phys2523 (Giancoli's 4th Ed, 2008) ANS 22-1

Chapter 22 Gauss's Law

I] Assignments

Questions: 1-4, 6, 7, 11, 12,

Problems: 1, 3, 4, 5, 6, 9, 12, 13, 15, 17, 18, 20, 21, 24, 26, 29, 33, 34, 35, 36, 39, 45, 50, 52, 56, 58, 61, 63.

II] Solution to Selected Problems

22-1) $\Phi \cong \vec{A}\cdot\vec{E} = AE\cos\theta$

where the direction of $\vec{A}$ is along the normal of the surface.

a) when the face is $\perp$ to the field lines. ie: $\vec{A} \parallel \vec{E} \Rightarrow \theta = 0°$

$$\therefore \Phi_a = AE = \pi r^2 \cdot E$$
$$= \pi \times (0.13\,m)^2 \times 5.8\times10^2 \,\frac{N}{C}$$
$$= 0.308\times10^2 \,\frac{N\cdot m^2}{C} = 30.8\,\frac{N\cdot m^2}{C}$$

(b) $\theta' = 90° - 45° = 45°$

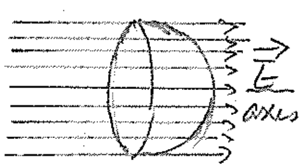
$$\Phi_b = \vec{A}\cdot\vec{E} = AE\cos\theta'$$
$$= AE\cos 45° = 30.8\times0.707\,\frac{N\cdot m^2}{C}$$
$$= 21.8\,\frac{N\cdot m^2}{C}$$

(c) the face $\parallel \vec{E} \Rightarrow \vec{A}$ (the normal) $\perp \vec{E}$

$\Rightarrow \theta'' = 90°$ $\quad \cos 90° = 0$

$$\therefore \Phi_c = AE\cos\theta'' = 0$$

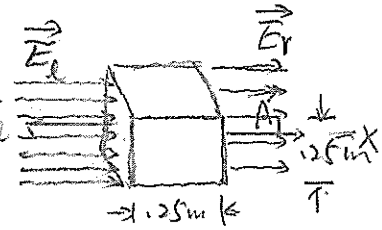
22-4)



Since $\vec{E}$ is $\parallel$ axis of the hemisphere & $\vec{E}$ is uniform. $\Rightarrow$ same amount of electric flux go through the circular flat surface and the curved hemisphere

$$\therefore \Phi_{\text{flat circle}} = EA_{\text{flat circle}}\cos 0° = \pi r^2 E$$

22-9) A cube of $\ell = 0.25\,m$



$E_\ell = 560\,\frac{N}{C}$, & $E_r = 410\,\frac{N}{C}$

$\vec{A}_\ell$ ($\perp$ to A_L & from inside out) is along $-\hat{i}$; & $\vec{A}_r$ is along $+\hat{i}$.

$$\therefore \Phi_\ell = \vec{E}_\ell\cdot\vec{A}_\ell = \hat{i}E_\ell\cdot(-\hat{i})A_\ell$$
$$= -E_\ell A_\ell = -560\,\frac{N}{C}\times(.25\,m)^2$$
$$= -35\,\frac{N\cdot m^2}{C}$$

("−" means flux enters the enclosed surface)

$$\Phi_r = \vec{E}_r\cdot\vec{A}_r = \hat{i}E_r\cdot(\hat{i})A_r$$
$$= E_rA_r = 560\,\frac{N}{C}\times(.25\,m)^2$$
$$= 35\,\frac{N\cdot m^2}{C}$$

For the other 4 surfaces, they all $\parallel$ to $\vec{E}$: that is the direction of these surfaces (along their respective normal) all $\perp$ to $\vec{E}$ $\Rightarrow \theta_{\vec{A}\cdot\vec{E}} = 90°$ $\quad \cos 90° = 0$

Φ the rest surfaces $= 0$

22-15) whenever r (distance from a wire) is $\ll$ L (the length of the wire), we can consider L as infinite.

use results of Example 22-6 or Gauss' Law

$$E(R) = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{R} \sim 2k\frac{\lambda}{R}$$

"Cylindrical Symmetry"

22-18) A metal sphere w/ $r_0 = 3.00\,m$, & $Q = -5.50\,\mu C = -5.50\times10^{-6}\,C$

