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Analysis of Groundwater Contaminants Using Aris Dispersion Model

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Abstract

The paper presents the study of dispersion of contaminants in unsteady laminar flow of an incompressible fluid (groundwater) bounded by an upper porous layer and lower impermeable layer with interphase mass transfer. An analytical solution of unsteady advection dispersion based on Aris-Barton method of moments is presented up to the second moment about the mean in axial direction.

Keywords: Interphase mass transfer; Aris dispersion; Groundwater; Contaminants

MSC 2010 No.: 76D05, 76S05, 76V05

1. Introduction

Groundwater is one of the most important sources for life. Nevertheless, today groundwater faces increasing threats from anthropogenic impacts, including contamination with pathogenic microorganisms and viruses. In recent years, problems which involve the flow of water and contaminants separately or simultaneously through saturated porous media have received much attention from sanitary landfills and agricultural practices. As contaminants are released into the subsurface environment, they infiltrate through the vadose zone which are consumed without prior conventional water treatment. Some contaminants are non-reactive and therefore act as passive tracers, moving with the water as it advects and disperses in the subsurface. The flow of water and contaminants through the porous media is considered as a bulk transport problem; it is described through the use of the differential equation known as the advection-dispersion equation.

Dispersion of a contaminant describes the spread of particles through random motion from regions of higher concentration to regions of lower concentration. Fluid flow at the interface region in systems which consists of a fluid saturated porous medium has received considerable attention. The work of Beavers and Joseph (1967) was one of the first attempts to study the fluid flow boundary condition at the interface region. They performed experiments and detected a slip in the velocity at the interface.

Neal and Nader (1974) presented one of the earlier attempts regarding the interface boundary condition in porous medium. Vafai and Kim (1990) developed an exact solution for the fluid flow at the interface between a porous medium and a fluid layer including the inertia and boundary effects.

Taylor (1953) first presented the idea of “shear effect” for the case of dispersion of passive contaminant in a viscous laminar flow through a circular tube. Aris (1956) extended Taylor’s theory to include longitudinal diffusion and developed a “method of moments” approach to analyze the convection process in steady flow using first few integral moments. Barton (1983) presented an approach for steady flow that resolved certain technical difficulties in the Aris method of moments and obtained the solutions of second and third moment equations which are valid for all time.

A lot of work has been done by various researchers on interphase mass transfer problems which is present in all branches of technology. Dumitrache and Frunzulica (2012) have studied the effects of interphase mass transfer on the unsteady convective diffusion in a fluid flow through a tube surrounded by porous medium. Meenapriya and Nirmala (2012) have developed a mathematical model to study the unsteady convective diffusion of atmospheric aerosols with interphase mass transfer in a couple stress fluid flow through a channel in the presence of electric field.

In this paper, we focus on the study of dispersion of contaminants for unsteady laminar flow of groundwater with interphase mass transfer under the influence of pressure gradient.

2. Mathematical Formulation

Consider a two-dimensional fully developed laminar flow of a viscous, incompressible fluid (groundwater) bounded by a porous (upper) and impermeable (lower) layer. The groundwater is assumed to be contaminated. The continuity and momentum equation for the fluid and contaminated particles are given below:

Fluid Particle

$$\frac{\partial u^f}{\partial x} + \frac{\partial v^f}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u^f}{\partial t} + u^f \frac{\partial u^f}{\partial x} + v^f \frac{\partial u^f}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u^f}{\partial x^2} + \frac{\partial^2 u^f}{\partial y^2} \right) + \frac{KN}{\rho}(u^s - u^f) - \frac{\nu}{k} u^f, \tag{2}$$

Contaminated Particle

$$\frac{\partial u^s}{\partial x} + \frac{\partial v^s}{\partial y} = 0, \tag{3}$$

$$\frac{\partial u^s}{\partial t} + u^s \frac{\partial u^s}{\partial x} + v^s \frac{\partial u^s}{\partial y} = \frac{K}{m}(u^f - u^s), \tag{4}$$

where u^f is the velocity of the fluid, u^s is the velocity of the contaminated particle, ρ is the density of the fluid, p is the pressure of the fluid, N is the number density of the contaminated particle, ν is the kinematic viscosity of the fluid particle, K is Stoke's resistance (drag coefficient), m is the mass of the contaminated particle, k is the permeability of porous medium, and t is time.

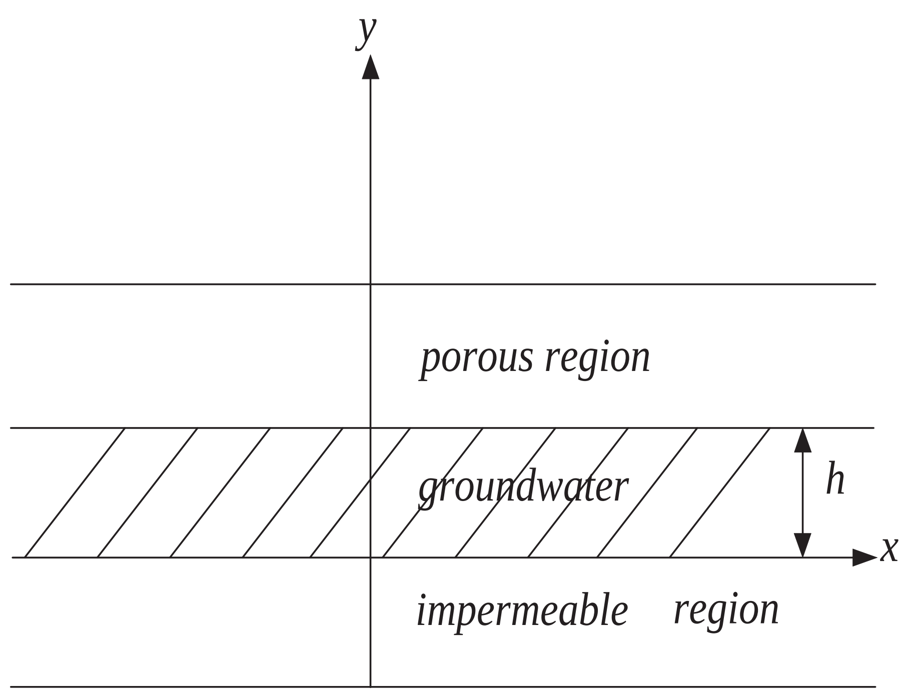

Figure 1: Physical configuration

Assuming the flow is unidirectional with constant pressure gradient, equations (2) and (4) are solved subject to the initial and boundary conditions,

$$u^f = 0, \quad u^s = 0 \quad \text{at } t = 0,$$

$$u^f = {u_a}^f + \epsilon e^{nt} {u_b}^f, \quad u^s = 0 \text{ at } y = 0, \tag{5}$$

$$\frac{du^f}{dy} = \frac{\alpha}{\sqrt{k}} u^f, \quad \frac{du^s}{dy} = \frac{\alpha}{\sqrt{k}} u^s \text{ at } y = h.$$

