Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Vol. 4, No. 1 (June 2009) pp. 40– 51

Applications and Applied Mathematics:
An International Journal
(AAM)

A New Approach to Improve Inconsistency in the Analytical Hierarchy Process

Morteza Rahmami*
Technology Development Institute (ACECR)
P.O. Box: 13445-1668
Tehran, Iran
rahmanimr@yahoo.com

Hamidreza Navidi
Department of Mathematics
Shahed University
P.O. Box: 18151-159
Tehran, Iran
navidi@shahed.ac.ir

Received: June 22, 2008; Accepted: March 13, 2009

Abstract

In this paper, a new approach based on the generalized Purcell method for solving a system of homogenous linear equations is applied to improve near consistent judgment matrices. The proposed method relies on altering the components of the pairwise comparison matrix in such a way that the resulting sequences of improved matrices approach a consistent matrix. The complexity of the proposed method, together with examples, shows less cost and better results in computation than the methods in practice.

Keywords: Inconsistency; Homogenous Linear Equations; Generalized Purcell Method; Judgment Matrix; Reciprocal matrix; AHP Method.

MSC (2000) No.: 90C08, 90C59

1. Introduction

Saaty (1977) has introduced a procedure for prioritizing decision alternatives based on a positive matrix, $A = (a_{ij})$ of pairwise comparisons for n > 2 items expressed on a ratio scale. This method, known as the Analytical Hierarchy Process (AHP), has been used extensively in related literature. It takes $a_{ij} > 0$ as an estimate of the perceived intensity of an agent's preference in

*Corresponding Author

favor of alternative i versus j. Thus, the judgment matrix $A = (a_{ij})$ is reciprocal, namely $a_{ij} = 1/a_{ji}$. In applications of the AHP to real decision-making problems, the entries in the above reciprocal matrix are taken from the finite set: $\{1/9, 1/8, \cdots, 1, 2, \cdots, 8, 9\}$, which has been suggested by Saaty (1996) to be appropriate. Saaty (1977) argues that an appropriate solution to this problem is given by the non-negative normalized column vector $w = (w_1, w_2, \cdots, w_n)^T$ that satisfies $Aw = \lambda_{\max} w$, in which $\lambda_{\max}$ is the largest modulus eigenvalue of the matrix A. The Perron-Frobenius theorem insures that $\lambda_{\max}$ is real and positive and the components of the corresponding right-eigenvector have same sign. The latter vector w is thus unique, under the constraints that its components are nonnegative and their sum is equal to one [Aupetit and Genest (1993)]. It is known that for a $n \times n$ positive reciprocal matrix A, $\lambda_{\max} \geq n$ (Saaty, 1996). Furthermore, A is said to be consistent if $a_{ij} = a_{ik} * a_{kj}$ for all $i, j, k = 1, 2, \cdots, n$. Using this definition, Saaty (2003) has shown that A is consistent matrix if only if $\lambda_{\max} = n$.

In order to find w, Saaty proposed the initial solution method based on solving an eigenvalue problem [Saaty (1977, 1980)]. Specific error measures were adopted in two other popular methods; The Least Squares Approach chooses w to minimize the sum of squared differences and the Row Geometric Mean method minimizes the sum of squares of differences of the logarithms of the values [Saaty and Vargas (1984), Moody (1998)].

Almost simultaneously, there were efforts to modify elements of a reciprocal pairwise comparison matrix in such a way that it approaches a consistent matrix. Harker (1987) derived explicit formula for first and second partial derivative of Perron root of a positive reciprocal matrix and used these results to direct a decision maker toward more consistent judgments. This problem has received some attention recently [Genets and Zhung (1996), Saaty (1998, 2003), Gass and Rapcsak (2004), and others]. Also, Dahl (2005) uses a multiplicative approach based on the entry wise logarithmic transformation to approximate a reciprocal matrix. The complexity of Dahl's algorithm is $O(n^3)$.

The rest of this paper is organized as follows: In section two, the theoretical background of a new method is given to solve the homogeneous system of linear equations. Section three will then show a new approach to determine the vector w based on the best nontrivial solution of $\min_{w>0} \|(A^{(i)} - nI)w\|_2$, where $A^{(0)} = A$. Then, we try to correct the most inconsistent judgment of $A^{(i)}$, $i = 0, 1, \cdots$, in a systematic manner by iterative steps. In final section, the numerical results and comparisons of a few examples will be presented [Saaty (2003, 1980), Dahl (2005)].

