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Heat and Mass Transfer in MHD Micropolar Fluid in The Presence of Diffusion Thermo and Chemical Reaction

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Abstract

This work is devoted to investigating the influence of diffusion thermo effect on hydromagnetic heat and mass transfer oscillatory flow of a micropolar fluid over an infinite moving vertical permeable plate in a saturated porous medium in the presence of transverse magnetic field and chemical reaction. The dimensionless equations are solved analytically using perturbation technique. The effects of the various fluid flow parameters entering into the problem on the velocity, microrotation, temperature and concentration fields within the boundary layer are discussed with the help of graphs. Also the local skin-friction coefficient, the wall couple stress coefficient, and the rates of heat and mass transfer coefficients are derived and shown in graphs. Comparison of the obtained numerical results is made with existing literature and is found to be in good agreement.

Keywords: Chemical reaction; Micropolar fluid; Diffusion thermo effect; MHD, Porous medium

MSC 2010 No.: 74F25, 74A35, 76R50, 76W05, 76S05

1. Introduction

The theory of micropolar fluids originally developed by Eringen (1964, 1966, 1972) has been a popular field of research in recent years. Micropolar fluids are those consisting of randomly oriented particles suspended in a viscous medium, which can undergo a rotation that can affect the hydrodynamics of the flow, making it a distinctly non-Newtonian fluid. Eringen's theory has provided a good model for studying a number of complicated fluids, such as colloidal fluids, polymeric fluids and blood: they have a non-symmetrical stress tensor. Raptis (2000) analyzed the boundary layer of micropolar fluids and their applications were considered by Ariman et al. (1973).

The unsteady hydrodynamic free convection flow of a Newtonian and polar fluid has been investigated by Helmy (1998). El-Hakien et al. (1999) studied the effect of the viscous and joule heating on MHD free convection flows with variable plate temperatures in a micropolar fluid. In many chemical engineering processes a chemical reaction between a foreign mass and the fluid does occur. These processes take place in numerous industrial applications, such as the polymer production, the manufacturing of ceramics or glassware, the food processing Cussler (1998), and so on. Chaudhary and Abhaykumar (2008) studied the effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime. Chambre and Young (1958) have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das et al. (1994) has studied the effects of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Heat and mass transfer effects on unsteady magnetohydrodynamic free convection flow near a moving vertical plate embedded in a porous medium was presented by Das and Jana (2010). Bakr (2011) presented an analysis on MHD free convection and mass transfer adjacent to a moving vertical plate for micropolar fluid in a rotating frame of reference in the presence of heat generation /absorption and chemical reaction. Mahmoud (2010) analyzed the effects of slip and heat generation/absorption on MHD mixed convective flow of a micropolar fluid over a heated stretching surface. Hayat (2011) studied the effects of heat and mass transfer on the mixed convective flow of a MHD micropolar fluid bounded by a stretching surface using Homotopy analysis method. Mansour (2007) discussed an analytical study on the MHD flow of a micropolar fluid due to heat and mass transfer through a porous medium bounded by an infinite vertical porous plate in the presence of a transverse magnetic field in slip-flow regime.

The Diffusion-thermo (Dufour) effect was found to be of a considerable magnitude such that it cannot be ignored as described by Eckert and Drake (1972) in their book. Dufour effect has been referred to as the heat flux produced by a concentration gradient. The Soret and Dufour effects are important for intermediate molecular weight gases in coupled heat and mass transfer in binary systems, often encountered in chemical process engineering and also in high speed aerodynamics. Postelnicu (2004) studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Alam and Rahman (2006) discovered the Dufour and Soret effect on unsteady MHD flow in a porous medium. Olajuwon (2007) examined convection heat and mass transfer in a hydromagnetic flow of a second grade fluid past a semi-infinite stretching sheet in the presence of thermal radiation and thermo-diffusion. Soret and Dufour effects on mixed convection in a non-Darcy porous medium saturated with micropolar fluids were studied by

Srinivasacharya and Ram Reddy (2011). Reena and Rana (2009) investigated double-diffusive convection in a micropolar fluid layer heated and soluted from below saturating a porous medium. Very recently, Prakash (2016) investigated the porous medium and diffusion-thermo effects on unsteady combined convection magneto hydrodynamics boundary layer flow of viscous electrically conducting fluid in the presence of first order chemical reaction and thermal radiation.

A mathematical model for the steady thermal convection heat and mass transfer in a micropolar fluid saturated Darcian porous medium in the presence of significant Dufour and Soret effects and viscous heating was presented by Rawat and Bhargava (2009). Hayat and Qasim (2010) studied heat and mass transfer on unsteady MHD flow in micropolar fluid with thermal radiation. Rashad et al. (2009) studied the heat and mass transfer oscillatory flow of a micropolar fluid over a vertical permeable plate. Seddeeket al. (2009) investigated the analytical solution for the effect of radiation on the flow of a magneto-micropolar fluid past a continuously moving plate with suction and blowing. Srinivasacharya and Upendar (2013) analyzed the flow, heat and mass transfer characteristics of the mixed convection on a vertical plate in a micropolar fluid in the presence of Soret and Dufour effects. Olajuwon and Oahimire (2013) investigated the effects of thermo-diffusion and thermal radiation on unsteady heat and mass transfer of free convective MHD micropolar fluid flow bounded by a semi- infinite porous plate in a rotating frame under the action of transverse magnetic field with suction.

The main object of the present investigation is to study the effects of diffusion-thermo and first order homogeneous chemical reaction on micropolar fluid flow over a vertical permeable plate in a porous medium.

2. Mathematical Formulation

An unsteady, two-dimensional incompressible laminar free convection flow of a viscous, electrically-conducting micropolar fluid over an infinite vertical porous moving permeable plate in a saturated porous medium has been considered. A uniform magnetic field of strength $B_0$is applied normal to the surface and the induced magnetic field effect is neglected. The x^* −axis is taken along the planar surface in the upward direction and the y^* −axis is taken to be normal to it. Since the plate is infinite, the flow variables are functions of y^* and the time t^* only. Initially, the fluid as well as the plate is at rest, but for time $t > 0$ the whole system is allowed to move with a constant velocity. At $t = 0$, the plate temperature and concentration are suddenly raised to T_w and C_w, and maintained constant thereafter.