We now introduce the following non-dimensional variables:

$$u^{f^*} = \frac{u^f}{v_0}; \; u^{s^*} = \frac{u^s}{v_0}; \; v^{f^*} = \frac{v^f}{v_0}; \; v^{s^*} = \frac{v^s}{v_0}; \; x^* = \frac{v_0 x}{\nu}; \; y^* = \frac{v_0 y}{\nu}; \; p^* = \frac{p}{\rho v_0^2}; \; t^* = \frac{t v_0^2}{\nu}. \tag{6}$$

The asterisks (*) denote the dimensionless quantities. Substituting equation (6) into equations (1) to (5) and, for simplicity, neglecting the asterisks we get,

$$\frac{\partial u^f}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u^f}{\partial y^2} + G_f(u^s - u^f) - \frac{1}{Da} u^f, \tag{7}$$

$$\frac{\partial u^s}{\partial t} = \frac{1}{G_p}(u^f - u^s), \tag{8}$$

where $G_f = \frac{KN\nu}{v_0^2 \rho}$ is the fluid particle parameter,

$G_p = \frac{K\nu}{m v_0^2}$ is the particle mass parameter, and

$Da = \frac{\nu^2}{k v_0^2}$ is the porosity parameter.

$$u^f = 0, u^s = 0 \text{ at } t = 0,$$

$$u^f = {u_a}^f + \epsilon e^{nt} {u_b}^f, \quad u^s = 0 \text{ at } y = 0, \tag{9}$$

$$\frac{du^f}{dy} = \frac{\alpha}{Da^2} u^f, \quad \frac{du^s}{dy} = \frac{\alpha}{Da^2} u^s \text{ at } y = 1.$$

Equations (7) and (8) together with the boundary conditions (9) are solved using the perturbation technique as given below,

$$u_i(x, y, t) = u_{0_i}(y) + \epsilon e^{\mathbf{i}(\lambda x + \omega t)} u_{1_i}(y) + o(\epsilon^2),$$

$$v_i(x, y, t) = \epsilon e^{\mathbf{i}(\lambda x + \omega t)} v_{1_i}(y) + o(\epsilon^2),$$

$$p(x,y,t) = p_0(x) + \epsilon e^{\mathbf{i}(\lambda x+\omega t)} p_1(y) + o(\epsilon^2),$$

$$\theta_i(x,y,t) = \theta_{0_i}(y) + \epsilon e^{\mathbf{i}(\lambda x+\omega t)} \theta_{1_i}(y) + o(\epsilon^2),$$

$$\phi_i(x,y,t) = \phi_{0_i}(y) + \epsilon e^{\mathbf{i}(\lambda x+\omega t)} \phi_{1_i}(y) + o(\epsilon^2),$$

where $i = f, s$ represents the fluid and solid particles, respectively, and **i** represents the imaginary part.

Considering the real part and neglecting the higher order terms (order of ϵ^2), we get the velocities of fluid and contaminated particle respectively as,

$$u^f = g_2 e^{\frac{1}{\sqrt{Da}}} y + g_3 e^{\frac{-1}{\sqrt{Da}}} y - Da g_1 + \epsilon e^{nt}(g_7 \cos \sqrt{g}_6 y + g_8 \sin \sqrt{g}_6 y), \tag{10}$$

$$u^s = g_2 e^{\frac{1}{\sqrt{Da}}} y + g_3 e^{\frac{-1}{\sqrt{Da}}} y - Da g_1 + \frac{\epsilon e^{nt}}{1+nG_p}(g_7 \cos \sqrt{g}_6 y + g_8 \sin \sqrt{g}_6 y). \tag{11}$$

The average velocity is given by

$$\overline{u}^f = \int_0^1 u^f dy,$$

$$\overline{u}^f = \sqrt{D}a \left(g_2 (e^{\frac{1}{\sqrt{Da}}} - 1) - g_3 (e^{\frac{-1}{\sqrt{Da}}} + 1) \right) - Da g_1 + g_9. \tag{12}$$

Similarly,

$$\overline{u}^s = \sqrt{D}a \left(g_2 (e^{\frac{1}{\sqrt{Da}}} - 1) - g_3 (e^{\frac{-1}{\sqrt{Da}}} + 1) \right) - Da g_1 + \frac{g_9}{1+nG_p}, \tag{13}$$

where

$$g_1 = \frac{dp_0}{dx},$$

$$g_2 = \frac{1}{e^{\frac{1}{\sqrt{Da}}}(\frac{\alpha}{Da^2} - \frac{1}{\sqrt{Da}}) - e^{\frac{-1}{\sqrt{Da}}}(\frac{\alpha}{Da^2} - \frac{1}{\sqrt{Da}})} \left((u_a^f + Da g_1)(e^{\frac{-1}{\sqrt{Da}}})(\frac{-\alpha}{Da^2} - \frac{1}{\sqrt{Da}}) \right),$$

$$g_3 = u_a^f - g_2 + Da g_1, \quad g_4 = \frac{G_f}{1+nG_p}, \quad g_5 = G_f + \frac{1}{Da} + n,$$

$$g_6 = g_4 - g_5, \quad g_7 = u_b^f,$$

$$g_8 = \frac{-1}{\frac{\alpha}{Da^2}\sin\sqrt{g_6}-\sqrt{g_6}\cos\sqrt{g_6}}\left(\frac{\alpha}{Da^2}u_b^f\cos\sqrt{g_6}+u_b^f\sqrt{g_6}\sin\sqrt{g_6}\right),$$
$$g_9 = \frac{1}{g_6}\epsilon(g_7\sin\sqrt{g_6}-g_8\cos\sqrt{g_6}-g_8),$$
$$g_{10} = \sqrt{D}a\left(g_2(e^{\frac{1}{\sqrt{Da}}}-1)-g_3(e^{\frac{-1}{\sqrt{Da}}}+1)\right)-Dag_1+g_9,$$
$$g_{11} = \sqrt{D}a\left(g_2(e^{\frac{1}{\sqrt{Da}}}-1)-g_3(e^{\frac{-1}{\sqrt{Da}}}+1)\right)-Dag_1+\frac{g_9}{1+nG_p}.$$

Dispersion.