2. Theoretical Background

In this section, first some properties of positive reciprocal matrices will be given. Then, a new method [Rahmani and Momeni (2009)], based on Purcell method for solving simultaneous linear systems [Purcell (1953)], will be provided to solve a homogeneous system of linear equations. In the following, all vectors are column vectors.

2.1. Properties of Positive Reciprocal Matrices

Perron's Theorem shows that any positive reciprocal matrix A has a largest eigenvalue $\lambda_{\max}$ that is real and positive. The corresponding eigenvalue problem $Ax = \lambda_{\max} x$ has a solution as x with $x_i > 0$ for all i. The eigenvalue $\lambda_{\max}$ always satisfies $\lambda_{\max} \geq n$, and $\lambda_{\max} = n$ if and only if consistency holds [Saaty (2003)].

Let s_k be the sum of the components of the column k of the reciprocal matrix A. Multiply each column k of A by the s_k^{-1}, and denote the resulting matrix by $\tilde{A}$. The following theorems show that the matrix $\tilde{A}$ and $\sum s_i^{-1}$ can be used to verify consistency of A. In results section, because of simplicity, we use $\sum s_i^{-1}$ to test the convergence of A to a consistent matrix [Stein and Mizzi (2007)].

Theorem 2.1.

Let A be a positive reciprocal matrix. The matrix A is consistent if and only if all columns of $\tilde{A}$ are identical.

Theorem 2.2.

Let A be a reciprocal matrix with s_j the sum of column j. Then, $\sum s_i^{-1} \leq 1$.

Theorem 2.3.

The reciprocal matrix A is consistent if only if $\sum s_i^{-1} = 1$.

The following theorem has been proved by Saaty (1996, p. 50). Here we present a new proof.

Theorem 2.4.

The consistent reciprocal matrix has $\lambda_{\max} = n$ and its rank is one.

Proof:

Suppose $\{e_i\}_{i=1}^n$ be the standard basis and $A = (a_{ij})$ be a consistent reciprocal matrix. Set $w_i = a_{i1}$. Thus, $a_{1i} = w_i^{-1}$ and $a_{ij} = w_i / w_j$. Now, let $W = (W_1, W_2, \cdots, W_n)^T$ such that

$$W_1 = w_1 e_1 \text{ and } W_i = w_i\,(e_1 + e_i)\,,\text{ for } i \neq 1.$$

Then,

$$W^{-1}AW = (ne_1, e_1, e_1, \cdots, e_1)\,.$$

Clearly, the matrix A has only one nonzero eigenvalue equal to n and the corresponding eigenvector is $w = We_1 = (w_1, w_2, \cdots, w_n)^T$. Also the rank of A is equal to 1.

2.2. Generalized Purcell Method for Solving a Homogeneous System of Linear Equations

To solve the homogeneous system of linear equations $\mathbf{A}_{n\times n}\mathbf{x} = 0$, let $\{\mathbf{A}_i : \ i = 1, 2, \cdots, n\}$ be the column vectors of $\mathbf{A}^T$ and $E^1 = \{\mathbf{e}_i^1 : i = 1,2,\cdots,n\}$ be the standard basis for space R^n. Let, by induction, in the step j we have

$$E^j = \{\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_{j-1}, \mathbf{e}_j^j, \mathbf{e}_{j+1}^j, \cdots, \mathbf{e}_n^j\}.$$

To make the set E^{j+1}, the vector $\mathbf{e}_j^j$ is selected and replaced by $\mathbf{A}_j$ such that $\mathbf{A}_j$ will be orthogonal to new vectors $\mathbf{e}_i^{j+1}$ in E^{j+1} for $i = j+1, \cdots, n$. For this purpose let

$$\alpha_{ij} = (\mathbf{e}_i^j, \mathbf{A}_j)/(\mathbf{e}_j^j, \mathbf{A}_j)$$

and set

$$\mathbf{e}_i^{j+1} = \mathbf{e}_i^j - \alpha_{ij}\mathbf{e}_j^j, \quad i = j+1, j+2, \cdots, n\,.$$

Define the set $E_\perp^j = \{\mathbf{e}_i^j \in E^j : \mathbf{A}\mathbf{e}_i^j = 0\}$ as the solution of $\mathbf{A}_{n\times n}\mathbf{x} = 0$ in the step j .

