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A PARTIALLY DISCRETIZED AGE-DEPENDENT POPULATION MODEL WITH AN ADDITIONAL STRUCTURE

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Abstract

A semi-discretization method for solving an age-dependent population dynamics model with an additional structure is proposed. This method, unlike previous ones, considers the partial discretization which reduces the model equation into a first order ordinary differential equation. The latter is then solved explicitly and conditions under which second order accuracy arises are given. While the approach adopted is basically analytical, the main result shows that the sum of errors is bounded. An extension to the non-trivial case where growth depends on the additional parameter leads to a Riccati equation, and the existence and convergence of solutions are proved.

Key words: age structure, discrete scheme, population dynamics, physiology, convergence.

MSC 2000: 92D25, 65M15, 65M25, 35L45

1. Introduction

Structured population models combine knowledge of individuals in the population – its basic unit – and the study of the higher organization level: the population (Abia, et al. 2004). In other words, their purpose consists in reflecting the dependence on the individual physiological states of the dynamics of the whole population, usually conceived as a frequency distribution of individuals which evolves over time (Abia, et al. 2004). This effect is introduced into the model by *structuring* the population, i.e. classifying the individuals by various continuous, internal variables which represent a particular physiological feature. Age-dependent population dynamics models are of interest in population ecology, since they are closer to reality and there is considerable literature on their analysis. Models of this type have been proposed by Sinko and Streifer (1967), Bell and Anderson (1967), Oster and Takahasi (1974), Chichia (1990) and the references therein).

Partial differential equations (PDEs) form the basis of very many mathematical models of physical, chemical and biological phenomena. Their use has spread more recently into economics, financial forecasting and other fields (Morton and Mayers, 1996). To investigate the predictions of these models, it is often necessary to approximate the solution of these PDEs numerically, combined with the analysis of simple special cases. Because of their practical applications, numerical approaches to the problem of population dynamics are very important and unavoidable for most realistic cases in order to get some quantitative information from the model. Also, knowledge of the qualitative behaviour of solutions requires numerical methods to approximate essential parameters. The motivation for this study comes from the fact that better methods that handle the Sinko-Streifer's equation would be of great value in modelling many biological systems. The following comments on related works provide the context for the present paper.

There has been much investigation into numerical methods for solving models with just age structure (Ayati, 2000). However, Slobodkin (1953) observed long ago that for many organisms or biological systems, a difference in age or size taken separately does not explain the differences in individual behaviour. The age variable alone has a limited practical value due to the fact that age is very difficult to measure experimentally in a large number of species (Abia, et al. 2004). This has led to physiologically structured population models (Kooi and Kelpin, 2003). An extensive study with discussion of the biological background of can be found in Metz and Diekmann (1986) and Zhao (2003).

Age and space structured models are applicable to problems in ecology, epidemiology, population genetics and cell growth. For models with both age and space structure, Milner (1990) developed a method for population that diffuse to avoid crowding. Kim (1996), Kim and Park (1995), and Lopez and Trigiante (1985) developed methods for random dispersal. All these methods involve uniform time and age discretizations, with age step chosen to equal the time step. Ayati (2000) allows for variable time steps and independent age and time discretizations with spatial diffusion. He argued that the use of moving age discretization that allows for a non uniform age and time preserves the important fact that age and time advance together, and the advantage of having variable time steps is the ability to adaptively choose the time steps to assure robustness and efficiency, while the advantages of independent age and time discretizations are fewer computations and less memory use when the dependence on age is weak relative to the dependence on time.

Extensive description of numerical methods for the time integration of structured population models are given in the literature (Kooi and Kelpin, 2003). In full discretization schemes, time and state space are discretized simultaneously while in classical finite difference schemes (Richtmeyer and Morton, 1967), derivatives are replaced by differential quotients based on Taylor series expansion in grid points. Sulsky (1993) used the classical Lax-Wendroff method (which is a very popular explicit method in fluid dynamics (Abia, et al. 2000)) for the non-linear density dependent age structured models and later for size-structured models (Sulsky, 1994). He applied a fixed grid and the resulting scheme is an adapted version of the classical Lax-Wendroff method. For age-structured models, the support for the density distributions is known for the initial