We consider the dispersion of contaminants in the laminar flow of groundwater under a constant pressure gradient. The concentration of contaminants c is governed by the advection-dispersion equation of the form

$$\frac{\partial c^q}{\partial t}+u^q\frac{\partial c^q}{\partial x}=D\left(\frac{\partial^2 c^q}{\partial x^2}+\frac{\partial^2 c^q}{\partial y^2}\right), \tag{14}$$

subject to the initial and boundary conditions

$$c^q(y,0)=c(y),$$

$$D\frac{\partial c^q}{\partial y}=0 \text{ at } y=0, \tag{15}$$

$$-D\frac{\partial c^q}{\partial y}=k_s c \text{ at } y=h,$$

where $c(y)$ is the initial concentration in every elementary volume of the medium, k_s is the reaction rate constant, and D is the coefficient of mass diffusivity.

In this paper we consider two situations, one is for dispersion by fluid ($q=f$) and the other dispersion by contaminated particle ($q=s$).

Introducing the following dimensionless quantities,

$$x^*=\frac{x}{L};\ y^*=\frac{y}{L};\ U^{q^*}=\frac{u^q}{\overline{u^q}};\ t^*=\frac{tD}{L^2};\ C^q=\frac{c^q}{c_0},$$

equation (14) becomes,

$$\frac{\partial C^q}{\partial t}+Pe\ U^q(y,t)\frac{\partial C^q}{\partial x}=\frac{\partial^2 C^q}{\partial x^2}+\frac{\partial^2 C^q}{\partial y^2}, \tag{16}$$

with $U^q = \dfrac{u^q - \bar{u^q}}{\bar{u^q}}$.

The Peclet number Pe is introduced here; this measures the relative characteristic time of the diffusion process to the convection process.

Using the dimensionless quantities, the initial and boundary conditions for the contaminant input become

$$C^q(y,0) = \mathscr{C}(y),$$

$$\frac{\partial C^q}{\partial y} = 0 \text{ at } y = 0, \tag{17}$$

$$\frac{\partial C^q}{\partial y} = -\beta C^q \text{ at } y = 1,$$

where $\mathscr{C}(y)$ is the dimensionalized initial concentration and $\beta = \dfrac{k_s L}{C_0}$ is the heterogeneous reaction rate parameter.

Following Aris (1956) we consider the p-th moment of the concentration distribution through y at time t as

$$C_p^q(y,t) = \int_{-\infty}^{\infty} x^p C^q(x,y,t)dx, \tag{18}$$

and the p-th moment of the distribution over the cross-section of the channel as

$$m_p^q(t) = \frac{1}{2}\int_{-1}^{1} C_p^q(y,t)dy = \bar{C}_p^q. \tag{19}$$

Using the definition (18), equation (16) reduces to

$$\frac{\partial C_p^q}{\partial t} = \frac{\partial^2 C_p^q}{\partial y^2} + p(p-1)C_{p-2}^q + Pe\ U^q(y,t)C_{p-1}^q. \tag{20}$$

The boundary and initial conditions to be applied to find the solutions are

$$C_p^q(y,0) = \mathscr{C}_p^q(y), \tag{21}$$

$$\frac{\partial C_p^q}{\partial y} = 0 \text{ at } y = 0,\ \frac{\partial C_p^q}{\partial y} = -\beta C_p^q \text{ at } y = 1, \tag{22}$$

Averaging equation (20) over the cross section of unit breadth, we find, after using equation (19) and the boundary conditions (22), that we get,

$$\frac{dm_p^q}{dt} = p(p-1)m_{p-2}^q + pPe\int_0^1 U^q(y,t)C_{p-1}^q dy. \tag{23}$$

Equation (20), subject to the conditions (21) and (22) for $p = 0, 1, 2, \cdots$, forms a sequence of inhomogeneous equations which can be solved for C_p^q. Once C_p^q is found, m_p^q can be obtained from equation (23).

According to Aris the above sequence of equations are solved, in principle to a sufficiently large value of p, but the first two or three moments are sufficient to describe the process of dispersion. In equation (23), for $p = 0$, $\frac{dm_0^q}{dt} = 0$, so that m_0 is a constant which is assumed to be unity indicating that the total quantity of contaminant is constant.

For $p = 0$, equation (20) reduces to

$$\frac{\partial C_0^q}{\partial t} = \frac{\partial^2 C_0^q}{\partial y^2}. \tag{24}$$

The solution of equation (24) subject to the conditions

$$C_0^q(y,0) = \mathscr{C}_0^q(y), \tag{25}$$

$$\frac{\partial C_0^q}{\partial y} = 0 \text{ at } y = 0, \tag{26}$$

$$\frac{\partial C_0^q}{\partial y} = -\beta C_0^q \text{ at } y = 1, \tag{27}$$

is written as

$$C_0^q(y,t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos(\lambda_n y)e^{-\lambda n^2 t}, \tag{28}$$

where

$$A_n = 2\int_0^1 C_0^q(y)\cos(\lambda_n y)dy, \tag{29}$$

and

$$A_0 = 2\int_0^1 C_0^q(y)dy. \tag{30}$$

The problem for C_1 is more complicated, however, since certain solvability conditions have to be satisfied to obtain the solution. For $p = 1$, equation (23) reduces to

$$\frac{dm_1^q}{dt} = Pe\int_0^1 U^q(y,t)C_0^q(y,t)dy. \tag{31}$$

The above procedure for $q = f, s$ representing the fluid and contaminated particle, respectively, gives the solution from equation (31) as