Lemma 2.1.

If $\mathbf{A}_j$ is a linearly dependent vector from $\{\mathbf{A}_k : 1 \le k < j\}$, then $E^{j+1} = E^j$.

In algorithm process to avoid division by zero, we select $\mathbf{A}_j$ such that $j = \arg\max_{j \le k \le n} \left|(\mathbf{A}_k, \mathbf{e}_j^j)\right|$. We call this selection process ***row pivoting***.

Lemma 2.2.

If $\max_{j\le k\le n}\left|\left(A_k,\mathbf{e}_j^j\right)\right|=0$ then $\mathbf{e}_j^j$ is a solution for $Ax=0$ and so $E_\perp^{j+1}=E_\perp^j\cup\{\mathbf{e}_j^j\}$.

Corollary 2.1.

If $Ax=0$ has non zero solution, then in step $j=n$ we have $\mathbf{e}_n^n\in E_\perp^n$, else $\mathbf{A}e_n^n=(A_n,e_n^n)$.

In order to assess the relative effectiveness of the method it is useful to have an upper bound for the relative error in the computed $\tilde{e}_n^n$. The theorem 5 gives this upper bound.

Theorem 2.5.

Let ε be a least upper bound of the computation error for α_{ij}, $\boldsymbol{\delta}_{\mathbf{e}_n^n}$ be the relative computation error of $\mathbf{e}_n^n$ and suppose $\beta=\max_{1\le i\le n}\frac{\|\tilde{e}_i^i\|_2}{\|\tilde{e}_n^n\|_2}$, then $\boldsymbol{\delta}_{\mathbf{e}_n^n}$ is less than or equal to $\beta\varepsilon(1-\varepsilon)^{-1}$.

Proof:

By row pivoting method we have

$$\left|\alpha_{ij}\right|=\left|\left(A_j,\mathbf{e}_i^j\right)\right|/\left|\left(A_j,\mathbf{e}_j^j\right)\right|<\infty.$$

Let $\alpha_{ij}=\tilde{\alpha}_{ij}+\varepsilon_{ij}$ for all i, j. Then,

$$\begin{aligned}\mathbf{e}_n^n&=\mathbf{e}_n^{n-1}-\alpha_{n-1,n-1}\mathbf{e}_{n-1}^{n-1}=\mathbf{e}_n^{n-1}-\left(\tilde{\alpha}_{n-1,n-1}+\varepsilon_{n-1,n-1}\right)\mathbf{e}_{n-1}^{n-1}\\&=\mathbf{e}_n^{n-1}-\tilde{\alpha}_{n-1,n-1}\mathbf{e}_{n-1}^{n-1}-\varepsilon_{n-1,n-1}\mathbf{e}_{n-1}^{n-1}=\tilde{\mathbf{e}}_n^n-\varepsilon_{n-1,n-1}\mathbf{e}_{n-1.}^{n-1}\end{aligned}$$

That $\tilde{\mathbf{e}}_i^i$ is the computational value of $\mathbf{e}_i^i$. So, by recursion we get

$$\mathbf{e}_n^n=\tilde{\mathbf{e}}_n^n-\varepsilon_{n-1,n-1}\tilde{\mathbf{e}}_{n-1}^{n-1}+\varepsilon_{n-1,n-1}\varepsilon_{n-2,n-2}\tilde{\mathbf{e}}_{n-2}^{n-2}-\ldots,$$

and finally

$$\mathbf{e}_n^n-\tilde{\mathbf{e}}_n^n=\sum_{i=1}^{n-1}(-1)^i\left(\prod_{j=1}^{i}\varepsilon_{n-j,n-j}\right)\tilde{\mathbf{e}}_{n-i}^{n-i},$$

or

$$\delta_{e_n^n} \cong \frac{\left\| \mathbf{e}_n^n - \tilde{\mathbf{e}}_n^n \right\|_2}{\left\| \tilde{e}_n^n \right\|_2} \leq \beta \sum_{i=1}^{n-1} \varepsilon^i = \beta\varepsilon(1-\varepsilon)^{-1} .$$

Remark

The number of operations in Purcell method in step j is equal to $n(2j+1)-2j^2$. So, $\sum_{j=1}^{n}(n(2j+1)-2j^2) \approx \frac{1}{3}n^3$. Thus, the complexity of Purcell method is $O(\frac{1}{3}n^3)$.