In the presence of chemical reaction and Diffusion thermo effects the dimensional governing equations for the flow are

$$\frac{\partial v^*}{\partial y^*} = 0\,, \qquad (1)$$

$$\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}=\left(v+v_r\right)\frac{\partial^2 u^*}{\partial y^{*2}}+2v_r\frac{\partial w^*}{\partial y^*}+g\beta_T\left(T-T_\infty\right)+g\beta_C\left(C-C_\infty\right)$$
$$-\frac{\sigma B_0{}^2}{\rho}u^*-\frac{v+v_r}{K_1}u^*, \tag{2}$$

$$\rho j^*\left(\frac{\partial w^*}{\partial t^*}+v^*\frac{\partial w^*}{\partial y^*}\right)=\gamma\frac{\partial^2 w^*}{\partial y^{*2}}, \tag{3}$$

$$\left(\frac{\partial T^*}{\partial t^*}+v^*\frac{\partial T^*}{\partial y^*}\right)=\alpha\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{D_m K_T}{C_s C_p}\frac{\partial^2 C}{\partial y^{*2}}, \tag{4}$$

$$\left(\frac{\partial C}{\partial t^*}+v^*\frac{\partial C}{\partial y^*}\right)=D\frac{\partial^2 C}{\partial y^{*2}}+\gamma_1^*\left(C-C_\infty\right). \tag{5}$$

Here, $u^*$and v^* are the components of velocity in the x^* and y^* respectively and w^* is the component of the angular velocity normal to the x^*y^* plane, T is temperature of the fluid, and C is the mass concentration of the species in the flow. $\rho, v, v_r, g, \beta_T, \beta_C, \sigma, K_1, j^*, \gamma, \alpha, D$, $\gamma_1^*, D_m, C_{p,}$ C_s and k_T are the density, kinematic viscosity, kinematic rotational viscosity, acceleration due to gravity, coefficient of volumetric thermal expansion of the fluid, coefficient of volumetric mass expansion of the fluid, electrical conductivity of the fluid, permeability of the medium, micro inertia per unit mass, spin gradient viscosity, thermal diffusivity, molecular diffusivity and the dimensional chemical reaction parameter, coefficient of mass diffusivity, specific heat at constant pressure, concentration susceptibility, and thermal diffusion parameter, respectively.

The boundary conditions for the problem are

$$u^*=u_p^*, w^*=-n_1\frac{\partial u^*}{\partial y^*}, T=T_\infty+\varepsilon(T_w-T_\infty)e^{n^*t^*},$$
$$C=C_\infty+\varepsilon(C_w-C_\infty)e^{n^*t^*} at y^*=0, \tag{6}$$
$$u^*\to 0, w^*\to 0, T\to T_\infty, C\to C_\infty \text{ as } y^*\to\infty.$$

The following comment should be made about the boundary condition used for the micro rotation term: when $n_1=0$, we obtain from the boundary condition stated in Equation (6), for the micro rotation, $w^*=0$. This represents the case of concentrated particle flows in which the microelements close to the wall are not able to rotate, Jena and Mathur (1982). The case corresponding to $n_1=0.5$ results in the vanishing of the anti-symmetric part of the stress tensor and represents weak concentrations, Ahmadi (1976), and suggests that the particle spin is equal to the fluid vorticity at the boundary for fine particle suspensions. As suggested by Peddieson (1972), the case corresponding to $n_1=1$ is representative of turbulent boundary layer flows. Thus, for $n_1=1$, the particles are not free to rotate near the surface. However, as $n_1=0.5$ and 1, the microrotation term gets augmented and induces flow enhancement.

On integrating the continuity Equation (1), we get

$$v^* = -V_0 , \tag{7}$$

where V_0 is the suction velocity, which has a non-zero positive constant.

We introduce the following dimensionless quantities

$$u^* = U_0 u,\ v^* = V_0 v,\ y^* = \frac{v}{V_0} y,\ u_p^* = U_0 U_p,\ w^* = \frac{U_0 V_0}{v} w ,$$
$$t^* = \frac{v}{V_0^2} t,\ T - T_\infty = (T_w - T_\infty)\theta,\ C - C_\infty = (C_w - C_\infty)\phi,\ n^* = \frac{V_0^2}{v} n,$$
$$j^* = \frac{v^2}{V_0^2} j,\ \Pr = \frac{v}{\alpha},\ Sc = \frac{v}{D},\ M = \frac{\sigma B_0^2 v}{\rho V_0^2},\ Gr_T = \frac{vg\beta_T (T_w - T_\infty)}{U_0 V_0^2},$$
$$Gr_C = \frac{vg\beta_C (C_w - C_\infty)}{U_0 V_0^2},\ \gamma = (\mu + \frac{\wedge}{2}) j^* = \mu j^* (1 + \frac{\beta}{2}),\ \beta = \frac{\wedge}{\mu} = \frac{v_r}{v},$$
$$K = \frac{K_1 V_0^2}{v^2}, \eta = \frac{\mu j^*}{\gamma} = \frac{2}{2+\beta}, \gamma_1 = \frac{v\gamma_1^*}{V_0^2}, Df = \frac{D_M K_T (C_w - C_\infty)}{vC_p C_s (T_w - T_\infty)}, \tag{8}$$

where U_0 is a scale of free stream velocity and β denotes the dimensionless viscosity ratio in which Λ is the coefficient of vertex viscosity. Pr, Sc, M, Gr_T, Gr_C, K, γ_1, and Df are the Prandtl number, Schmidt number, Magnetic field parameter, thermal and solutal Grashof number, permeability parameter, the dimensionless chemical reaction parameter, and Dufour number, respectively.