$$\frac{dm_1^f}{dt} = Pe\int_0^1 U^f(y,t)C_0^f(y,t)dy, \tag{32}$$

$$\frac{dm_1^s}{dt} = Pe\int_0^1 U^s(y,t)C_0^s(y,t)dy. \tag{33}$$

Integrating equation (32) we get,

$$\begin{aligned}
m_1^f \;=\; & \frac{1}{2g_{11}\sqrt{g_6}(1+nG_p)\lambda_1(-g_6+\lambda_1^2)(1+Da\lambda_1^2)} e^{-\frac{1}{\sqrt{Da}}-t\lambda_1^2} \\
& (\lambda 1(-A_0 e^t\lambda_1^2(\sqrt{Da}(e^{\frac{1}{\sqrt{Da}}}(\sqrt{Da}\ g_1+g_2-e^{\frac{1}{\sqrt{Da}}}g_2-g_3)+g_3)\sqrt{g_6}(1+nGp) \\
& +e^{\frac{1}{\sqrt{Da}}+nt}(g_{11}\sqrt{g_6}-g_8)\epsilon)(-g_6+\lambda_1^2)(1+Da\lambda_1^2)+2A_1 e^{\frac{1}{\sqrt{Da}}}\sqrt{g_6}(\sqrt{Da}(g_2-g_3) \\
& (1+nG_p)(g_6-\lambda_1^2)-e^{nt}\sqrt{g_6}g_8\epsilon(1+Da\lambda_1^2)))) \\
& +e^{\frac{1}{\sqrt{Da}}+nt}\epsilon\lambda_1(1+Da\lambda_1^2)\cos[\sqrt{g_6}](-A_0 e^{t\lambda_1^2}+g_8(-g_6+\lambda_1^2)+2A_1\ g_6\ g_8\ \cos(\lambda_1) \\
& +2A_1\sqrt{g_6}g_7\lambda_1\sin(\lambda_1))+e^{\frac{1}{Da}+nt}\ e\lambda_1(1+Da\lambda_1^2)\sin(\sqrt{g_6}) \\
& (A_0 e^{t\lambda_1^2}g_7(-g_6+\lambda_1^2)-2A_1 g_6 g_7\cos[\lambda_1]+2A_1\sqrt{g_6}g_8\lambda_1\sin(\lambda_1)) \\
& +2A_1\sqrt{g_6}(g_6-\lambda_1^2)(e^{\frac{1}{\sqrt{Da}}+nt}g_1 1\epsilon(1+Da\lambda_1^2)\sin(\lambda_1) \\
& +\sqrt{Da}(1+nG_p)(\sqrt{Da}e^{\frac{1}{\sqrt{Da}}}g_1(1+Da\lambda_1^2)\sin(\lambda_1)+ \\
& g_3\lambda_1(\cos(\lambda_1)-\sqrt{Da}\lambda_1\sin(\lambda_1))-e^{\frac{2}{Da}g_2\lambda_1}(\cos(\lambda_1)+\sqrt{Da}\lambda_1\sin(\lambda_1)))).
\end{aligned}$$

Integrating equation (33) we get,

$$
\begin{aligned}
m_1^s = & \frac{1}{2g_{10}\sqrt{g_6}\lambda_1(g_6-\lambda_1^2)(1+Da\lambda_1^2)} e^{-t\lambda_1^2} \\
& (e^{\frac{2}{\sqrt{Da}}\lambda_1}(A_0 e^{t\lambda_1^2}(\sqrt{Da}\ (e^{\frac{1}{\sqrt{Da}}}(\sqrt{Da}g_1+g_2-e^{\frac{1}{\sqrt{Da}}g_2-g_3})+g_3)\sqrt{g_6} \\
& +e^{\frac{1}{\sqrt{Da}}+nt}(g_{10}\sqrt{g_6}-g_8)\epsilon)(-g_6+\lambda_1^2)(1+Da\lambda_1^2) \\
& +2A_1 e^{\frac{1}{\sqrt{Da}}}\sqrt{g_6}(-\sqrt{Da}(g_2-g_3)(g_6+\lambda_1^2)+e^{nt}\sqrt{g_6}g_8\epsilon(1+Da\lambda_1^2) \\
& A0 e^{\frac{1}{\sqrt{Da}}+t(n+\lambda_1^2)} g_7\epsilon(-g_6+\lambda_1^2)\sin(\sqrt{g_6})(e^{\frac{1}{Da}+nt}\epsilon\lambda_1^2)\cos(\sqrt{g_6}) \\
& (A_0^{t(n+\lambda_1^2)}-2A_1g_6g_8\cos(\lambda_1)-2A_1\sqrt{g_6}g_7\lambda_1\sin(\lambda_1) \\
& +2A_1\sqrt{g_6}(g_6-\lambda_1^2)(\sqrt{Da}(e^{\frac{2}{\sqrt{Da}}g_2-g_3})\lambda_1\cos(\lambda_1)+(-e^{\frac{1}{\sqrt{Da}}+nt}g_{10}\epsilon(1+Da\lambda_1^2) \\
& +Da(e^{\frac{2}{Da}g_2\lambda_1^2}+g_3\lambda_1^2-e^{\frac{1}{\sqrt{Da}}}g_1(1+Da\lambda_1^2)))\sin(\lambda_1) \\
& e^{\frac{1}{Da}+nt}\epsilon\lambda_1\sin(\sqrt{g_6})(-A0Dae^{t\lambda 1^2}g_7\lambda_1^2(-g_6+\lambda_1^2) \\
& +2A_1\sqrt{g_6}(1+Da\lambda 1^2)(\sqrt{g_6}g_7\cos(\lambda_1)-g_8\lambda_1\sin(\lambda_1))))).
\end{aligned}
\tag{34}
$$

For $q=f$ and $q=s$, equation (20) gives

$$\frac{\partial C_1^f}{\partial t}=\frac{\partial^2 C_1^f}{\partial y^2}+Pe\ U^f(y,t)C_0^f(y,t), \tag{35}$$

$$\frac{\partial C_1^s}{\partial t}=\frac{\partial^2 C_1^s}{\partial y^2}+Pe\ U^s(y,t)C_0^s(y,t), \tag{36}$$

where C_0^f and C_0^s are obtained by substituting $q=f$ in equation (28).