3. Improving Inconsistency

We assume that the judgment matrix A is obtained as a small perturbation of an underlying consistent matrix constructed from a ratio scale $w = (w_1, \cdots, w_n)$. A near consistent matrix is a small reciprocal multiplicative perturbation of a consistent matrix. It is given $A = (\frac{w_i}{w_j}\varepsilon_{ij})$ where $\varepsilon_{ji} = \varepsilon_{ij}^{-1}$, and all elements of (ε_{ij}) are close to one [Saaty (2003)]. In order to obtain a consistent matrix, the most inconsistent entries of the near consistent matrix A, sequentially are corrected, in such a way that its eigenvalue approaches n. In this way, a sequence of near consistent matrices $A^{(0)}, A^{(1)}, A^{(2)}, \cdots$ has been constructed, such that

$$\lambda_{\max}(A^{(0)}) \geq \lambda_{\max}(A^{(1)}) \geq \lambda_{\max}(A^{(2)}), \cdots$$

and

$$\lim_{i\to\infty} \lambda_{\max}(A^{(i)}) = n .$$

Theorem 3.1.

Let $\{A^{(i)}\}$ be a convergent sequence of near consistent matrices such that

$$\overline{A} = \lim_{i\to\infty} A^{(i)} \text{ and } \lambda_{\max}(\overline{A}) = n .$$

Thus, $\overline{A}$ is a consistent matrix.

Proof:

Let $w=(w_1,w_2,\cdots,w_n)^T$ be the eigenvector of $A^{(i)}$ corresponding to $\lambda_{\max}(A^{(i)})$ and let (ε_{ij}) be such that $(a_{ij})^{(i)}=(\frac{w_i}{w_j}\varepsilon_{ij})$. By definition of $A^{(i)}w=\lambda_{\max}w$, for component l of $A^{(i)}w$ we have

$$\sum_{j=1}^{n}a_{lj}w_j=\lambda_{\max}w_l.$$

Thus, we have a simple straightforward relation

$$\lambda_{\max}=\tfrac{1}{n}\sum_{l=1}^{n}\tfrac{\lambda_{\max}w_l}{w_l}=\tfrac{1}{n}\sum_{l=1}^{n}\tfrac{1}{w_l}\sum_{j=1}^{n}a_{lj}w_j=\tfrac{1}{n}\sum_{l=1}^{n}\sum_{j=1}^{n}a_{lj}\tfrac{w_j}{w_l}=\tfrac{1}{n}\sum_{l=1}^{n}\sum_{j=1}^{n}\varepsilon_{lj}$$

$$=1+\tfrac{1}{n}\sum_{l>j}(\varepsilon_{lj}+\varepsilon_{lj}^{-1})\geq 1+\tfrac{1}{n}(n^2-n)=n.$$

So, $\lambda_{\max}$ is a convex function of ε_{ij}, greater than or equal to n and reaches its minimum point n at $\varepsilon_{ij}=1$ for all i,j. This means that $\overline{A}$ is a consistent matrix. Refer to Saaty (2003) for alternative proof with syntax error.

In practice, we try to find a nonzero solution of near homogenous equations $(A^{(i)}-nI)x\approx 0$ by the row pivoting method. Let $x^{(i)}$ be such solution and set $\beta_i=\left\|(A^{(i)}-nI)x^{(i)}\right\|_2$. Then, by altering appropriately the entries of $A^{(i)}$ under positive reciprocal condition, we expect that the sequence β_i satisfies $\beta_0\geq\beta_1\geq\beta_2\geq\cdots$ and $\lim_{i\to\infty}\beta_i=0$.

Corollary 3.1.

If the sequence $A^{(i)}$ is near consistent matrices and $\lim_{i\to\infty}\beta_i=0$ then $\overline{A}$ is consistent matrix and $w=\lim_{i\to\infty}x^{(i)}$ is its principal eigenvector.