Using Equation (8), Equations (1) - (7) reduce to the following initial-value problem:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = (1+\beta)\frac{\partial^2 u}{\partial y^2} + 2\beta \frac{\partial w}{\partial y} + Gr_T \theta + Gr_C \phi - Mu - \frac{1+\beta}{K} u , \tag{9}$$

$$\frac{\partial w}{\partial t} - \frac{\partial w}{\partial y} = \frac{1}{\eta}\frac{\partial^2 w}{\partial y^2} , \tag{10}$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Df \frac{\partial^2 \phi}{\partial y^2} , \tag{11}$$

$$\frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} + \gamma_1 \phi , \tag{12}$$

with the following boundary conditions:

$$u = U_p, w = -n_1 \frac{\partial u}{\partial y}, \theta = 1 + \varepsilon e^{nt}, at\ y = 0,$$
$$u \to 0, w \to 0, \theta \to 0, \phi \to 0, as\ y \to \infty . \tag{13}$$

3. Method of solution

The closed form solutions to Equations (9) to (12) are difficult to obtain and so we assume that the unsteady flow is superimposed on the mean steady flow so that in the neighborhood of the plate, we use the following linear transformations for small values of ε see Kim and Lee (2003):

$$\begin{aligned} u(y,t) &= u_0(y) + \varepsilon e^{nt} u_1(y) + O(\varepsilon^2), \\ w(y,t) &= w_0(y) + \varepsilon e^{nt} w_1(y) + O(\varepsilon^2), \\ \theta(y,t) &= \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + O(\varepsilon^2), \\ \phi(y,t) &= \phi_0(y) + \varepsilon e^{nt} \phi_1(y) + O(\varepsilon^2). \end{aligned} \tag{14}$$

After substituting the expressions (14) into Equations (9) - (13), we get

$$(1+\beta)u_0'' + u_0' - (M + \frac{1+\beta}{K})u_0 = -Gr_T\theta_0 - Gr_C\phi_0 - 2\beta w_0', \tag{15}$$

$$(1+\beta)u_1'' + u_1' - (n + M + \frac{1+\beta}{K})u_1 = -Gr_T\theta_1 - Gr_C\phi_1 - 2\beta w_1', \tag{16}$$

$$w_0'' + \eta w_0' = 0, \tag{17}$$

$$w_1'' + \eta w_1' - n\eta w_1 = 0, \tag{18}$$

$$\theta_0'' + \Pr\theta_0' = -Df\,\Pr\phi_0'', \tag{19}$$

$$\theta_1'' + \Pr\theta_1' - n\Pr\theta_1 = -Df\,\Pr\phi_1'', \tag{20}$$

$$\phi_0'' + Sc\phi_0' + \gamma_1 Sc\phi_0 = 0, \tag{21}$$

$$\phi_1'' + Sc\phi_1' + Sc(\gamma_1 - n)\phi_1 = 0, \tag{22}$$

with the boundary conditions

$$\begin{aligned} &u_0 = U_p, u_1 = 0, w_0 = -n_1 u_0', w_1 = -n_1 u_1', \\ &\theta_0 = 1, \theta_1 = 1, \phi_0 = 1, \phi_1 = 1, at\ y = 0, \\ &u_0 = 0, u_1 = 0, w_0 = 0, w_1 = 0, \\ &\theta_0 = 0, \theta_1 = 0, \phi_0 = 0, \phi_1 = 0, as\ \ y \to \infty. \end{aligned} \tag{23}$$

Solving Equations (15) - (22) with the boundary conditions (23) and substituting the solutions into Equations (14), we get

$$\begin{aligned} u = {} & a_3 c_2 (e^{-\eta y} - e^{-h_2 y}) + a_1 e^{-h_2 y} - a_2(1+h_7)e^{-\Pr y} + e^{-h_5 y} a_5 (Gr_T h_7 - Gr_C) \\ & + \varepsilon(b_3 e^{-h_4 y} - (b_3 + (Gr_T h_8 - Gr_C)b_4 + b_1 c_3)e^{-h_3 y} \\ & + (Gr_T h_8 - Gr_C)b_4 e^{-h_6 y} + b_1 c_3 e^{-h_1 y})e^{nt}, \end{aligned} \tag{24}$$

$$w = c_2 e^{-\eta y} + \varepsilon(c_3 e^{-h_1 y})e^{nt}, \tag{25}$$

$$\theta = e^{-\Pr y} + h_7(e^{-\Pr y} - e^{-h_5 y}) + \varepsilon(e^{-h_4 y} + h_8(e^{-h_4 y} - e^{-h_6 y}))e^{nt}, \tag{26}$$

$$\phi = e^{-h_5 y} + \varepsilon(e^{-h_6 y})e^{nt}. \tag{27}$$

The local friction coefficient, local wall Couple stress coefficient, local Nusselt number, and local Sherwood number are important physical quantities for this type of heat and mass transfer problem. These are defined as follows:

The wall shear stress may be written as

$$\begin{aligned}\tau_w^* &= (\mu + \wedge)\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} + \wedge w^*\big|_{y^*=0} \\ &= \rho U_0 V_0 [1 + (1-n_1)\beta] u'(0).\end{aligned} \tag{28}$$

Therefore, the local skin-friction coefficient is

$$\begin{aligned}C_f &= \frac{2\tau_w^*}{\rho U_0 V_0} = 2[1 + (1-n_1)\beta]u'(0) \\ &= 2(1 + (1-n_1)\beta)[a_3 c_2 (h_2 - \eta) - a_1 h_2 + a_2 \Pr(1 + h_7) \\ &\quad - a_5 h_5 (Gr_T h_7 - Gr_C) \\ &\quad + \varepsilon e^{nt}\{h_3(b_3 + (Gr_T h_8 - Gr_C)b_4 + b_1 c_3) - h_4 b_3 - h_6 b_4 (Gr_T h_8 - Gr_C) \\ &\quad - b_1 h_1 c_3\}].\end{aligned} \tag{29}$$