Applying Laplace transformation and taking inverse Laplace using Cauchy's residue theorem, the solution of (35) is given by

$$
\begin{aligned}
C_1^f &= e^{\alpha-\sqrt{\beta}+y\sqrt{\beta}+t\beta} + y^{\alpha+t\alpha^2-\sqrt{\alpha^2}-y\sqrt{\alpha^2}}k - e^{t\alpha^2}k + \frac{A_0 e^{\sqrt{n}}(-1+\sqrt{n}t+y)g_8 Pe\epsilon \sin(\sqrt{g_6})}{g_{10}(g_6+n)} \\
&\quad \frac{+A_0 Dae^{\frac{1}{Da}} Pe(e^{\frac{t}{Da}+\sqrt{\frac{1}{Da}}(-1+y)}(1+e^{\frac{2}{\sqrt{Da}}})g_2 + e^{\frac{1}{\sqrt{Da}}}(g_2+g_3-e^{\frac{t}{Da}(g_2+g_3)+g_1 t}))}{g_{10}} \\
&\quad + \frac{e^{-t\lambda_1^2} Pe\epsilon(A_1 e^{nt} - A_0(e^{t\lambda_1^2-e^{t}(n+\lambda_1^2)+e^{nt+\sqrt{n}(-1+y)+t\lambda_1^2}}) - A_1 e^{nt+(-1+y)\sqrt{n-\lambda_1^2}}\cos(\lambda_1))}{n} \\
&\quad + \frac{1}{(g_{10}((g_6+n)^2-4g_6\lambda_1^2))}\Bigg(A_1 e^{nt-(t+\lambda_1^2)+(-1+y)+\sqrt{n-\lambda 1^2}g_8} Pe\epsilon \\
&\quad (-2\sqrt{g_6}\lambda_1\cos(\lambda_1)\sin(\sqrt{g_6})) + (g_6+n)\cos(\sqrt{g_6})\sin(\lambda_1)\Bigg) \\
&\quad + \frac{1}{(g_6+n)^2-4g_6\lambda_1^2} A_1 e^{t}(n-\lambda_1^2)g_7 Pe\epsilon \\
&\quad (-g_6-n+e^{(-1+y)\sqrt{n-\lambda 1^2}}((g_6+n)\cos\sqrt{g_6}\cos(\lambda_1) + 2\sqrt{g_6}\lambda_1\sin\sqrt{g_6}\sin(\lambda_1))).
\end{aligned}
\tag{37}
$$

Similarly,

$$
\begin{aligned}
C_1^s &= \frac{e^{\alpha-\sqrt{\beta}} + y\sqrt{\beta} + t\beta k}{2g_{11}\beta^2} + \frac{e^{\alpha+t\alpha^2-\sqrt{\alpha^2}} + y\sqrt{\alpha^2}}{k}(2g_{11}\alpha^4) - \frac{e^{t\alpha^2}k}{\alpha^2} \\
&\quad + \frac{A_0 e^{\sqrt{n}(-1+\sqrt{n}t+y)} g_8 Pe\epsilon\sin(\sqrt{g_6})}{2g_{11}(g_6+n)(1+nG_p)} \\
&\quad + \frac{A_0 Dae^{-\frac{1}{Da}} Pe(e^{\frac{t}{Da}}\sqrt{\frac{1}{Da}(-1+y)}(e^{\frac{1}{\sqrt{Da}}}(g_2+g_3-e^{\frac{t}{Da}}(g_2+g_3))g_1 t))}{2g_{11}} \\
&\quad + \frac{1}{(g_{10}g_{11}(1+nG_p)((g_6+n)^2-4g_6\lambda_1^2))}\Bigg(A_1 e^{nt-(t+\lambda 1^2)+(-1+y)+\sqrt{n-\lambda 1^2}} g_8 Pe\epsilon \\
&\quad (-2\sqrt{g_6}\lambda_1\cos(\lambda_1)\sin(\sqrt{g_6}) + (g_6+n)\cos(\sqrt{g_6})\sin(\lambda_1))\Bigg) \\
&\quad + \frac{1}{(g_{11}+(1+nG_p)((g_6+n)^2-4g_6\lambda_1^2))}\Bigg((A_1 e^{t(n-\lambda_1^2)} g_7 Pe\epsilon(-g_6-n+e^{-1+y\sqrt{n-\lambda_1^2}} \\
&\quad (g_6+n)\cos\sqrt{g_6}\cos(\lambda_1) + 2\sqrt{g_6}\lambda_1\sin\sqrt{g_6}\sin(\lambda_1)))\Bigg) \\
&\quad + \frac{Pe\epsilon A_0(1-e^{nt}+e^{\sqrt{n}(-1+\sqrt{n}t+y)} + 2A_1 e^{t(n-\lambda_1^2)})(-1+e^{(-1+y)\sqrt{n-\lambda_1^2}\cos(\lambda_1)})}{2n(1+nG_p)}.
\end{aligned}
\tag{38}
$$