Here, we give an experimental approach based on minimizing the maximum error to improve the elements of near consistent matrix $A^{(i)}$ in order to achieve a consistent matrix. By using the method of section 2, let w and v are the solution of near homogenous equations $(A^{(i)}-nI)x\approx 0$ and $(A^{(i)}-nI)^T x\approx 0$, respectively. Then, we can set $a_{ij}\approx\frac{w_i}{w_j}\varepsilon_{ij}$ and $a_{ij}\approx\frac{v_j}{v_i}\varepsilon_{ij}$ or

$$\left(\varepsilon_{ij}\right)\approx\tfrac{1}{2}(a_{ij}\tfrac{w_j}{w_i}+a_{ij}\tfrac{v_i}{v_j})=\tfrac{1}{2}a_{ij}(\tfrac{w_j}{w_i}+\tfrac{v_i}{v_j})$$

Let $B=2\varepsilon$, then

$$B=A^{(i)}\circ(vv^{-T}+w^{-1}w^T),$$

in which $\circ$ implies the Hadamard product. Now, select $kl = \arg\max_{ij} b_{ij}$. If $A^{(i)}$ is consistent, then $b_{kl} = 2$, else inconsistency of a_{kl} is greater than other elements of $A^{(i)}$. To improve a_{kl}, replace a_{kl} in $A^{(i+1)}$ by geometric means of $\frac{w_k}{w_l}$ and $\frac{v_l}{v_k}$ i.e. $\sqrt{(w_k v_l)/(w_l v_k)}$, and continue the process. This Algorithm is given in Table 1.

Table 1. Algorithm to Improve the Elements of Near Consistent Matrix

Step1: *Compute vectors* $w \neq 0$ *and* $v \neq 0$ *such that* $(A^{(i)} - nI)w \approx 0$ *and* $(A^{(i)} - nI)^T v \approx 0$.

Step2: *Set* $kl = \arg\max_{ij} a_{ij} (\frac{v_i}{v_j} + \frac{w_j}{w_i})$.

Step3: *Set* $a_{kl} = \min\left(\max\left(\sqrt{(w_k v_l)/(w_l v_k)}, 1/M\right), M\right)$ *and* $a_{lk} = 1/a_{kl}$ *in matrix* $A^{(i+1)}$, *where* $1 \leq M \leq 9$ *is a problem-specific bound.*

Step4: *If* $\lim_{i\to\infty} \beta_i \approx 0$ *set* $\overline{A} = A^{(i+1)}$ *and stop, else set* $i = i+1$ *and go to step 1, where* $\beta_i = \min \left\| (A^{(i)} - nI) w \right\|_2$.

In the following examples, the consistency ratio is defined as $CR = CI / RI$, in which $CI = (\lambda_{\max} - n)/(n-1)$ and RI are called the consistency index and random consistency Index, respectively. *RI* is the mean consistency index of randomly generated reciprocal matrices (Saaty (1996)). Table 2 represents the random consistency index for matrices of orders $n \leq 10$.

Table 2. The random consistency indexes for $n \leq 10$

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

4. Numerical Examples

To elucidate this presentation and test the accuracy of the proposed method, here we consider a few examples.

a) In the first example, prioritizing the wealth of nations through their world influence Wealth of Nations (1976) is the problem. Table 3 indicates the pairwise comparisons of the seven countries with respect to wealth [Saaty (1996)].

Table 3. Wealth of Nations (1976)

	U.S.	U.S.S.R.	China	France	U.K.	Japan	W. Germany
U.S.	1	4	9	6	6	5	5
U.S.S.R.	.25	1	7	5	5	3	4
China	.11	0.14	1	0.2	0.2	0.14	0.2
France	0.17	0.2	5	1	1	0.33	0.33
U.K.	0.17	0.2	5	1	1	0.33	0.33
Japan	0.2	0.33	7	3	3	1	2
W. Germany	0.2	0.25	5	3	3	0.5	1

Table 4 presents the normalized eigenvector that used by Saaty (1996) along with the results of the proposed method. The actual Gross National Products fraction (GNP) is given in the last column.

Table 4. Priorities of Nation's Wealth

	Normalized Eigenvector (Saaty,1996)	Iteration 1.	Iteration 2.	Iteration 3.	Iteration 4.	Iteration 5.	Iteration 8.	Fraction of GNP
U.S.	0.427	0.446	0.411	0.415	0.420	0.420	0.396	0.413
U.S.S.R.	0.230	0.234	0.253	0.237	0.219	0.220	0.207	0.225
China	0.021	0.011	0.012	0.013	0.014	0.015	0.017	0.043
France	0.052	0.049	0.051	0.052	0.053	0.053	0.058	0.069
U.K.	0.052	0.049	0.051	0.052	0.053	0.053	0.058	0.055
Japan	0.123	0.120	0.125	0.134	0.137	0.128	0.140	0.104
W. Germany	0.094	0.092	0.096	0.097	0.105	0.111	0.124	0.091
λ_{max}	7.608	7.608	7.516	7.480	7.432	7.401	7.366	
$C.R.$	0.08	0.08	0.065	0.061	0.055	0.052	0.046	
$\sum s_i^{-1}$	0.945	0.945	0.966	0.969	0.973	0.976	0.980	
The most inconsistent element	-------	(1,2)	(2,6)	(2,7)	(6,7)	(1,6)	(2,4)	

b) Consider an example involving the prioritization of criteria used to buy a house for a family [Saaty (2003)]. Table 5 gives the pairwise comparison matrix.