The wall couple stress can be written as:

$$M_w = \gamma \left.\frac{\partial w^*}{\partial y^*}\right|_{y=0}. \tag{30}$$

Thus, the local couple stress coefficient is

$$\begin{aligned}C_w{}' &= \frac{M_w v^2}{\gamma U_0 V_0^2} = w'(0) \\ &= -c_2\eta - \varepsilon e^{nt} c_3 h_1.\end{aligned} \tag{31}$$

The rate of heat transfer at the surface in terms of the local Nusselt number can be written as:

$$N_u = x\frac{\left(\partial T / \partial y^*\right)_{y=0}}{T_\infty - T_w}, \tag{32}$$

$$\begin{aligned}N_u \operatorname{Re}_x^{-1} &= -\theta'(0) \\ &= \Pr + h_7(\Pr - h_5) + \varepsilon e^{nt}\{h_4 + h_8(h_4 - h_6)\},\end{aligned}$$

where $\mathrm{Re}_x = \dfrac{xV_0}{\nu}$ is the local Reynolds number.

The rate of mass transfer at the surface in terms of the local Sherwood number is given by

$$Sh = x\frac{(\partial C/\partial y^*)_{y^*=0}}{C_\infty - C_w}$$

$$Sh\,\mathrm{Re}_x^{-1} = -\phi'(0) = h_5 + h_6 \varepsilon e^{nt}. \tag{33}$$

4. Results and discussion

The analytical solutions are obtained for concentration, temperature, velocity and microrotaion for different values of fluid flow parameters such as Schmidt number Sc, chemical reaction parameter Kr, Dufour number Df, magnetic field parameter M, permeability parameter K, thermal Grashof number Gr_T and mass Grashof number Gr_C which are presented in figures 1-13. Throughout the calculations the parametric values are chosen as $t=1, \varepsilon=0.1, n=0.1$, $\beta=1, Gr_T=4,\ Gr_C=2, U_p=0.5,\ n_1=0.5, \mathrm{Pr}=0.71, \eta=0.1$.

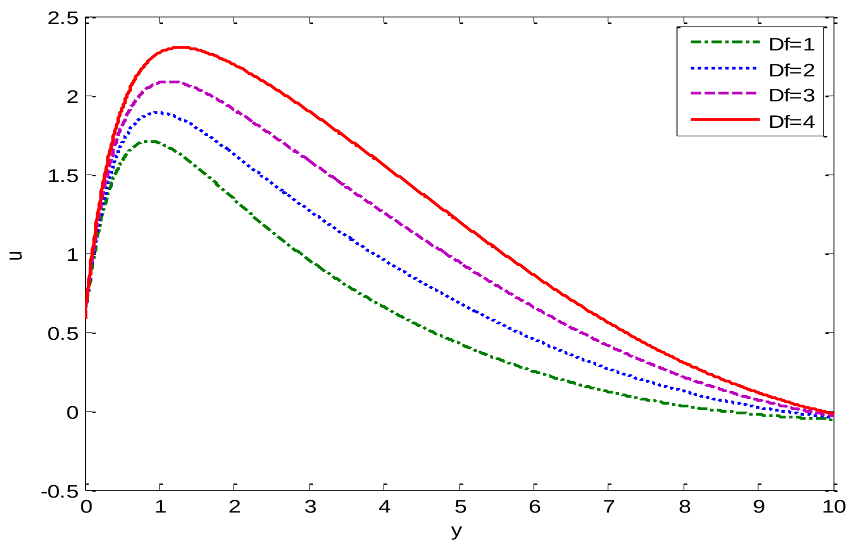

Figure 1. Velocity Profiles for different values of Dufour number Df with $Sc=0.2, \gamma_1=0.5, M=2, K=5$.

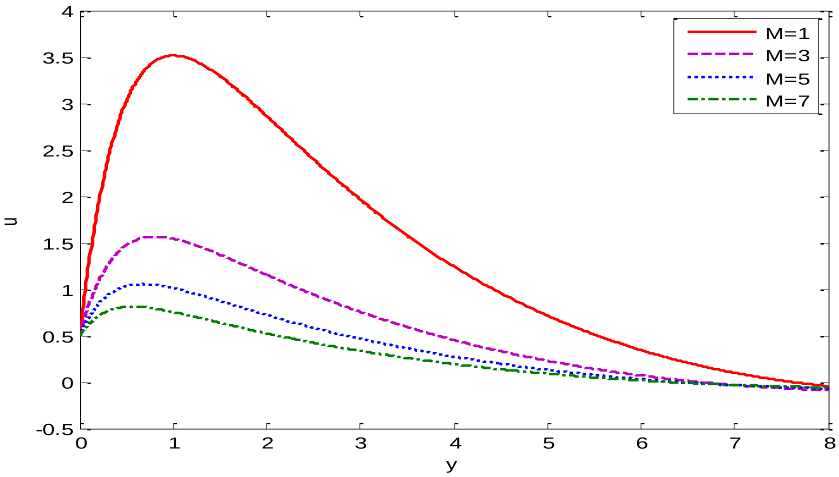

Figure 2. Velocity Profiles for different values of magnetic field parameter M with $Sc = 0.2, \gamma_1 = 0.5, Df = 0.5, K = 5.$