Putting $p = 2$ in equation (23) and recalling that $m_0 = 1$, we get

$$
\frac{dm_2^q}{dt} = 2 + 2Pe\int_0^1 U^q(y,t)C_1^q(y,t)dy. \tag{39}
$$

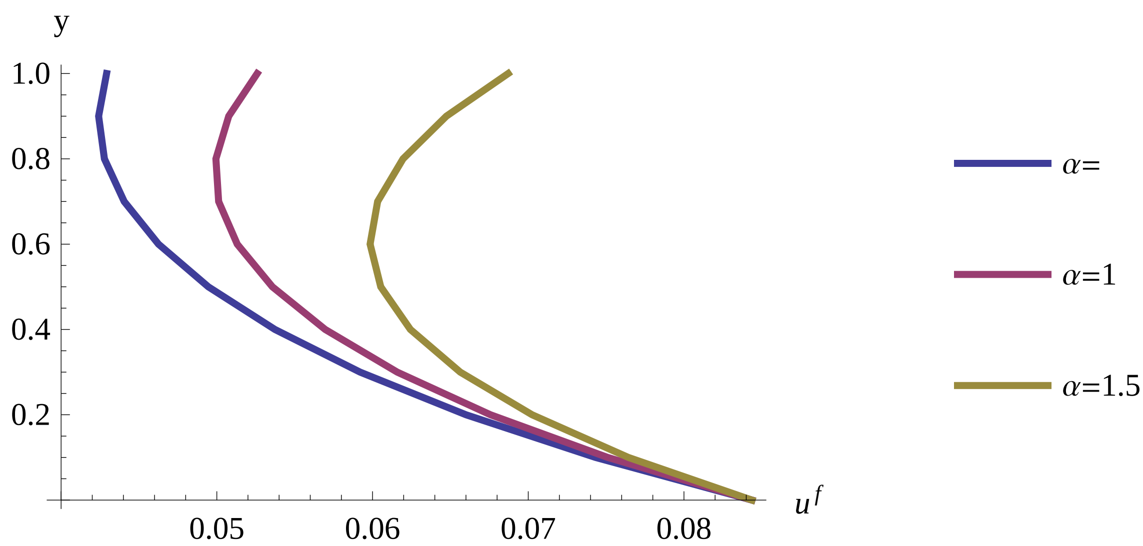

Figure 2: u^f varying with dimensionless time for different α

Therefore,

$$\frac{1}{2}\frac{dm_2^q}{dt} = 1 + Pe\int_0^1 U^q(y,t)C_1^q(y,t)dy. \tag{40}$$

According to Aris, the left-hand side of equation (40) is reasonable to use the definition of the measure of the effective dispersion coefficient. Therefore the measure of the effective dispersion coefficient E^q/De becomes

$$E^q/De = 1 + Pe\int_0^1 U^q(y)C_1^q(y,t)dy, \tag{41}$$

where

E^q is the effective dispersion coefficient,
De is the eddy diffusion coefficient,
$q = f, s$ represents the effective dispersion of fluid and solid phases, respectively.

3. Results and Discussion

Figures 2 and 3 represent the velocity profiles of groundwater and the contaminant particle in the groundwater at a given instant of time for different values of slip parameter (α), respectively. The graph shows that groundwater velocity and contaminant particle velocity intends with an increase in slip parameter which enhances the fluid flow mean that the slip parameter has the

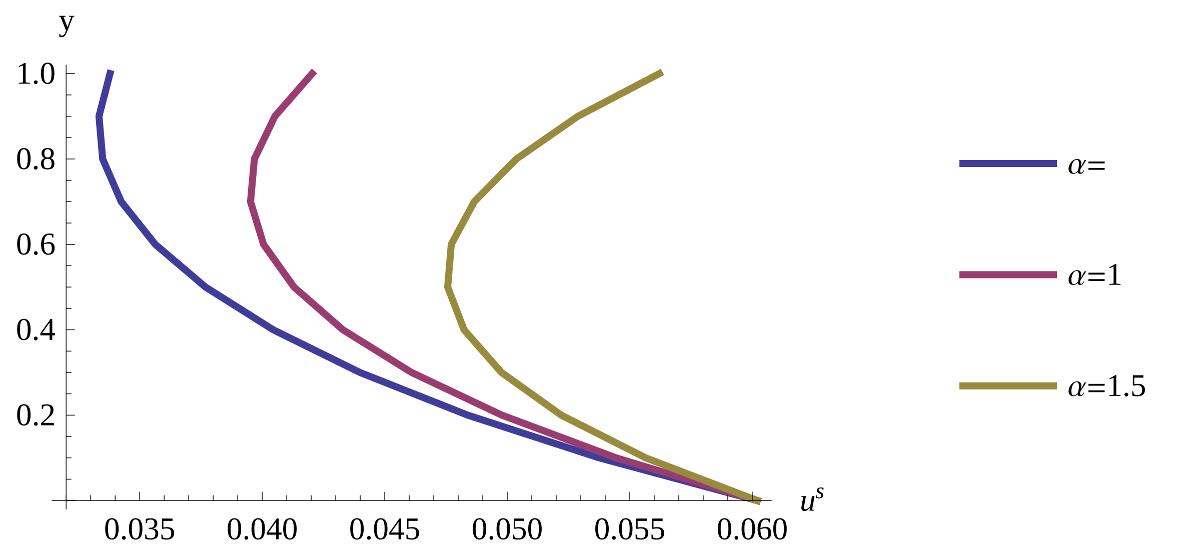

Figure 3: u^s varying with dimensionless time for different α

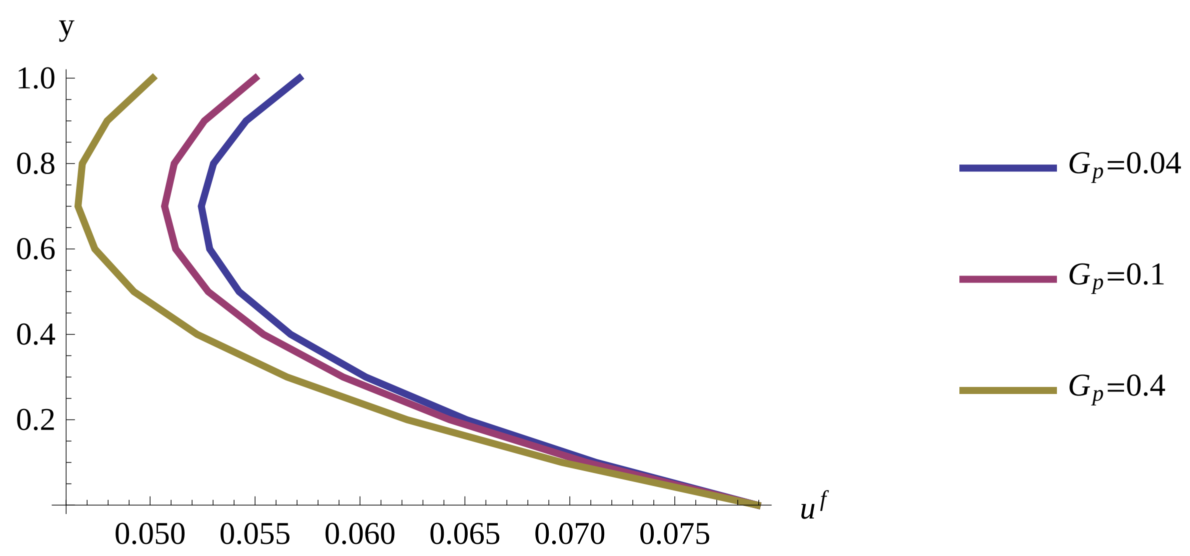

Figure 4: u^f varying with dimensionless time for different G_p

tendency to reduce friction forces which increases the velocity of groundwater and contaminant particle.