Table 5. Pairwise Comparison Matrix to Buy a House

	Size	Trans.	Nbrhd	Age	Yard	Modern	Cond.	Finance
Size	1	5	3	7	6	6	1/3	1/4
Trans.	1/5	1	1/3	5	3	3	1/5	1/7
Nbrhd	1/3	3	1	6	3	4	6	1/5
Age	1/7	1/5	1/6	1	1/3	1/4	1/7	1/8
Yard	1/6	1/3	1/3	3	1	1/2	1/5	1/6
Modern	1/6	1/3	1/4	4	2	1	1/5	1/6
Cond.	3	5	1/6	7	5	5	1	1/2
Finance	4	7	5	8	6	6	2	1

In Table 6, the first column is the principal normalized eigenvector of the pairwise comparison matrix of Table 5. The second column is the principal normalized eigenvector of the modified pairwise comparison matrix that used the method proposed by Saaty (2003). Other columns are the results of the proposed method.

Table 6. Results of Prioritization of Criteria to Buy a House

	Normalized Eigenvector (Saaty,2003)	Modified Nor.Eig. (Saaty,2003)	Iteration 1.	Iteration 2.	Iteration 3.	Iteration 4.	Iteration 5.	Iteration 6.
Size	0.173	0175	0.175	0.177	0.158	0.159	0.159	0.170
Trans.	0.054	0.062	0.043	0.056	0.059	0.053	0.054	0.049
Nbrhd	0.188	0.103	0.200	0.119	0.130	0.131	0.117	0.117
Age	0.018	0.019	0.010	0.004	0.006	0.007	0.009	0.010
Yard	0.031	0.034	0.027	0.031	0.033	0.033	0.034	0.036
Modern	0.036	0.041	0.029	0.037	0.039	0.042	0.042	0.043
Cond.	0.167	0.221	0.167	0.209	0.207	0.207	0.220	0.220
Finance	0.333	0.345	0.370	0.366	0.369	0.368	0.366	0.365
$\lambda_{\max}$	9.669	8.811	9.669	8.912	8.818	8.739	8.669	8.61
$C.R.$	0.17	0.08	0.17	0.09	0.08	0.08	0.07	0.06
$\sum s_i^{-1}$	0.847	--------	0.847	0.929	0.940	0.942	0.958	0.959
The most inconsistent element	(3,7)	---------	(3,7)	(1,3)	(2,6)	(3,7)	(2,5)	(1,2)

c) As a final example, consider the Saaty's matrix, which is related to a decision-making situation for national welfare including criteria such as inflation, unemployment, growth, domestic stability and foreign relation [Dahl (2005)]

$$A = \begin{pmatrix} 1.00 & 3.00 & 5.00 & 4.00 & 6.00 \\ 0.33 & 1.00 & 4.00 & 4.00 & 6.00 \\ 0.20 & 0.25 & 1.00 & 2.00 & 2.00 \\ 0.25 & 0.25 & 0.50 & 1.00 & 2.00 \\ 0.17 & 0.17 & 0.50 & 0.50 & 1.00 \end{pmatrix}.$$

Priority vectors shown in table 7 obtained using the eigenvector method, the approach suggested by Farkas et al. (2003), Dahl's (2005) approach and the proposed method respectively. The solution obtained from the proposed method resembles that of Saaty's method.