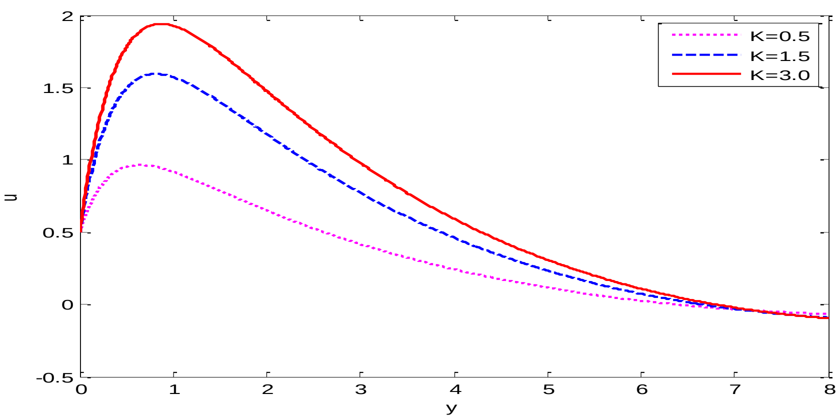

Figure 3. Velocity Profiles for different values of permeability parameter K with $Sc = 0.2, \gamma_1 = 0.5, Df = 0.5, M = 2.$

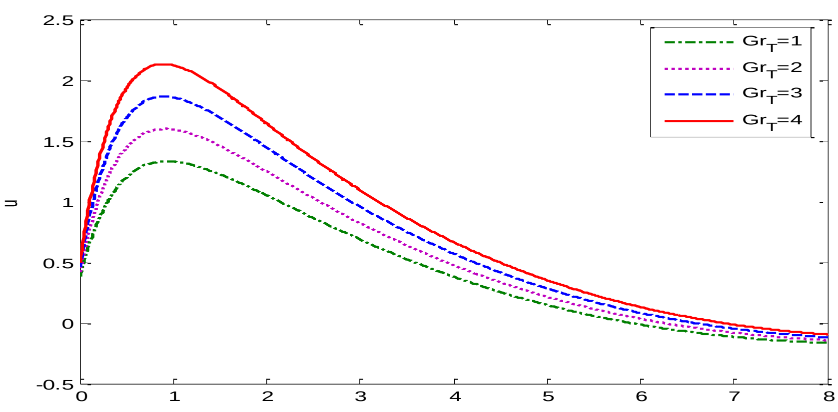

Figure 4. Velocity Profiles for different values of thermal Grashof number Gr_T with $Sc = 0.2, \gamma_1 = 0.5, Df = 0.5, M = 2, K = 5.$

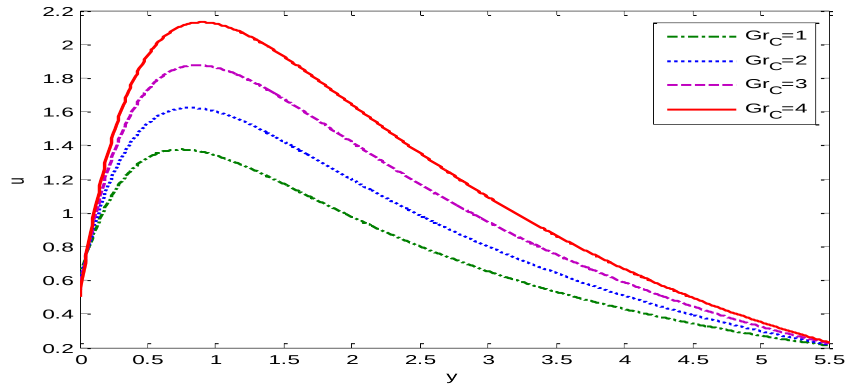

Figure 5. Velocity Profiles for different values of Mass Grashof number Gr_C for $Sc = 0.2, \gamma_1 = 0.5, Df = 0.5, M = 2, K = 5.$

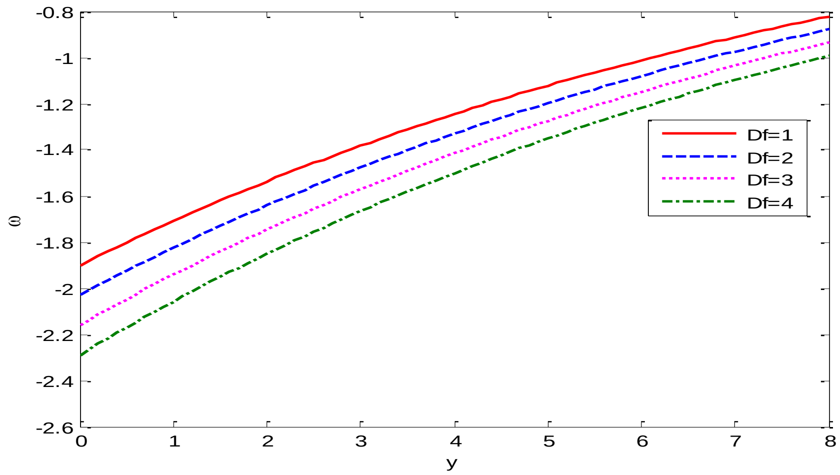

Figure 6. Micro rotation profiles for different values Dufour number Df with $Sc = 0.2, \gamma_1 = 0.5, M = 2, K = 5.$

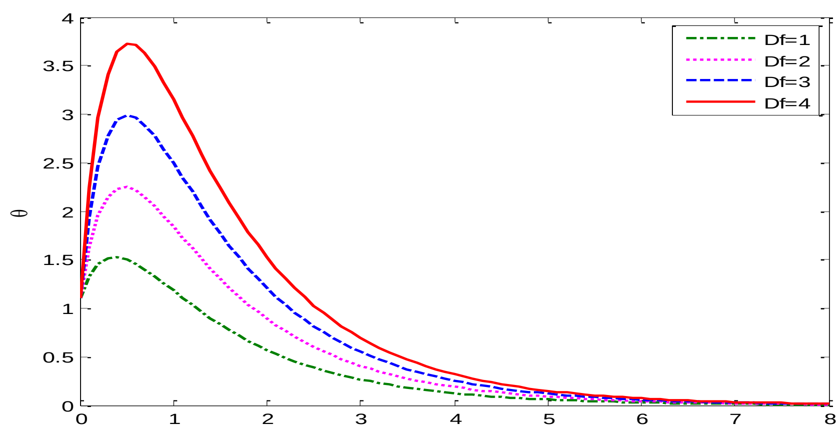

Figure 7. Temperature Profiles for different various values of Dufour number Df with $Sc = 2, \gamma_1 = 0.2.$