⇒ ① Spherical symmetry; ② $E(r) = 0$, when $r \le 3.0\,m$

③ $E(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$ when $r \geq 3.00\,m$

(a) $r = 0.250\,m < r_0$ $E = 0$

(b) $r = 2.90\,m < r_0$ $E = 0$

(c) $E(3.10m) = k\frac{Q}{r^2} = 9.0\times10^{9}\frac{Nm^2}{C^2}\frac{-5.50\times10^{-6}C}{(3.10m)^2}$
$= -\frac{9.0\times5.50}{3.10^2}10^{9-6}\frac{N}{C} = 5.15\times10^{3}\frac{N}{C}$

(d) $E(8.00m) = 9.0\times10^{9}\frac{Nm^2}{C^2}\times\frac{-5.5\times10^{-6}C}{(8.00m)^2}$
$= -\frac{9.0\times5.5}{(8.00)^2}\times10^{9-6}\frac{N}{C} = 773\frac{N}{C}$

(e) If it was a thin shell. Ans will be the same from (a) to (d)

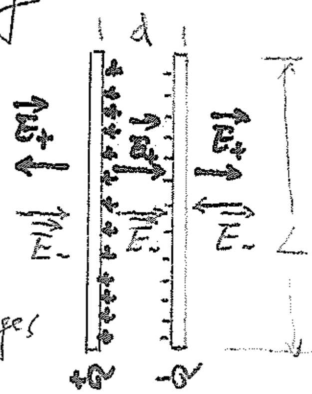
(f) If it was a nonconductor solid uniformly charged throughout

Then in (a) & (b) $r < r_0$
use (p596) $E = k\frac{Q}{r_0^3}r$ $(r < r_0)$
and in (c) & (d) $r > r_0$
use (p596) $E = k\frac{Q}{r^2}$ $(r > r_0)$

22-24) Planar Symmetry

Note: for planar charge if $d << L$
$E = \frac{\sigma}{2\epsilon_0}$

So the E created by the "+" charges & "–" charges in the 3 regions are shown. (and right)



(b) at left of the plates $\vec{E}_+ + \vec{E}_- = 0$

(a) in between plates $\vec{E} = \vec{E}_+ + \vec{E}_- = \hat{i}\,2\frac{\sigma}{2\epsilon_0} = \hat{i}\frac{\sigma}{\epsilon_0}$

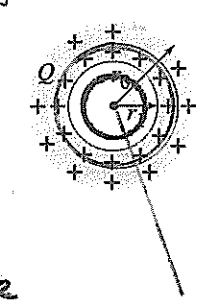
22-24) Continued

(b) at the right side of the plates
$\vec{E}_+ + \vec{E}_- = 0$

(c) If the two plates were non-conductors The results will be the same as long the thickness of the plate is very small compare to its height and width.

Also note Electric field inside the nonconducting plate is not zero.

22-29) A non-conducting spherical shell w/ inside radius r_1 and outside radius r_0.



(a) for $E(r)$ & $0 < r < r_1$

Consider a concentric sphere of $r < r_1$ using spherical symmetry (like 22-18) ② $E(r) = 0$ $0 < r < r_1$

(b) for $E(r')$ & $r_1 < r' < r_0$, consider a concentric sphere of $r_1 < r' < r_0$.

The key is to find Qenclosed in terms of total charge Q

$Q = \rho V = \rho\frac{4\pi}{3}(r_0^3 - r_1^3)$ &

$Q_{enclosed} = \rho V' = \rho\frac{4\pi}{3}(r'^3 - r_1^3)$

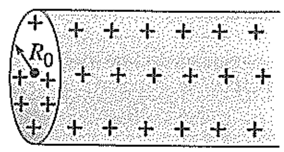
∴ $Q_{enclosed} = \left(\frac{r'^3 - r_1^3}{r_0^3 - r_1^3}\right)Q$

$\oint \vec{E}\cdot d\vec{A} =$
$= E(4\pi r'^2) = \frac{Q_{enclosed}}{\epsilon_0}$
$= \frac{r'^3 - r_1^3}{r_0^3 - r_1^3}Q/\epsilon_0$

∴ $E(r_1 < r' < r_0) = \frac{1}{4\pi\epsilon_0}\frac{(r'^3 - r_1^3)}{(r_0^3 - r_1^3)}\frac{Q}{r'^2}$

(c) for $r > r_0$ $E(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$

22-34) Cylindrical Symmetry: a long solid nonconducting cylinder is uniformly charged $\rho = Q/V$