Table 7. National Welfare Criteria and Their Priorities

	Inflation	Unemployment	Growth	Domestic stability	Foreign relation	Distance from Saaty's method
Farkas et al.'s method	0.4027	0.3531	0.0895	0.0929	0.0617	0.1747
Saaty's method	0.4767	0.2865	0.1029	0.0819	0.0520	--------
Dahl's method	0.5020	0.2619	0.1025	0.0802	0.0535	0.0535
Proposed method	0.4873	0.2869	0.1035	0.0817	0.0407	0.0231

By using the proposed method, the consistent matrix $\overline{A}$, which is obtained in 32 steps is as follows

$$\overline{A} = \begin{pmatrix} 1.00 & 1.00 & 3.99 & 4.00 & 6.00 \\ 0.99 & 1.00 & 3.98 & 4.00 & 6.00 \\ 0.25 & 0.25 & 1.00 & 1.01 & 1.51 \\ 0.25 & 0.25 & 0.99 & 1.00 & 1.50 \\ 0.17 & 0.17 & 0.66 & 0.67 & 1.00 \end{pmatrix}.$$

Principal Eigenvector of $\overline{A}$ is $w = (0.375 \;\; 0.374 \;\; 0.094 \;\; 0.094 \;\; 0.062)$ and Table 8 presents the $\sum s_i^{-1}$ values.

Table 8. Values of $\sum s_i^{-1}$

Steps 1-4	0.963795	0.984936	0.988476	0.995556
Steps 5-8	0.996240	0.997857	0.998167	0.998387
Steps 9-12	0.998674	0.999440	0.999531	0.999626
Steps 13-16	0.999673	0.999812	0.999848	0.999872
Steps 17-20	0.999897	0.999957	0.999966	0.999973
Steps 21-24	0.999976	0.999977	0.999992	0.999994
Steps 25-28	0.999994	0.999994	0.999998	0.999998
Steps 28-32	0.999999	0.999999	0.999999	1.000000

5. Conclusion

In this paper a new method is presented to improve an inconsistent judgment matrix. Computational experiments show that in our approach the $\sum s_i^{-1}$ as consistency criteria is monotonically increasing to one. Our next research will be to try finding the systematic relationship between the behaviors of $\sum s_i^{-1}$ and the new approach to improve inconsistency.

REFERENCES

Aupetit, B. and Genest, C. (1993). On some useful properties of the Perron eigenvalue of a positive reciprocal matrix in the context of analytic hierarchy process. European Journal of Operational Research,Vol. 70, pp. 263-268.

Dahl, G. (2005). A method for approximating symmetrically reciprocal matrices by transitive matrices. Linear Algebra and Its Applications, Vol. 403, pp. 207-215.

Farkas, A., Lancaster, P. and Rozsa, P. (2003). Consistency adjustments for pairwise comparison matrices. Numerical Linear Algebra Applications, Vol. 10, pp. 689-700.

Gass, S.I. and Rapcsak, T. (2004). Singular value decomposition in AHP. European journal of operation research, Vol. 154, pp. 573-584.

Genets, C. and Zhung, S. (1996). A graphical analysis of ratio- scaled paired comparison data. Management Science, Vol. 42, No 3, pp. 335-349.

Harker, P. T. (1987). Derivatives of the Perron root of a positive reciprocal matrix: with application to the analytic hierarchy process. Applied Mathematics and Computation, Vol. 22, pp. 217-232.

Moody, Chu, T. (1998). On the optimal consistent approximation to pair wise comparison matrices. Linear Algebra and Its Applications, Vol. 272, pp. 155-168.

Purcell, E. W. (1953). The vector method of solving simultaneous linear systems, J. Math. Phys. Vol. 32 pp. 180-183.

rahmani M., and. Momeni-Masuleh, S.H. (2009). Generalized Purcell method and its applications. International Journal of Applied Mathematics, Vol. 22, No. 1, pp. 103-115.

Saaty T. L. (2003). Decision making with the AHP: why is the principal eigenvector necessary? European journal of operation research, Vol. 145, pp. 85-91.

Saaty, T. L. (1998). Ranking by eigenvector versus other methods in the analytic hierarchy process, Appl. Math. Lett. Vol. 11, No. 4, pp.121-125.

Saaty T. L. (1977). A scaling method for priorities in hierarchical structures. Journal of Mathematical Psychology, Vol. 15, pp. 234-281.

Saaty, T. L. (1996). *The analytic hierarchy process*. RWS Publications, Pittsburgh.

Saaty, T. L. and Vargas, L. g. (1984). Comparison of eigenvalue, logarithmic least squares and least squares methods in estimating ratios. Mathematical Modeling, Vol. 5, pp. 309- 324.

Stein, W. E. and Mizzi, P. J. (2007). The harmonic consistency index for the analytic hierarchy process. European Journal of Operational Research, Vol. 177, pp. 488-497.