The effects of the particle mass parameter G_p on the velocity distribution of groundwater and contaminant particle are shown in Figures 4 and 5, respectively. The graph illustrates that the fluid phase velocity and solid phase velocity detends with an increase in particle mass parameter. This is due to the presence of drag force which slows down the motion of the groundwater and contaminants.

Figures 6 and 7 shows the effect of Peclet number on the dispersion coefficient for the case of

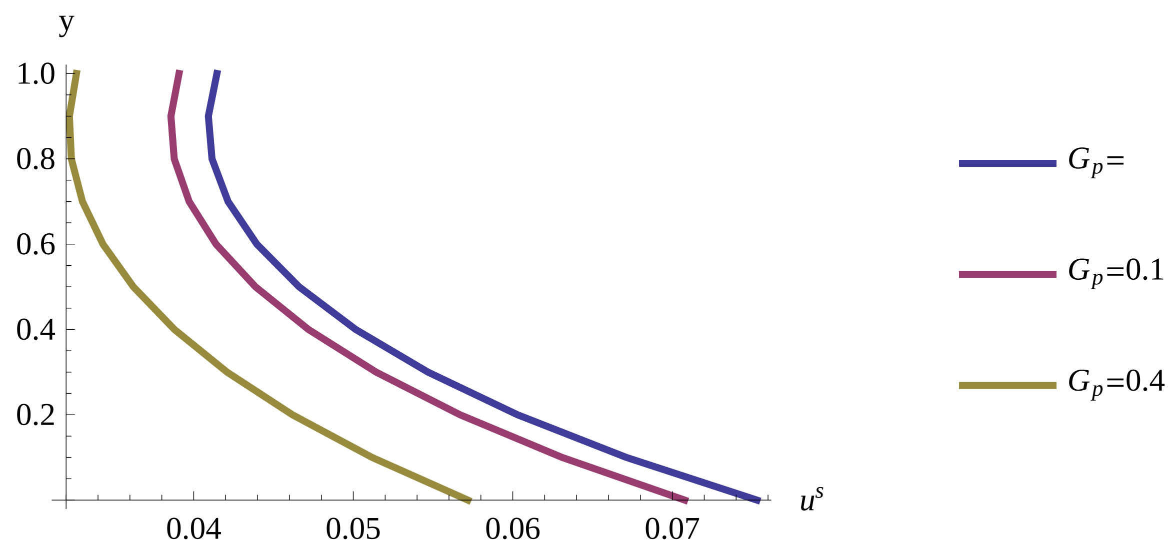

Figure 5: u^s varying with dimensionless time for different G_p

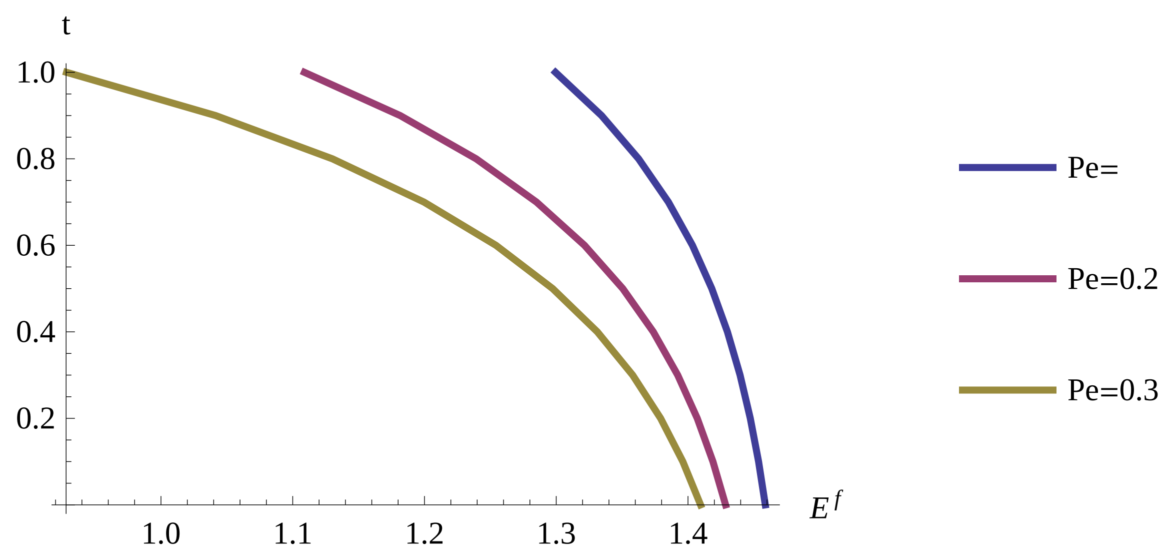

Figure 6: E^f varying with dimensionless time for different Pe

groundwater and contaminants. For a given time t, the effective dispersion coefficient diminishes with rise in Pe, because the increase in absorption at the boundaries helps to enlarge the number of moles in the reactive material undergoing chemical reaction and changing the amount of mass material across the channel.

Figure 8 is a plot of dispersion coefficient against the time of fluid for different values of reaction rate parameter β. For a given time t, the effective dispersion coefficient increases with rise in β. Figure 9 illustrates the effect of reaction rate parameter β on dispersion coefficient of contaminants. Here the dispersion goes along with the flow with rise in the reaction rate parameter, due to the shear effect of high frequency on dispersion.