(a) for $E(R>R_0)$ consider a co-axial cylinder of length l and radius $R>R_0$

$\oint \vec{E}\cdot d\vec{A} = E(2\pi R l)$ just like Example 22-6

and $Q_{encl} = \rho V = \rho(\pi R_0^2)\cdot l = \lambda l$

Since $\oint \vec{E}\cdot d\vec{A} = Q_{encl}/\epsilon_0$

$\therefore \quad E(2\pi R \cancel{l}) = \rho\pi R_0^2 \cancel{l}/\epsilon_0$

$\therefore \quad E(R>R_0) = \frac{1}{2\pi\epsilon_0}\frac{\rho\pi R_0^2}{R} = \frac{\lambda/R}{2\pi\epsilon_0}$

(b) for $E(R'<R_0)$, the evaluation of $\oint \vec{E}\cdot d\vec{A}$ is the same $\Rightarrow E(2\pi R')l$

but $Q_{encl} = \rho(\pi R'^2)l$

Since $\oint \vec{E}\cdot d\vec{A} = Q_{encl}/\epsilon_0$

$\therefore \quad E(2\pi R')\cancel{l} = \frac{\rho(\pi R'^2)\cancel{l}}{\epsilon_0}$

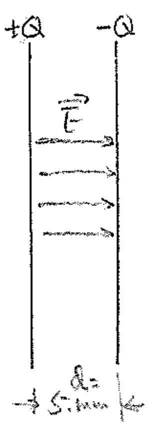
$\therefore \; E(R<R_0) = \frac{1}{2\pi\epsilon_0}(\rho\pi)R'$

22-45) $Q = 15\,\mu C = 1.5\times10^{-5}\,C$

$A = 1.0\,m^2 \quad \therefore \sigma = \frac{Q}{A} = 1.5\times10^{-5}\,\frac{C}{m^2}$

$\therefore E = \frac{\sigma}{\epsilon_0} = \frac{1.5\times10^{-5}\,\frac{C}{m^2}}{8.85\times10^{-12}\,\frac{C^2}{N\cdot m^2}}$

$= 1.69\times10^{6}\,\frac{N}{C}$



(a) $F = QE = 1.5\times10^{-5}\,C \times 1.695\times10^{6}\,\frac{N}{C}$

$= 25.4\,N$ attractive

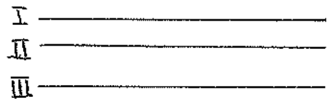
(b) E does not depends on d, as long as d << 1.0 m, so is F

$\therefore W = \int_i^f \vec{F}\cdot d\vec{l} \overset{\vec{F}\parallel d\vec{l}}{=} \int_i^f F\,dl \overset{F=C}{=} F\int_i^f dl$

$= F\cdot D = 25.4\,N \times (10-5)\times10^{-3}\,m$

$= 127\times10^{-3}\,J = 0.127\,J$

22-58) Three parallel sheets, all charged.



$\sigma_1 = 6.5\times10^{-9}\,C/m^2$

$\sigma_2 = -2.0\times10^{-9}\,C/m^2$

$\sigma_3 = 5.0\times10^{-9}\,C/m^2$

Let's just calculate the Force per unit area on sheet II. It will be attracted upward by σ_1 " " down by σ_3

$E_1(\downarrow) = \frac{\sigma_1}{\epsilon_0} = \frac{6.5\times10^{-9}\,C/m^2}{8.85\times10^{-12}\,\frac{C^2}{N\,m^2}} = 734\,\frac{N}{C}$

$E_3(\uparrow) = \frac{\sigma_3}{\epsilon_0} = \frac{5.0\times10^{-9}\,C/m^2}{8.85\times10^{-12}\,\frac{C^2}{N\,m^2}} = 565\,\frac{N}{C}$

$\therefore \; F_{on\,II} = Q_2(E_3 - E_1)$

$F_{II}/A = \frac{Q_2}{A_2}(E_3 - E_1) \cdot \sigma_2(E_3 - E_1)$

$= -2.0\times10^{-9}\,\frac{C}{m^2}(565-734)\,\frac{N}{C}$

$= +338\times10^{-9}\,\frac{N}{m^2} = 3.38\times10^{-7}\,\frac{N}{m^2}\,(\uparrow)$

The force on sheet I & III will be similarly calculated.