12}	**15**	**2**		J_2	**15**	**2**
	M_{22}	**23**	**2**		J_3	**9**	**3**
	M_{23}	**15**	**2**		J_4	**37**	**3**
	M_{24}	**23**	**2**				
Conway Groups	Co_1	**39**	**2**	Fischer Groups	Fi_{22}	**11**	**2**
	Co_2	**15**	**2**		Fi_{23}	**15**	**2**
	Co_3	**11**	**2**		Fi_{24}	**23**	**2**
Held Group	He	**14**	**2**	Higman-Sims Group	HS	**20**	**2**
Rudvalis Group	Ru	**14**	**2**	McLaughlin Group	M^cL	**7**	**2**
Lyons Group	Ly	**31**	**2**	Harada Norton Group	HN	**35**	**2**
Suzuki Group	Suz	**9**	**2**	Thompson Group	Th	**39**	**2**
O'Nan Group	$O'N$	**20**	**2**	Baby Monster Group	B	**47**	**2**
Monster Group	M	**92**	**2**				

The suitable special conjugacy classes with reductions for all the 26 sporadic groups, are stored in Table 7, therefore, each sporadic group is an unmaturated, see Moghani et al. (2011), Safarisabet et al. (2013) and Moghani (2016) for further details.☐

4. Conclusion

In this paper, we considered all simple Janko groups J_1, J_2, J_3 and J_4, with the Q-conjugacy relationship. With the above equivalence relation on Janko groups, we found all dominant classes enabling us to find all possible Q-conjugacy characters for the Janko groups.

Finally, we proved that for each of the 26 simple sporadic groups (Mathieu groups (M_{11}, M_{12}, M_{22}, M_{23}, M_{24}), Janko groups (J_1, J_2 J_3, J_4), Conway groups (Co_1, Co_2, Co_3), Fischer groups (Fi_{22}, Fi_{23}, Fi_{24}), Higman–Sims group (HS), McLaughlin group (M^cL), Held group (He), Rudvalis group (Ru), Suzuki group (Suz), O'Nan group (O'N), Harada–Norton group (HN), Lyons group (Ly), Thompson group (Th), Baby Monster group (B) and Monster group (M)), there is at least one presented reduction in the form of union of conjugacy classes in its corresponding character table in Table 7 such that $t \geq 2$. Thus, each sporadic group is an unmaturated group.

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