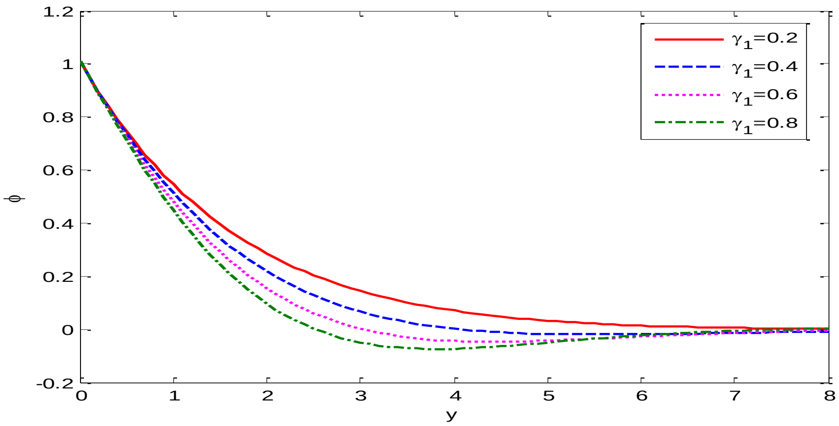

Figure 8. Concentration profiles for different values of Chemical reaction parameter γ_1 with $Sc = 0.6$.

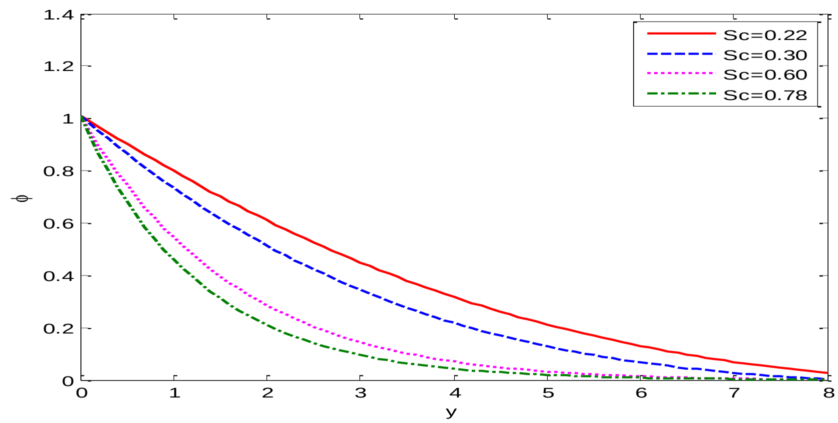

Figure 9. Concentration profiles for different values of Schmidt number Sc with $\gamma_1 = 0.2$.

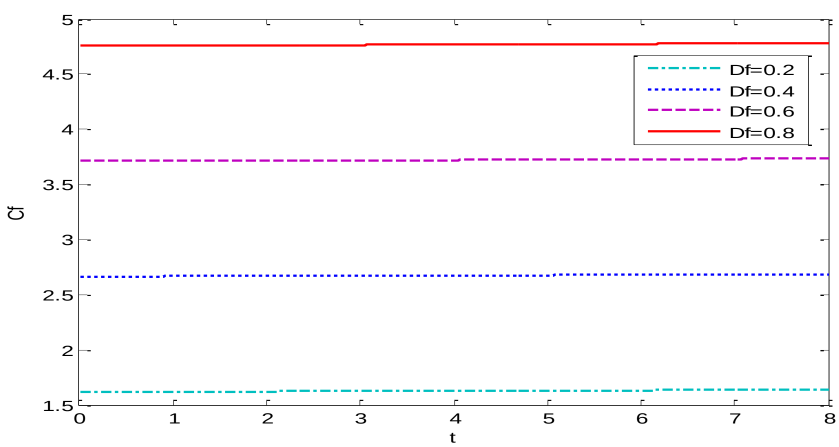

Figure 10. Local friction factor for various values of Dufour number Df against time t with $Sc = 2, \gamma_1 = 0.1, \ \eta = 0.01, \ M = 2, K = 2, \ Pr = 1, Gr_T = 2, Gr_C = 1, Up = 0.5$.

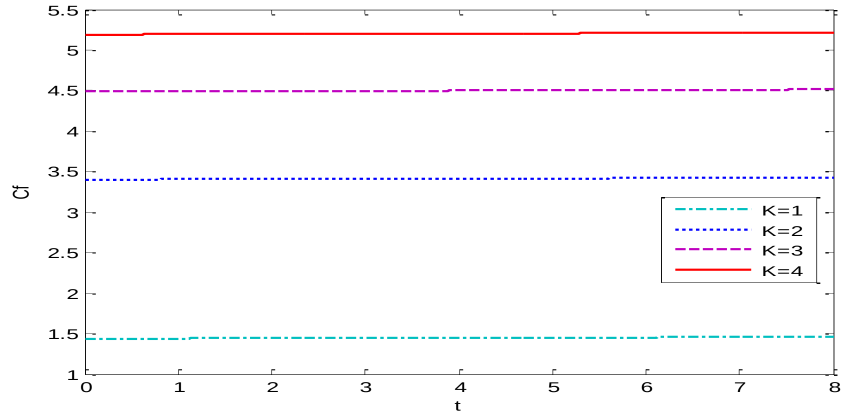

Figure 11. Local friction factor for various values of Porous permeability parameter K against time, t with $Sc = 2, \gamma_1 = 0.1, \ M = 2, Df = 0.5, \ Pr = 1, Gr_T = 2, Gr_C = 1, Up = 0.5.$