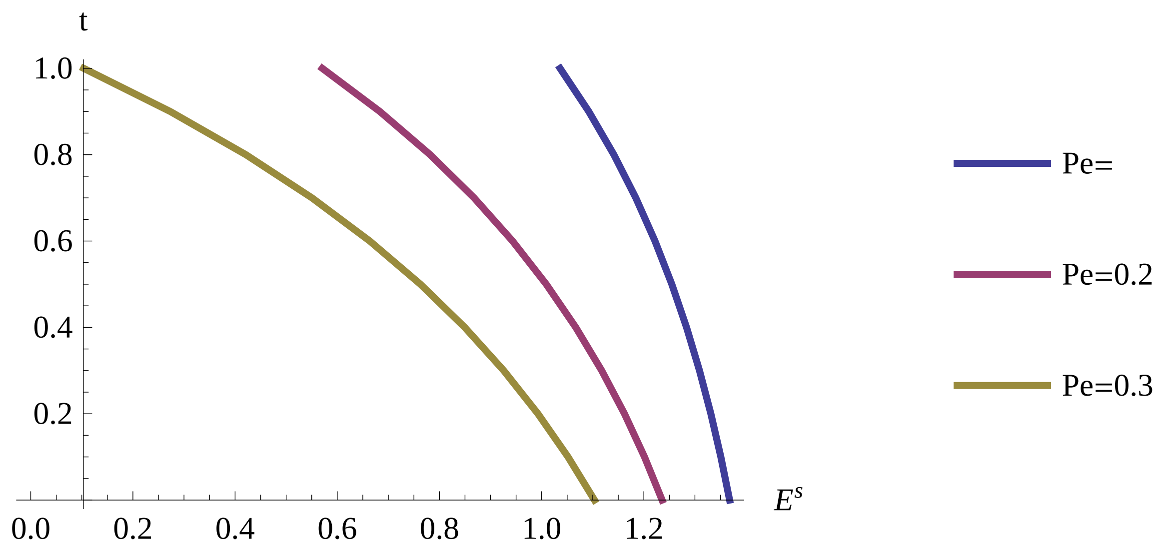

Figure 7: E^s varying with dimensionless time for contaminants with different Pe

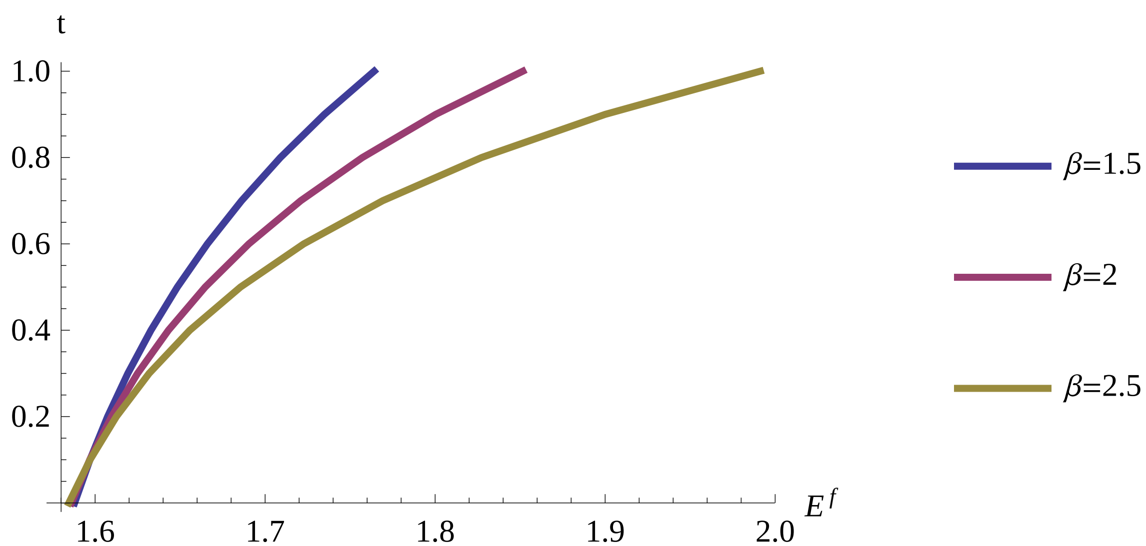

Figure 8: E^f varying with dimensionless time for different β

4. Conclusion

A contaminant transport model was developed for unsteady, saturated, geochemically heterogeneous porous media. The contaminant transport model in porous medium remains a key issue in the area of hydrology because various microorganisms frequently enter the aquifers either by accident and affects the environment. Aris method of moments is used in calculating the second moment about the mean of a dispersing contaminant. The reaction rate parameter β has strong impact on the transport of contaminant. Microorganisms migrating into and through soil from sources on the land surface may cause a serious threat to both ground and surface waters. The

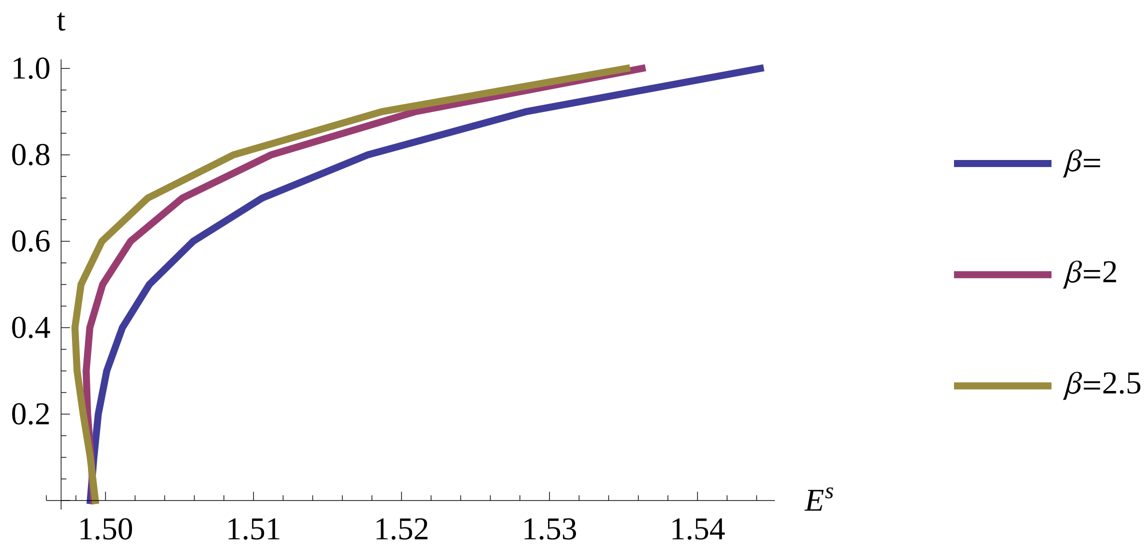

Figure 9: E^s varying with dimensionless time for contaminants with different β

solutions are useful for analyzing the possible prevention of the spread of chemical reacting contaminants over a flow of fresh water.

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