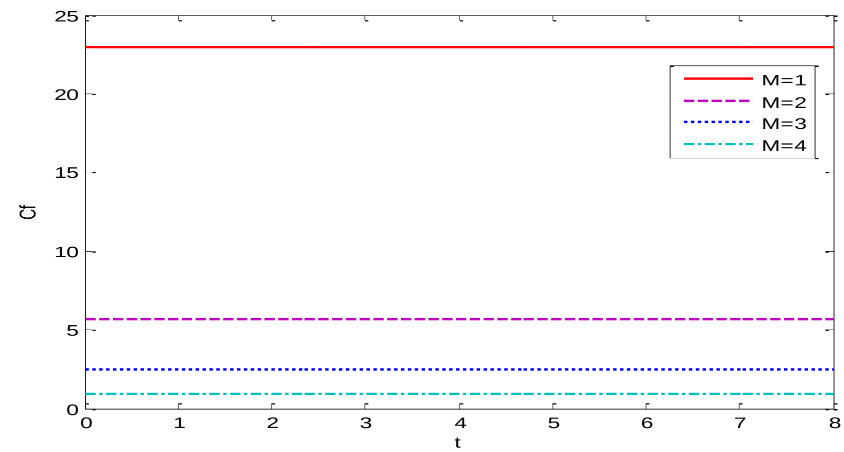

Figure 12. Local Skin friction coefficient for various values of Magnetic field parameter M against time t with $Sc = 2, \gamma_1 = 0.1, \ K = 5, Df = 0.5, \ Pr = 1, Gr_T = 2, Gr_C = 1, Up = 0.5.$

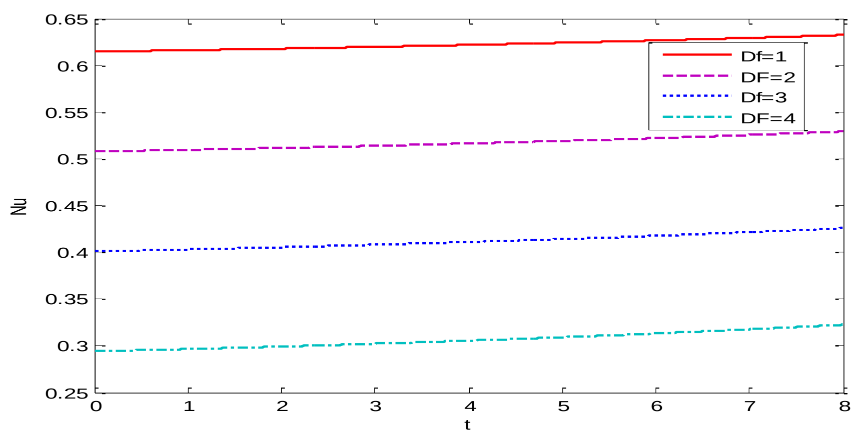

Figure 13. Local Nusselt number for various values of Dufour number Df against time t with $Sc = 0.6, \gamma_1 = 0.1, \ K = 5.$

Table 1. Comparison of the present result of Nusselt number and Sherwood number with Modather (2009) for various values of t when $n_1 = 0.5,\ n = 0.1, Gr_T = 2, Gr_C = 1,$ $K' = 5, \gamma_1 = 0.1, Up = 0.5, \varepsilon = 0.01, Du = 0, \beta = 1,\ M = 2, \Pr = 1, Sc = 2.$

Modather Results (2009)			Present Results	
t	$Nu\,\mathrm{Re}_x^{-1}$	$Sh\,\mathrm{Re}_x^{-1}$	$Nu\,\mathrm{Re}_x^{-1}$	$Sh\,\mathrm{Re}_x^{-1}$
0	1.00887	1.91217	1.0089	1.9122
1	1.00981	1.91404	1.0098	1.9140
3	1.01198	1.91838	1.0120	1.9184
5	1.01463	1.92369	1.0146	1.9237
10	1.02412	1.94267	1.0241	1.9427
20	1.06556	2.02555	1.0656	2.0256
30	1.17822	2.25086	1.1782	2.2509
40	1.48445	2.86332	1.4844	2.8633
50	2.31687	4.52816	2.3169	4.5282

Table 2. Comparison of the present result of Nusselt number and Sherwood number with Modather (2009) for various values of $\Pr$, n, Sc and γ_1 when $n_1 = 0.5,\ Gr_T = 2, Gr_C = 1,$ $K' = 5,\ Up = 0.5, \varepsilon = 0.01, Du = 0, \beta = 1,\ M = 2.$

				Modather Results(2009)		Present Results	
Pr	n	Sc	γ_1	$Nu\,\mathrm{Re}_x^{-1}$	$Sh\,\mathrm{Re}_x^{-1}$	$Nu\,\mathrm{Re}_x^{-1}$	$Sh\,\mathrm{Re}_x^{-1}$
0.7	0.1	-	-	0.70640	-	0.7055	-
1	0.1	-	-	1.00981	-	1.0098	-
1	0	-	-	1.01000	1.91337	1.0100	1.9134
1	0.05	-	-	1.00996	1.91374	1.0100	1.9137
1	0.1	-	-	1.00981	1.91404	1.0098	1.9140
1	0.15	-	-	1.00948	1.91426	1.0095	1.9143
-	0.1	1	-	-	0.89530	-	0.8953
-	0.1	2	-	-	1.91404	-	1.9140
-	0.1	2	0	-	2.02094	-	2.0209
-	0.1	2	0.1	-	1.91404	-	1.9140
-	0.1	2	0.2	-	1.79264	-	1.7926

The effect of Dufour number on velocity, microrotation and temperature are shown Figures 1, 6 and 7, respectively. It is seen that the fluid velocity and temperature increase with increasing values of *Df*. Physically, the Dufour term that appears in the temperature equation measures the contribution of concentration gradient to thermal energy flux in the flow domain. It has a vital role in the ability to increase the thermal energy in the boundary layer. The microrotation decreases with increase in Dufour number.

The effect of the magnetic parameter M on the boundary layer velocity is shown in Figure 2. It is observed that increasing magnetic field parameter reduces the velocity. This is due to an increase in the Lorentz force which acts against the flow if the magnetic field if applied in the normal direction.

Figure 3 illustrates the effects of permeability of the porous medium parameter K on fluid velocity. It is clear that as permeability parameter increases, the velocity increases along the boundary layer thickness which is expected since when the holes of porous medium become larger, the resistivity of the medium may be neglected and hence the momentum boundary layer thickness increases.

The velocity profiles in the boundary layer for various values of the thermal Grashof number Gr_T are shown in Figure 4. It is noticed that an increase in Gr_T leads to a rise in the fluid velocity due to enhancement in buoyancy force. Here, the positive values of Gr_T correspond to cooling of the plate. In addition, it is observed that the velocity increases sharply near the wall as Gr_T increases and then decays to the free stream value.

Figure 5 depicts the velocity profiles for different values of solutal Grashof number Gr_C. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach a free stream value. It is expected that the fluid velocity increases and the peak value becomes more distinctive due to increase in the buoyancy force represented by Gr_C.

Figure 8 displays the effect of chemical reaction parameter γ_1 on species concentration. From this figure it is understood that an increase in γ_1 will suppress the concentration of the fluid. Higher values of γ_1 amount to a fall in the chemical molecular diffusivity. They are obtained by species transfer. An increase in γ_1 will suppress species concentration. The concentration distribution decreases at all points of the flow field with the increase in the reaction parameter.

Effect of the Schmidt number Sc on concentration is displayed in Figure 9. Here, both the concentration profiles and the boundary layer thickness decrease when the Schmidt number Sc increases. From a physical point of view, the Schmidt number is dependent on mass diffusion D and an increase in Schmidt number corresponds to a decrease in mass diffusion and the concentration profile reduces.

Figures 10 and 13 show the variation of Skin friction coefficient and heat transfer rate on Dufour number against time t. It is noticed that the friction factor increases with an increase in the Dufour number while the heat transfer rate decreases with the increasing values of Dufour number.

The effects of magnetic field parameter and porous permeability parameter on skin friction coefficient against time t are shown in Figures 11 and 12. It is clear that the Skin friction coefficient at the wall increases with increase in Porous permeability parameter while the opposite trend is observed with the increasing values of Magnetic field parameter.

Tables 1 and 2 show the comparison of Nusselt number and Sherwood number for various values of flow parameters t, $\Pr$, n Sc and γ_1 respectively. On comparison it is observed that the results of the present study agree well with the results accomplished by Modather (2009).

5. Conclusions

The effects of Diffusion-thermo and chemical reaction on MHD free convection heat and mass transfer flow of an incompressible, micropolar fluid along an infinite-vertical porous moving permeable plate embedded in a saturated porous medium have been studied. A perturbation method is used in finding the solution. The results are discussed through graphs and tables for different values of fluid flow parameters. In addition, the results obtained showed that these parameters have significant influence on the fluid flow, heat and mass transfer. The conclusions are summarized as follows:

- The translational velocity distribution across the boundary is increased with increasing values of K, Gr_T, Gr_C, and Df while they show opposite trend with an increasing values of M.
- The magnitude of microrotation decreases with increasing value of Df.
- Inclusion of Dufour effect is to increase the skin-friction, while an opposite trend is noticed for Nusselt number.
- The temperature profiles increase with an increasing value of Dufour number, and it reaches the maximum peak value near the plate. Thus, the boundary layer thickness increases for higher values of the Dufour number.
- An increase in the chemical reaction parameter implies decrease in the species concentration.

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Appendix

$$h_1 = \frac{\eta + \sqrt{\eta^2 + 4\eta n}}{2},\ h_2 = \frac{1 + \sqrt{1 + 4(1+\beta)(M + \frac{1+\beta}{K'})}}{2(1+\beta)},\ h_3 = \frac{1 + \sqrt{1 + 4(1+\beta)(n + M + \frac{1+\beta}{K'})}}{2(1+\beta)},$$

$$h_4 = \frac{\Pr + \sqrt{\Pr^2 + 4\Pr n}}{2},\ h_5 = \frac{Sc + \sqrt{Sc^2 - 4Sc\gamma_1}}{2},\ h_6 = \frac{Sc + \sqrt{Sc^2 - 4Sc(\gamma_1 - n)}}{2},$$

$$h_7 = \frac{Df \Pr h_5^2}{h_5^2 - \Pr h_5}, \qquad h_8 = \frac{Df \Pr h_6^2}{h_6^2 - \Pr h_6 - n\Pr}, \quad c_7 = 1 + h_7, c_8 = 1 + h_8,$$

$$a_2 = \frac{Gr_T}{(1+\beta)\Pr^2 - \Pr - (M + \frac{(1+\beta)}{K'})},\quad a_3 = \frac{2\beta\eta}{(1+\beta)\eta^2 - \eta - (M + \frac{(1+\beta)}{K'})},$$

$$a_5 = \frac{1}{(1+\beta)h_5^2 - h_5 - (M + \frac{(1+\beta)}{K'})},$$

$$b_1 = \frac{2\beta h_1}{(1+\beta)h_1^{\,2} - h_1 - (n + M + \frac{(1+\beta)}{K'})}, b_3 = \frac{-Gr_T(1+h_8)}{(1+\beta)h_4^{\,2} - h_4 - (n + M + \frac{(1+\beta)}{K'})},$$

$$b_4 = \frac{1}{(1+\beta)h_6^{\,2} - h_6 - (n + M + \frac{(1+\beta)}{K'})},\ c_2 = \frac{n_1[h_2 a_1 - a_2 \Pr(1+h_7) + h_5 a_5 (Gr_T h_7 - Gr_C)]}{(1 - a_3 n_1(\eta - h_2))},$$

$$c_3 = \frac{-n_1 h_3 b_3 - n_1 h_3 b_4 (Gr_T h_8 - Gr_C) + n_1 h_4 b_3 + n_1 h_6 b_4 (Gr_T h_8 - Gr_C)]}{(1 + n_1 h_3 b_1 - n_1 h_1 b_1)}.$$